Problem-Based Mathematics I

Mathematics Department
Shady Side Academy
Pittsburgh, PA
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[Problems originated with the Mathematics Department at Phillips Exeter Academy, NH.]

SHADY SIDE ACADEMY SENIOR SCHOOL
DEPARTMENT OF MATHEMATICS

Problem-Based Mathematics I

MISSION/HISTORY: As part of an on-going curriculum review the Mathematics Department of Shady Side Academy sent two members of the senior school math faculty to visit Phillips Exeter Academy [PEA] in 2008 to observe their classes. After this observation and much reflection, the department decided to adopt this problem-based curriculum. The materials used in the Mathematics I and II courses are taken directly from PEA. We thank the teachers at PEA for the use of their materials.

RATIONALE: The Shady Side Academy Mathematics Department Goals are as follows:

Students will develop the habit of using mathematical reasoning based on logical thinking.
Students will develop adequate skills necessary to solve problems mathematically.
Students will recognize that the structure and order of mathematics can be discovered in the world around us.
Students will recognize the connections of mathematics to other disciplines.
Students will express themselves clearly in mathematical discourse.
Students will be familiar with and proficient in appropriate technology.
Students will achieve their highest mathematical goals.
Students will gain an appreciation for the study of mathematics.

In addition, the teachers in the Department of Mathematics want you to be an articulate student of mathematics. We want you to be able to speak and write mathematics well. We want you to be a fearless problem solver so that you approach problems with curiosity and not trepidation. The Mathematics I classroom is student-centered. The curriculum is problem-based with an integrated design. You will continually learn new material while reviewing prior topics.

EXPECTATIONS: In order for you to be successful in this course, the Mathematics Department has the following suggestions and expectations. First, we expect you to attempt every problem. More than merely writing the problem number, write an equation or draw a picture or write a definition; in other words, indicate in some way that you have thought about and tried the problem. Next, seek help wherever you can find it. We expect you to cooperate with your peers and teachers. The Mathematics Department is a team of teachers striving to help all students reach their potential. You are encouraged to ask any teacher for help if your own is not available. Finally, as stated in the Student Handbook on page 13: “Homework for Forms III and IV normally is limited to 45 minutes of homework per night per subject on days when that class meets.” We expect you to spend 45 minutes on mathematics homework to prepare for each class meeting.

To the Student

Contents: Members of the PEA Mathematics Department have written the material in this book. As you work through it, you will discover that algebra and geometry have been integrated into a mathematical whole. There is no Chapter 5, nor is there a section on tangents to circles. The curriculum is problem-centered, rather than topic-centered. Techniques and theorems will become apparent as you work through the problems, and you will need to keep appropriate notes for your records — there are no boxes containing important theorems. You will begin the course with this binder of problems, graph paper, and a protractor. All of your solutions are to be kept in this binder. It will be periodically collected and will factor into your term grade. There is no index in your binder but the reference section at the end should help you recall the meanings of key words that are defined in the problems (where they usually appear italicized).

Comments on problem-solving: You should approach each problem as an exploration. Reading each question carefully is essential, especially since definitions, highlighted in italics, are routinely inserted into the problem texts. It is important to make accurate diagrams whenever appropriate. Useful strategies to keep in mind are: create an easier problem, guess and check, work backwards, and recall a similar problem. It is important that you work on each problem when assigned, since the questions you may have about a problem will likely motivate class discussion the next day.

Problem-solving requires persistence as much as it requires ingenuity. When you get stuck, or solve a problem incorrectly, back up and start over. Keep in mind that you're probably not the only one who is stuck, and that may even include your teacher. If you have taken the time to think about a problem, you should bring to class a written record of your efforts, not just a blank space in your notebook. The methods that you use to solve a problem, the corrections that you make in your approach, the means by which you test the validity of your solutions, and your ability to communicate ideas are just as important as getting the correct answer. Proper spelling is essential for clear written communication.

About technology: Many of the problems in this book require the use of technology (graphing calculators or computer software) in order to solve them. Moreover, you are encouraged to use technology to explore, and to formulate and test conjectures. Keep the following guidelines in mind: write before you calculate, so that you will have a clear record of what you have done; store intermediate answers in your calculator for later use in your solution; pay attention to the degree of accuracy requested; refer to your calculator's manual when needed; and be prepared to explain your method to your classmates. Also, if you are asked to "graph $y = (2x - 3)/(x + 1)$", for instance, the expectation is that, although you might use your calculator to generate a picture of the curve, you should sketch that picture in your notebook or on the board, with correctly scaled axes.

Shady Side Academy
Introductory Math Guide for Students

Homework

First, we expect you to attempt every problem. More than merely writing the problem number in your notebook, write an equation or draw a picture or write a definition; in other words, indicate in some way that you have thought about and tried the problem. As stated in the Student Handbook on page 13: "Homework for Forms III and IV normally is limited to 45 minutes of homework per night per subject on days when that class meets." We expect you to spend 45 minutes on mathematics homework to prepare for each class meeting.

Going to the Board

It is very important to go to the board to put up homework problems. Usually, every homework problem is put on the board at the beginning of class, presented, and then discussed in class. By doing this, you will develop your written and oral presentation skills.

Plagiarism

You can get help from almost anywhere, but make sure that you cite your help, and that all work shown or turned in is your own, even if someone else showed you how to do it. Never copy work from others. Teachers do occasionally give problems/quizzes/tests to be completed at home. You may not receive help on these assessments, unless instructed to by your teacher; it is imperative that all the work is yours. More information about plagiarism can be found on page vi in your binder.

Math Extra-Help

Getting help is an integral part of staying on top of the math program here at Shady Side Academy. It can be rather frustrating to be lost and stuck on a problem. Teachers, peer tutors, study groups, the internet, your resource book and classmates are all helpful sources.

Teachers and Meetings

The very first place to turn for help should be your teacher. Teachers at SSA are always eager to help you succeed. The Math Department office is located on the 3rd floor of Rowe Hall. Individual meetings can be arranged with teachers during study halls, free periods, or after school. You can always ask or email any teacher in the department for help. Getting help from your teacher is the first and most reliable source to turn to for extra help.

SSA Student Quotes

"This program really helped me learn and understand the concepts of Algebra II. It helped us as a group because we covered materials together. We all said our own ideas and accepted when they were wrong. It gave each individual confidence in their understanding of the material. Some days we did not check over every problem like I would have liked to, but this allowed me to be a frequent visitor in the math office. It was a different approach that ran very smoothly in this class."
--Betsy Vuchinich, '12

"I came into the year unsure of what to think about this approach to mathematics. I had criticism and positive words about the packet, and I didn't know what to expect. Though sometimes I was confused, in the end, everything worked out."
--Erin Gorse, '12

"I loved Mathematics II this year because the curriculum was completely different from anything I've previously encountered in math. We didn't use a book for the majority of the year, instead we focused on more complicated word problems and worked together in small groups to solve these difficult problems. This forced us to think through the problems and think about "Why?" more so than "How?" and this was a much different look for a math class. Working with your peers in a setting that promoted group work was refreshing, and I enjoyed it very much. I hope the Math Department continues to use this curriculum."
--Jonathan Laufe, '12

"The curriculum for Mathematics II under Ms. Whitney was not easy, but the use of a packet full of word problems that challenged our minds to apply concepts previously learned really expanded our knowledge much easier than traditional out of the book teaching. The packet introduced to me a new way of learning that I was not familiar with, but even if students are struggling to understand concepts of problems then teachers make themselves available to work with you very often. You will not get by easily in this course by daydreaming, but this hands on experience in the classroom of interacting with your classmates and teacher will show how much easier learning is because you stress previously learned material and open windows to other, more complex problems."
--Christopher Bush, '13

"The curriculum definitely took some getting used to but once you figure it out, it has a balance of being challenging and easy at the same time."
--Elijah Williams, '13

"I thought the word problems were unnecessary at first, then I found my mind starting to expand."
--Guy Philips, '13

"Math I is a great way to learn and if I had to describe it in one word it would be 'Utopian': The classroom environment motivates me to do better and it teaches you to either accept your method or to abandon it for a better one. The fact that the teachers act as moderators in the classroom makes it a better way to learn because it really gets you to think. I loved Math One and look forward to doing more Exeter problems in Math 2."
--Adam D'Angelo, '14

"I think Math II will teach you a lot about not only math. Even coming out of a year of "the binder", Math I, I found that I actually learned math a lot differently this year than last; this required me to be malleable with how I approached things and studied. "Doing math differently", for lack of a better word, was something that confronted me this year, and it challenged things I already did in a healthy way: communicating via email, asking questions, organizing things differently, learning how to take notes in new ways, and meeting new people. A beneficial thing I recommend is visiting the math office even once a week after school to meet with your teacher (or any teacher) and go over homework, do practice problems, and chat. There are a lot of cool people in that office, and the more you communicate with them, the more you will enjoy and feel confident with math. Overall, you will probably grow a lot as a person throughout taking Math II, and learn many valuable life skills; be excited for that."
--Felicia Reuter '17

This class was very difficult in the beginning, as I had never had a class taught in this format before. However, as the year progressed, I learned to use my teacher and answer key as vital tools and began to understand the format much better. I then realized that from this format I, to quote Mrs. Ament, "struggled a little and learned a lot."
--Jolie Rosenberg, '18

SHADY SIDE ACADEMY SENIOR SCHOOL
DEPARTMENT OF MATHEMATICS

Policy on Plagiarism and Cheating

At the beginning of each course, each teacher in the Mathematics Department will explain to the class what is expected with regard to the daily completion of homework, the taking of in-class tests, make-up tests, and take-home tests. Students will be told whether or not they may use books and/or other people when completing in-class or out-of class assignments/tests. The consequences listed below will take effect if a teacher suspects that a student is in violation of the instructions given for a particular assignment or test.

PLAGIARISM

Plagiarism is the act of representing something as one's own without crediting the source. This may be manifest in the mathematics classroom in the form of copying assignments, fabricating data, asking for or giving answers on a test, and using a "cheat sheet" on an exam.

CHEATING

If, during an in-class test, the teacher in that room considers that a student has violated the teacher's instructions for the test, the teacher will instruct the student that there is a suspicion of cheating and the teacher will initiate the consequences below. If a student is taking a make-up test out of class and any teacher considers that the student is, or has been, cheating the teacher will bring the issue to the notice of the Department Chair, and initiate the consequences below. Sharing the content of a particular test with an individual who has not taken the test is considered by the department to be cheating by both parties.

CONSEQUENCES

When a teacher suspects plagiarism or academic dishonesty, the teacher and Department Chair will speak with the student. The Department Chair, in conjunction with the Dean of Student Life, will determine whether plagiarism or academic dishonesty has occurred. If plagiarism or academic dishonesty is determined, the Dean of Student Life and the Department Chair make the decision about the appropriate response to the situation, which will likely include referral to the Discipline Committee. The Department Chair will contact the family to discuss the infraction and consequence. If a Discipline Committee referral is made, the Dean of Student Life will follow up with the family as well.

In any case of plagiarism or cheating, the student concerned will likely receive a failing grade for that piece of work, as well as any other appropriate steps deemed necessary by the Department Chair and the Dean of Academic Life.

Math I Guided Notes

The following pages are a place for you to organize the concepts, topics, formulas and ideas you learn this year. You can use them in any way you wish. It is suggested that when you come upon an important finding or result in class or on your own, that you write it in these notes so that it is easily accessible when it comes time to study for an exam or review material. These notes are not a substitute for taking notes in other ways, and the Mathematics Department encourages you to use a notebook to have a record of your work, corrections and any notes you get in class. We hope this is useful to you, and we welcome any feedback.

--SSA Senior School Math Department

Area/Perimeter/Volume:

Sketch	Area	Perimeter
Square with sides *s*		
Rectangle with sides *w* and *l*		
Triangle with base *b* and height *h*		

Two Characteristics of Direct Variation are:

1. ____

2. ____

Lines:

. The *slope-intercept form* of a line is:

where b = __________________ and m = _________________

. The *point-slope form* of a line is:

where h = __________________, k = ___________________, and m = _________________

. The *general form* of a line is:

Linear Systems of Equations

The two methods for solving a system of linear equations are:

1. ______________________________ and
2. ______________________________

Absolute Value

Absolute value has to do with __________________________.

Notes on solving absolute value equations:

Notes on solving absolute value inequalities:

Quadratic Equations:

The three ways we have of solving quadratics when they are set equal to zero are:

- ______________________________________
- ______________________________________
- ______________________________________

Radicals

"Like radicals" are radicals with the same:

1. ____________________

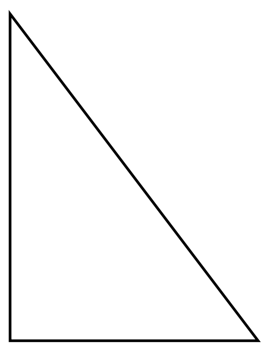
2. ____________________

"Rationalizing a denominator" means

The Pythagorean Theorem states: In a right triangle, ________________________________

____.

Label the right triangle below to match your definition above.



Triangles: Draw an example of an altitude, a median and an angle bisector. Are these ever the same?

Triangles
Special right triangles:

Type	Ratio/Notes/Drawing
45-45-90	
30-60-90	

Geometry - Quadrilaterals

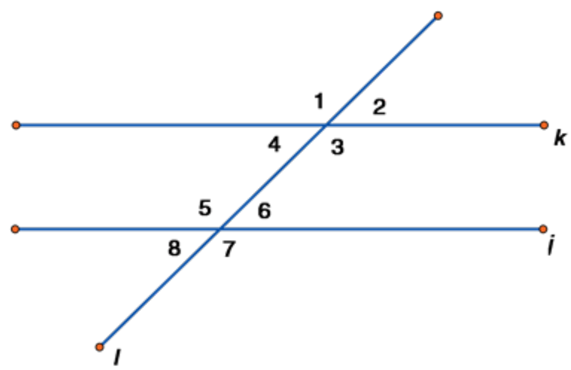
	Parallelogram	Rectangle	Square	Rhombus	Kite
Sketch					

Angles of Polygons

Interior angles of a polygon:

Exterior angles of a polygon:

Parallel Lines



Proofs

List all of the triangle congruence postulates and draw an example of each.

Theorems

The main geometric theorems we have learned this year and a description (in my own words) and/or sketch of the theorem are:

Theorem	**Notes**

Distance and Equidistance

- You find the distance between two points by:

- You find a point equidistant from two points by:

- You find the set of all points equidistant from two points by:

Distance, Work, Rate, Time

Sample chart: What kinds of things do you put in each box and how do they relate to each other?

Exponents

Exponent rule	Explanation/Proof/Reason	Exponent rule	Explanation/Proof/Reason

Problem-Based Mathematics I

1. (I.1.1) Light travels at 186282 miles per second, and the Sun is about 93 million miles from the Earth. How many minutes does it take light to reach the Earth from the Sun?

2. (I.1.4) Many major-league baseball pitchers can throw the ball at 90 miles per hour. At that speed, how long does it take a pitch to travel from the pitcher's mound to home plate, a distance of 60 feet 6 inches? Give your answer to the nearest hundredth of a second. There are 5280 feet in a mile.

3. (I.1.7) Your class sponsors a benefit concert and prices the tickets at $8 each. Kim sells 12 tickets, Andy 16, Pat 13, and Morgan 17. Compute the total revenue brought in by these four persons. Notice that there are two ways to do the calculation.

4. (I.1.8) Kelly telephoned Brook about a homework problem. Kelly said, "Four plus three times two is 14, isn't it?" Brook replied, "No, it's 10." Did someone make a mistake? Can you explain where these two answers came from?

5. (I.1.9) It is customary in algebra to omit multiplication symbols whenever possible. For example, $11x$ means the same thing as $11 \cdot x$. Which of the following can be condensed by leaving out a multiplication symbol? (a) $4 \cdot \frac{1}{3}$ (b) $1.08 \cdot p$ (c) $24 \cdot 52$ (d) $5 \cdot (2 + x)$

6. (I.1.10) Wes bought some school supplies at an outlet store in Maine, a state that has a 6.5% sales tax. Including the sales tax, how much did Wes pay for two blazers priced at $49.95 each and 3 pairs of pants priced at $17.50 each?

7. (I.1.11) (Continuation) A familiar feature of arithmetic is that *multiplication distributes over addition*. Written in algebraic code, this property looks like $a(b+c) = ab + ac$. Because of this property, there are two equivalent methods that can be used to compute the answer in the previous problem. Explain, using words and complete sentences.

8. (I.2.1) Woolworth's had a going-out-of-business sale. The price of a telephone before the sale was $39.98. What was the price of the telephone after a 30% discount? If the sale price of the same telephone had been $23.99, what would the (percentage) discount have been?

9. (I.2.2) Pick any number. Add 4 to it and then double your answer. Now subtract 6 from that result and divide your new answer by 2. Write down your answer. Repeat these steps with another number. Continue with a few more numbers, comparing your final answer with your original number. Is there a pattern to your answers?

10. (I.2.3) Using the four *integers* 2, 3, 6 and 8 once each—in any order—and three arithmetic operations selected from among addition, subtraction, multiplication, and division, write expressions whose values are the target numbers given below. You will probably need to use parentheses. For example, to hit the target 90, you could write $90 = (3 + 6) \cdot (8 + 2)$.
(a) 3 (b) 24 (c) 36 (d) 30

1. (I.2.5) On a recent episode of *Who Wants to Be a Billionaire*, a contestant was asked to arrange the following five numbers in increasing order. You try it, too.

(a) 2/3 (b) 0.6666 (c) 3/5 (d) 0.666 (e) 0.67

2. (I.2.6) The area of a circle whose radius is *r* is given by the expression πr^2. Find the area of each of the following circles to the nearest tenth of a square unit of measure:

(a) a circle whose radius is 15 cm (b) a circle whose radius is 0.3 miles

3. (I.2.7) Choose any number. Double it. Subtract six and add the original number. Now divide by three. Repeat this process with other numbers, until a pattern develops. By using a variable such as *x* in place of your number, show that the pattern does not depend on which number you choose initially.

4. (I.2.9) Given the information $w = 4$ inches and $h = 7$ inches, find two ways to *evaluate* $2w + 2h$. What is the geometric significance of this calculation?

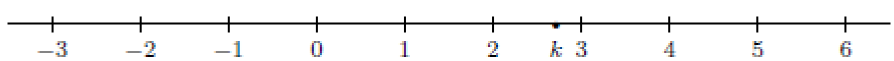
5. (I.2.10) Simplify $x+2+x+2+x+2+x+2+x+2+x+2+x+2+x+2+x+2$.

6. (I.3.1) What is the value of 3 + (−3)? What is the value of (−10.4) + 10.4? These pairs of numbers are called *opposites*. What is the sum of a number and its opposite? Does every number have an opposite? State the opposite of:

(a) −2.341 (b) 1/3 (c) x (d) $x+2$ (e) $x-2$

7. (I.3.2) As shown on the number line below, *k* represents an unknown number between 2 and 3. Plot each of the following, extending the line if necessary:

(a) $k+3$ (b) $k-2$ (c) $-k$ (d) $6-k$



8. (I.3.3) You are already familiar with operations involving positive numbers, but much mathematical work deals with negative numbers. Common uses include temperatures, money, and games. It is important to understand how these numbers behave in arithmetic calculations. First, consider addition and subtraction. For each of the following, show how the answer can be visualized using a number-line diagram:

(a) The air temperature at 2 pm was12°. What was the air temperature at 8 pm, if it had dropped 15° by then?

(b) Telescope Peak in the Panamint Mountain Range, which borders Death Valley, is 11045 feet above sea level. At its lowest point, Death Valley is 282 feet below sea level. What is the vertical distance from the bottom of Death Valley to the top of Telescope Peak?

(c) In a recent game, I had a score of 3. I then proceeded to lose 5 points and 7 points on my next two turns. On the turn after that, however, I gained 8 points. What was my score at this moment in the game?

1. (I.3.5) Locate the following numbers relative to each other on a number line:
   (a) 3.03 (b) 3.303 (c) 3.033 (d) 3.333 (e) 3.33

2. (I.3.6) The area of the surface of a sphere is described by the formula $S = 4\pi r^2$, where r is the radius of the sphere. The Earth has a radius of 3960 miles and dry land forms approximately 29.2% of the Earth's surface. What is the area of the dry land on Earth? What is the surface area of the Earth's water?

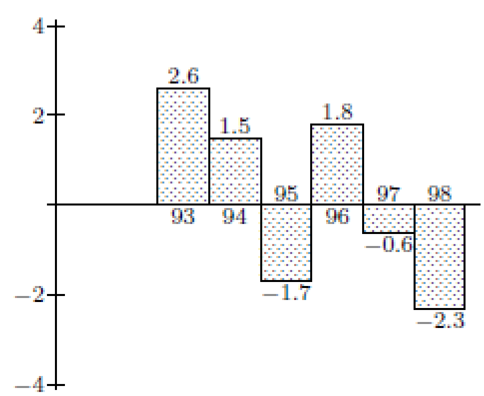
3. (I.3.7) Mark a random number x between 1 and 2 (at a spot that only you will think of) on a number line. Plot the *opposite* of each of the following:
   (a) x (b) $x+5$ (c) $x-4$ (d) $6-x$

4. (I.4.1) At 186282 miles per second, how far does light travel in a year? Give your answer in miles, but use *scientific notation*, which expresses a large number like 93400000 as 9.34×10^7 (which might appear on your calculator as 9.34 E7 instead). The answer to this question is called a *light year* by astronomers, who use it to measure huge distances. Other than the Sun, the star nearest the Earth is Proxima Centauri, a mere 4.2 light years away.

5. (I.4.2) Before you are able to take a bite of your new dark chocolate bar (yum), a friend comes along and takes 1/4 of the bar. Then another friend comes along and you give this person 1/3 of what you have left. Make a diagram that shows the part of the bar left for you to eat.

6. (I.4.3) Later you have another dark chocolate bar. This time, after you give away 1/3 of the bar, a friend breaks off 3/4 of the remaining piece. What part of the original chocolate bar do you have left? Answer this question by drawing a diagram.

7. (I.4.4) Profits and losses for the Whirligig Sports Equipment Company for the six years indicated are graphed on the chart at the right. The vertical scale is in millions of dollars. What was the change in profit and losses from:
(a) 1993 to 1994?
(b) 1994 to 1995?
(c) 1997 to 1998?
For the six years graphed, did the company make an overall profit or sustain an overall loss? How much was the net profit or net loss?



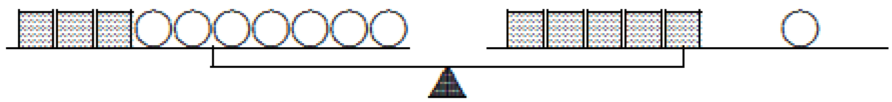
8. (I.4.5) The temperature outside is dropping 3° per hour. If the temperature at noon was 0°, what was the temperature at 1 pm? at 2 pm? at 3 pm? at 6 pm? What was the temperature t hours after noon?

9. (I.4.8) The product of two negative numbers is always a positive number. How would you explain this rule to a classmate who does not understand why the product of two negative numbers must be positive?

1. (I.4.9) Let k represent some unknown number between −4 and −5. Locate between two *consecutive integers* each of the following:

(a) $-k$ (b) $-k+5$ (c) $\frac{k}{2}+2$ d) $\frac{k+2}{2}$

2. (I.5.1) Use the *balance diagram* below to find how many marbles it takes to balance one cube.



3. (I.5.2) (Continuation) Using c to stand for the weight of one cube and m for the weight of one marble, write an equation that *models* the picture in the previous problem. Use this equation to find how many marbles it takes to balance one cube.

4. (I.5.3) The division problem $12 \div \frac{3}{4}$ is equivalent to the multiplication problem $12 \cdot \frac{4}{3}$. Write each of the following division problems as equivalent multiplication problems:

(a) $20 \div 5$ (b) $20 \div \frac{1}{5}$ (c) $20 \div \frac{2}{5}$ (d) $a \div \frac{b}{c}$ (e) $\frac{b}{c} \div a$

5. (I.5.4) What is the value of $\frac{2}{3} \cdot \frac{3}{2}$? What is the value of $4 \cdot \frac{1}{4}$? These pairs of numbers are called *reciprocals*. What is the product of a number and its reciprocal? Does every number have a reciprocal? State the reciprocal of the following:

(a) $\frac{5}{3}$ (b) $-\frac{1}{2}$ (c) 2000 (d) $\frac{a}{b}$ (e) x

6. (I.5.7) Jess takes a board that is 50 inches long and cuts it into two pieces, one of which is 16 inches longer than the other. How long is each piece?

7. (I.5.8) Consider the sequence of numbers 2, 5, 8, 11, 14,..., in which each number is three more than its predecessor.

(a) Find the next three numbers in the sequence.

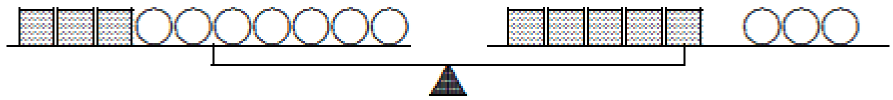
(b) Find the 100th number in the sequence.

(c) Using the variable n to represent the position of a number in the sequence, write an expression that allows you to calculate the n^{th} number. The 200th number in the sequence is 599. Verify that your expression works by evaluating it with n equal to 200.

8. (I.5.9) A group of ten persons were planning to contribute equal amounts of money to buy some pizza. After the pizza was ordered, one person left. Each of the other nine persons had to pay 60 cents extra as a result. How much was the total bill?

Problem-Based Mathematics I

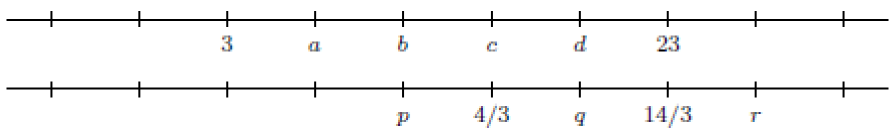
1. (I.6.1) In the balance diagram below, find the number of marbles that balance one cube.



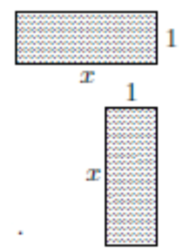
2. (I.6.2) For each of the following, find the value of x that makes the equation true. The usual way of wording this instruction is *solve for* x:

(a) $2x = 12$ (b) $-3x = 12$ (c) $ax = b$

3. (I.6.3) On each of the following *number lines*, all of the labeled points are evenly spaced. Find *coordinates* for the seven points designated by the letters.



4. (I.6.5) A rectangle whose length is x and whose width is 1 is called an x-*block*. The figure below shows two of them.



(a) What is the area of an x-block?
(b)What is the combined area of two x-blocks?
(c) Show that there are two different ways to combine two x-blocks to form a rectangle whose area is $2x$.
(d) Draw two different rectangular diagrams to show that $x + 2x = 3x$.

5. (I.6.6) Use the distributive property to explain why $3x + 2x$ can be simplified to $5x$.

6. (I.6.7) (Continuation) Write each of the following as a product of x and another quantity:

(a) $16x + 7x$ (b) $12x - 6x$ (c) $ax + bx$ (d) $px - qx$

7. (I.6.8) Solve each of the following equations for x:

(a) $16x + 7x = 46$ (b) $12x - 6x = 3$ (c) $ax + bx = 10$ (d) $px - qx = r$

8. (I.6.9) Draw a balance diagram that is modeled by the *equation* $c + m + c + 7m + c = 2c + 2m + 3c$. How many marbles will one cube balance?

9. (I.7.1) In baseball statistics, a player's slugging ratio is defined to be $\frac{s + 2d + 3t + 4h}{b}$ where s is the number of singles, d the number of doubles, t the number of triples and h the number of home runs obtained in b times at bat. Dana came to bat 75 times during the season, and hit 12 singles, 4 doubles, 2 triples, and 8 home runs. What is Dana's slugging ratio, rounded to three decimal places?

1. (I.7.2) Make a dot somewhere between 0 and 0.5 on a number line, and label it k. Place each of the following on the same number line as accurately as you can.
(a) $-k$ (b) $2k$ (c) k^2 (d) $k-2$ (e) $\sqrt{k}$

2. (I.7.3) Simplify each of the following:
(a) the sum of $6x+2$ and $-8x+5$;
(b) the result of subtracting $5x-17$ from $8x+12$;
(c) the product of $7x$ and $4x-9$.

3. (I.7.4) Solve $\frac{2}{3}(3x+14)=7x+6$ by first multiplying both sides of the equation by 3 then applying the distributive property to the left side of the equation.

4. (I.7.5) Because $12x^2+5x^2$ is equivalent to $17x^2$, the expressions $12x^2$ and $5x^2$ are called like terms. Explain. Why are $12x^2$ and $5x$ called *unlike terms*? Are $3ab$ and $11ab$ *like terms*? Explain. Are $12x^2$ and $5y^2$ like terms? Explain. Are $12x^2$ and $12x$ like terms? Explain.

5. (I.7.6) In each of the following, use appropriate algebraic operations to remove the parentheses and combine like terms. Leave your answers in a simple form.
(a) $x(x+5)+2(x+5)$ (b) $2x(5x-2)+3(5x-2)$ (c) $5m(3m-2n)+4n(3m-2n)$

6. (I.7.8) Jess has just finished telling Lee about learning a wonderful new algebra trick: $3+5x$ can be simplified very neatly to just $8x$, because $a+bx$ is the same as $(a+b)x$. Now Lee has to break some bad news to Jess. What is it?

7. (I.7.9) Find whole numbers m and n that fit the equation $3m+6n=87$. Is it possible to find whole numbers m and n that fit the equation $3m+6n=95$? If so, find an example. If not, explain why not.

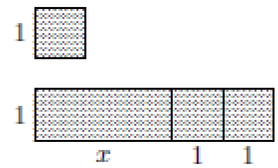
8. (I.7.12) Solve $9x+2=\frac{3}{4}(2x+11)$.

9. (I.8.2) Simplify the expression $k-2(k-(2-k))-2$ by writing it without using parentheses.

10. (I.8.4) Last year the price of an iPod touch was \$150.
(a) This year the price increased to \$180. By what percent did the price increase?
(b) If the price next year were 5% more than this year's price, what would that price be?
(c) If the price dropped 5% the year after that, show that the price would not return to \$180. Explain the apparent paradox.

Problem-Based Mathematics I

1. (I.8.6) Which number is closer to zero, $-\frac{4}{5}$ or $\frac{5}{4}$?



2. (I.8.8) The figure shows some more algebra blocks. The 1-by-1 square is called a *unit block* or a *1-block*. Below the 1-block is a representation of $x+2$, formed from an x-block and two 1-blocks. Draw a diagram using the appropriate number of x-blocks and 1-blocks to illustrate the distributive property $3(x+2)=3x+6$.

3. (I.8.9) Often it is necessary to rearrange an equation so that one variable is expressed in terms of others. For example, the equation $D=3t$ expresses D in terms of t. To express t in terms of D, divide both sides of this equation by 3 to obtain $\frac{D}{3}=t$.
(a) Solve the equation $C=2\pi r$ for r in terms of C.
(b) Solve the equation $p=2w+2h$ for w in terms of p and h.
(c) Solve the equation $3x-2y=6$ for y in terms of x.

4. (I.8.10) On a number line, what number is halfway between (a) -4 and 11? (b) m and n?

5. (I.9.1) Temperature is measured in both Celsius and Fahrenheit degrees. These two systems are of course related: the Fahrenheit temperature is obtained by adding 32 to 9/5 of the Celsius temperature. In the following questions, let C represent the Celsius temperature and let F represent the Fahrenheit temperature.
(a) Write an equation that expresses F in terms of C.
(b) Use this equation to find the value of F that corresponds to $C = 20$.
(c) On the Celsius scale, water freezes at 0° and boils at 100°. Use your formula to find the corresponding temperatures on the Fahrenheit scale. Do you recognize your answers?

6. (I.9.3) The Millers must make a 70-mile Thanksgiving trip to visit their grandparents. Pat Miller believes in driving at a steady rate of 50 miles per hour.
(a) With Pat in the driver's seat, how much time will the trip take?
(b) How many miles will the Millers travel in 18 minutes?
(c) Write an expression for the number of miles they will cover in t minutes of driving.
(d) After t minutes of driving, how many miles remain to be covered?

7. (I.9.5) Solve for x: (a) $3x-4=11$ (b) $-2x+5=-1$ (c) $ax+b=c$

8. (I.10.2) Percent practice: (a) 25% of 200 is what number? (b) 200 is 25% of what number? (c) Express 2/25 as a decimal; as a percent. (d) Express 24% as a decimal; as a fraction.

9. (I.8.3) Coffee beans lose 12.5% of their weight during roasting. In order to obtain 252 kg of roasted coffee beans, how many kg of unroasted beans must be used?

1. (I.9.4) The length of a certain rectangle exceeds its width by exactly 8 cm, and the perimeter of the rectangle is 66 cm. What is the width of the rectangle? Although you may be able to solve this problem using a method of your own, try the following approach, which starts by guessing the width of the rectangle. Study the first row of the table below, which is based on a 10-cm guess for the width. Then make your own guess and use it to fill in the next row of the table. If you have not guessed the correct width, use another row of the table and try again.

guess	*length*	*perimeter*	*target*	*check?*
10	10 + 8 = 18	2(10) + 2(18) = 56	66	no
w				

Now use the experience gained by filling in the table to write an equation for the problem: Write w in the *guess* column, fill in the length and perimeter entries in terms of w, and set your expression for the perimeter equal to the target perimeter. Solve the resulting equation. This approach to creating equations is called the *guess-and-check* method.

2. (I.10.1) *Number-line graphs*. Observe the following conventions, which may already be familiar:

- To indicate an interval on the number line, thicken that part of the number line.
- To indicate that an endpoint of an interval is included, place a solid dot on the number.
- To indicate that an endpoint is not included, place an open circle on the number.

For example, the diagram illustrates those numbers that are greater than −2 and less than or equal to 3.

Draw a number line for each of the following and indicate the numbers described:
(a) All numbers that are exactly two units from 5.
(b) All numbers that are more than two units from 5.
(c) All numbers that are greater than −1 and less than or equal to 7.
(d) All numbers that are less than four units from zero.

3. (I.10.4) Ryan earns x dollars every seven days. Write an expression for how much Ryan earns in one day. Ryan's spouse Lee is paid twice as much as Ryan. Write an expression for how much Lee earns in one day. Write an expression for their combined daily earnings.

4. (I.10.5) Solve for x: (a) $2(x-3)=4$ (b) $-3(2x+1)=5$ (c) $a(bx+c)=d$

5. (I.11.1) Atiba walked to a friend's house, m miles away, at an average rate of 4 mph. The m-mile walk home was at only 3 mph, however. Express as a fraction
(a) the time Atiba spent walking home; (b) the total time Atiba spent walking.

1. (I.10.6) Day student Avery just bought 10 gallons of gasoline, the amount of fuel used for the last 355 miles of driving. Being a curious sort, Avery wondered how much fuel had been used in city driving (which takes one gallon for every 25 miles) and how much had been used in freeway driving (which takes one gallon for each 40 miles). Avery started by guessing 6 gallons for the city driving, then completed the first row of the guess-and-check table below. Notice the failed check. Make your own guess and use it to fill in the next row of the table.

city g	*freeway g*	*city mi*	*freeway mi*	*total mi*	*target*	*check*
6	10 – 6 = 4	6(25) = 150	4(40) = 160	150 + 160 = 310	355	no
c						

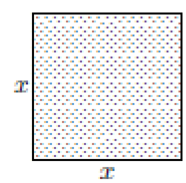
Now write *c* in the city-gallon column, fill in the remaining entries in terms of *c*, and set your expression for the total mileage equal to the target mileage. Solve the resulting equation.

2. (I.10.7) On a number line, graph all numbers that are closer to 5 than they are to 8.

3. (I.11.4) Solve for *x*: (a) $2(x-1)=3(x+2)$ (b) $-4(2x-2)=3(x+1)$

4. (I.11.7) Write an expression that represents the number that
(a) is 7 more than x ; (b) is 7 less than x ; (c) is x more than 7;
(d) exceeds x by 7; (e) is x less than 7; (f) exceeds 7 by x .

5. (I.11.8) The x^2-block, shown at right, is another member of the algebra-block family. Draw an algebra-block diagram that shows that $x(x+2)=x^2+2x$.



6. (I.11.9) There are 396 persons in a theater. If the *ratio* of women to men is 2:3, and the ratio of men to children is 1:2, how many men are in the theater?

7. (I.11.10) On a number line, graph a number that is twice as far from 5 as it is from 8. How many such numbers are there?

8. (I.12.1) Intervals on a number line are often described using the symbols < ("less than"), > ("greater than"), ≤ ("less than or equal to"), and ≥ ("greater than or equal to"). As you graph the following inequalities, remember the *endpoint convention* regarding the use of the dot • and the circle ◦ for included and excluded endpoints, respectively:
(a) $x < 5$ (b) $x \ge -6$ (c) $-12 \ge x$ (d) $4 < x < 8$ (e) $x < -3$ or $7 \le x$

9. (I.12.2) Solve the equation $A = P + Prt$ for *r*. Solve the equation $A = P + Prt$ for *P*.

10. (I.12.3) Using a number line, describe the location of $\frac{x+y}{2}$ in relation to the locations of *x* and *y*. Is your answer affected by knowing whether *x* and *y* are positive or not?

Problem-Based Mathematics I

1. (I.12.5) Evaluate the formula $36y + 12f + i$ when $y = 2.5$, $f = 2$, and $i = 5$. Find an interpretation for this formula.

2. (I.12.6) The indicator on the oil tank in my home indicated that the tank was one-eighth full. After a truck delivered 2400 liters of oil, the indicator showed that the tank was half full. What is the capacity of the oil tank, in liters?

3. (I.12.8) Graph on a number line the intervals described below:
(a) All numbers that are greater than 1 or less than −3.
(b) All numbers that are greater than −5 and less than or equal to 4.
(c) All numbers whose squares are greater than or equal to 1.

4. (I.12.9) Use mathematical notation to represent the intervals described below.
(a) All numbers that are greater than 1 or less than −3.
(b) All numbers that are greater than −5 and less than or equal to 4.
(c) All numbers whose squares are greater than or equal to 1.

5. (I.13.1) Randy and Sandy have a total of 20 books between them. After Sandy loses three by leaving them on the bus, and some birthday gifts double Randy's collection, their total increases to 30 books. How many books did each have before these changes?

6. (I.13.2) Combine the following fractions into a single fraction. Express each of your answers in lowest terms.
(a) $\frac{27}{5} + \frac{3y}{4}$ (b) $\frac{4m}{5} - \frac{2}{3}$ (c) $2 + \frac{x}{3}$ (d) $\frac{x}{2} + \frac{2x}{3} - \frac{3x}{4}$

7. (I.13.3) Solve the following for x:
(a) $4 - (x+3) = 8 - 5(2x-3)$ (b) $x - 2(3-x) = 2x + 3(1-x)$

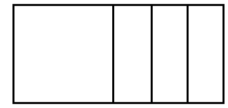
8. (I.13.6) (a) I am thinking of 6 consecutive positive integers, the smallest of which is 7. What is the largest of these integers?

(b) I am thinking of 6 consecutive positive integers, the smallest of which is m. What formula, in terms of m, represents the largest of these integers?

(c) I am thinking of n consecutive positive integers, the smallest of which is m. What formula, in terms of m and n, represents the largest of these integers?

9. (I.15.2) Graph on a number line the intervals corresponding to these two signs on the highway.
(a) The maximum speed is 65 mph and the minimum speed is 45 mph.
(b) The maximum speed is 55 mph.

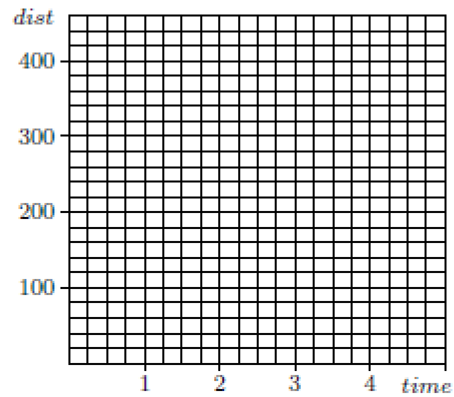
1. (I.15.3) Label the figure at right so that it provides a geometric representation of $x(x+3)$. Notice that this question is about *area*.



2. (I.15.4) It is sometimes necessary to write fractions with variables in the denominator. Without using your calculator, rewrite each of the following as a single fraction. This is called *combining over a common denominator*.

(a) $\frac{3}{a}+\frac{7}{a}$ (b) $\frac{3}{a}-\frac{7}{2a}$ (c) $\frac{3}{a}+\frac{7}{b}$ (d) $3+\frac{7}{b}$

3. (I.16.2) Crossing a long stretch of the Canadian plains, passenger trains maintain a steady speed of 80 mph. At that speed, what distance is covered in half an hour? How much time is needed to cover 200 miles? Fill in the missing entries in the table below, and plot points on the grid at right.



time	0	1/2		1	2		3		4	t
distance			60			200		300		

4. (I.15.7) Which of the following eight expressions does not belong in the list?

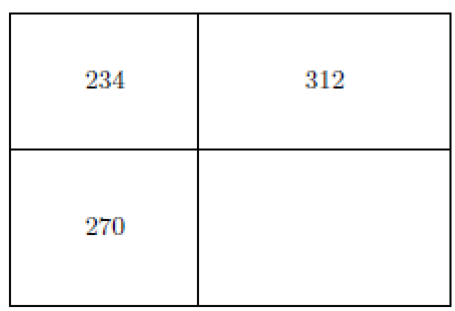
$a-b+c$ $c-b+a$ $c-(b-a)$ $-b+a+c$

$c+a-b$ $a-(b-c)$ $b-(c-a)$ $a+c-b$

5. (I.15.8) Last week, Chris bought a DVD for \$10.80 while the store was having a 25%-off sale. The sale is now over. How much would the same DVD cost today?

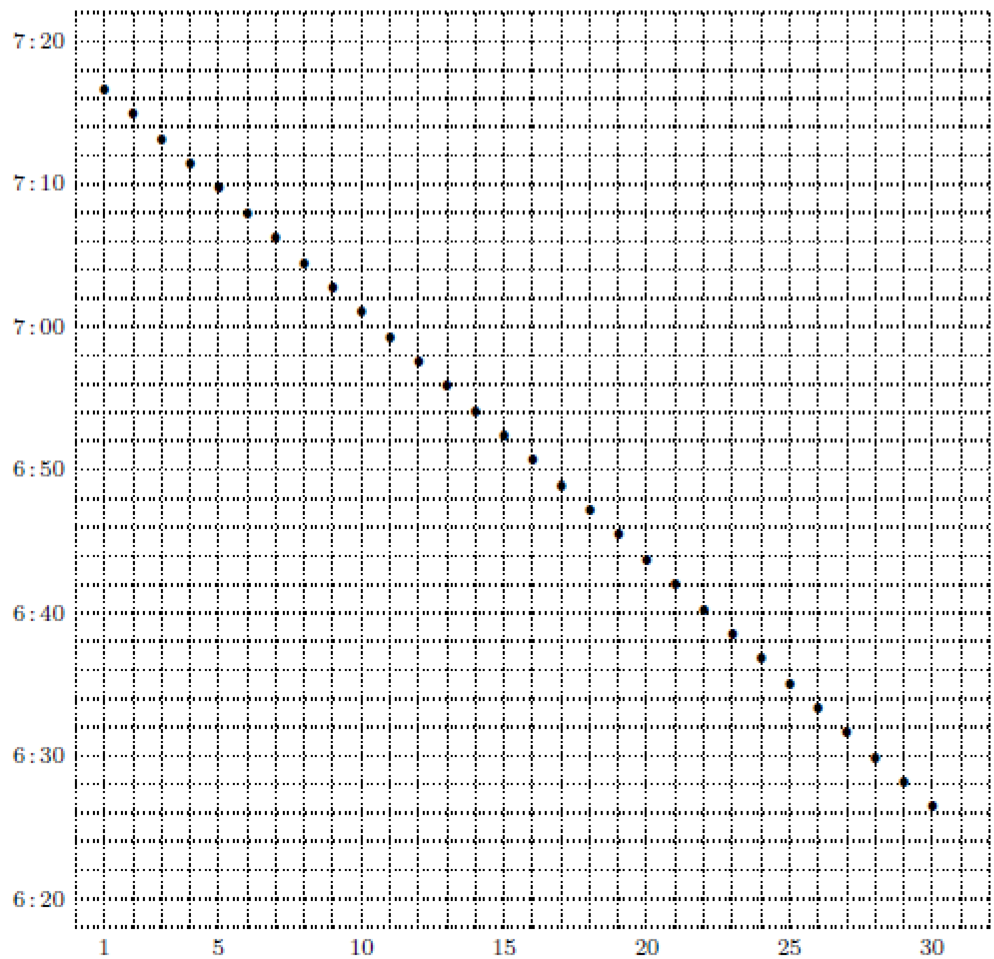
6. (I.16.1) The statement "x is between 13 and 23" defines an interval using two simultaneous inequalities: $13 < x$ and $x < 23$. The statement "x is not between 13 and 23" also uses two inequalities, but they are non-simultaneous: $x \le 13$ or $23 \le x$. Graph these two examples on a number line. Notice that there is a compact form $13 < x < 23$ for only one of them.

7. (I.16.7) The rectangle shown at right has been broken into four smaller rectangles. The areas of three of the smaller rectangles are shown in the diagram. Find the area of the fourth one.



8. (I.16.8) Tickets to a school play cost \$1.50 if bought in advance, and \$2.00 if bought at the door. By selling all 200 of their tickets, the players brought in \$360. How many of the tickets were sold in advance?

9. (I.16.5) Atiba walked to a friend's house, m miles away, at an average rate of 4 mph. The m-mile walk home was only at 3 mph. Atiba spent 2 hours walking in all. Find the value of m. You may consider drawing a diagram to help.



The graph displays the time of sunset at Exeter, New Hampshire during September. Some questions:

1. (I.14.1) At what time did the sun set on the 5^{th} of September? on the 30^{th} of September?

2. (I.14.2) On what day does the sun set at 6:54? at 7:08? at 6:30?

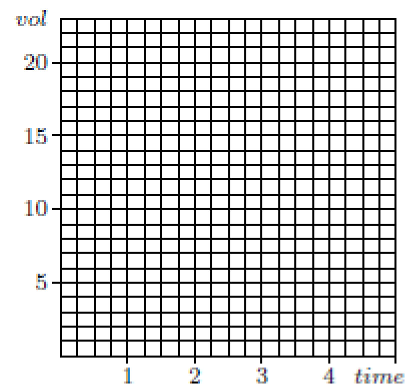
3. (I.14.3) Guess the time of sunset on the 1^{st} of October and on the 31^{st} of August.

4. (I.14.4) What is the average daily change of sunset time during the month of September?

5. (I.14.5) The dots in the graph form a pattern. Jess thinks that this pattern continues into October, November, and December. What do you think? Make a graph that shows how the time of sunset at Exeter changes during an entire year. A good source for such data is the U.S. Naval Observatory site http://aa.usno.navy.mil.

6. (I.14.6) What happens on the Autumnal Equinox, which is the 22^{nd} of September? Guess what time the sun rises on this day.

1. (I.15.5) It takes one minute to fill a four-gallon container at a local spring. How long does it take to fill a six-gallon container? Fill in the missing entries in the table below, and plot points on the grid at right.



time	1			2		3		4		5
volume	4	5	6		11		14		19	

Notice that it makes sense to connect the dots you plotted (thereby forming a *continuous* pattern). Is the same true of the sunset-time graph you looked at recently? Explain.

2. (I.16.3) The problems about the local spring and the Canadian plains contain relationships that are called *direct variations*. In your own words, describe what it means for one quantity to *vary directly* with another. Which of the following describe direct variations?

(a) The gallons of water in a tub and the number of minutes since the tap was opened.
(b) The height of a ball and the number of seconds since it was thrown.
(c) The length of a side of a square and the perimeter of the square.
(d) The length of a side of a square and the area of the square.

3. (I.16.4) (Continuation) Sketch graphs for each of the situations described above. Be sure to include meaningful descriptions and scales for each axis.

4. (I.16.6) The sides of a rectangle in the coordinate plane are parallel to the axes. Two of the vertices of the rectangle are $(3, -2)$ and $(-4, -7)$. Find coordinates for the other two vertices. Find the area of the rectangle.

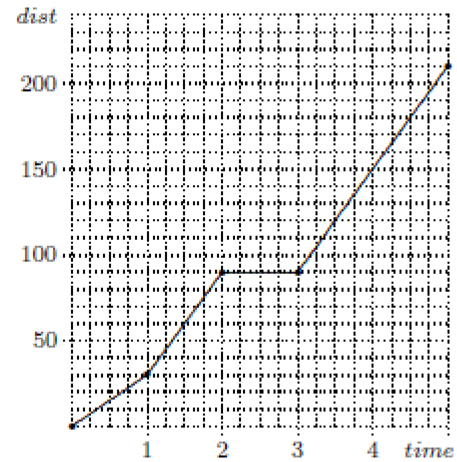
5. (I.21.6) The distance from SSA to downtown is 10 miles. If you bike from SSA to downtown in 40 minutes, what is your average speed for the trip? What does this mean?

6. (I.21.7) (Continuation) On the return trip from town, you pedal hard for the first ten minutes and cover 4 miles. Tired, you slow down and cover the last 6 miles in 36 minutes. What is your *average speed* for the return trip?

7. (I.17.1) Chandler was given \$75 for a birthday present. This present, along with earnings from a summer job, is being set aside for a mountain bike. The job pays \$6 per hour, and the bike costs \$345. To be able to buy the bike, how many hours does Chandler need to work?

8. (I.17.2) (Continuation) Let h be the number of hours that Chandler works. What quantity is represented by the expression $6h$? What quantity is represented by the expression $6h + 75$?
(a) Graph the solutions to the inequality $6h + 75 \geq 345$ on a number line.
(b) Graph the solutions to the inequality $6h + 75 < 345$ on a number line. What do the solutions to the inequality $6h + 75 \geq 345$ signify?

1. (I.17.3) Sandy recently made a 210-mile car trip, starting from home at noon. The graph at right shows how Sandy's distance from home (measured in miles) depends on the number of hours after noon. Make up a story that accounts for the four distinct parts of the graph. In particular, identify the speed at which Sandy spent most of the afternoon driving.



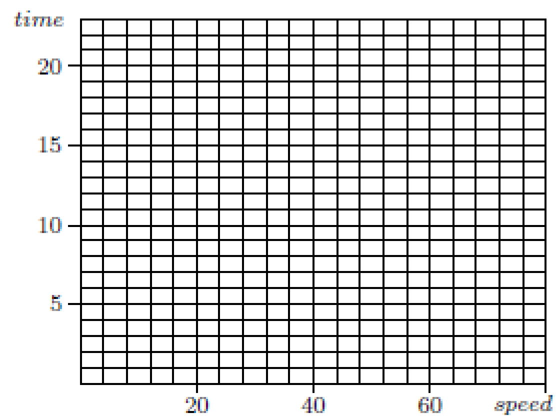
2. (I.17.7) Solve the inequality $3 - x > 5$ using only the operations of addition and subtraction. Is $x = 0$ a solution to the inequality?

3. (I.18.4) The line through (1, 6) and (0, 3) passes through every quadrant except one. Which one?

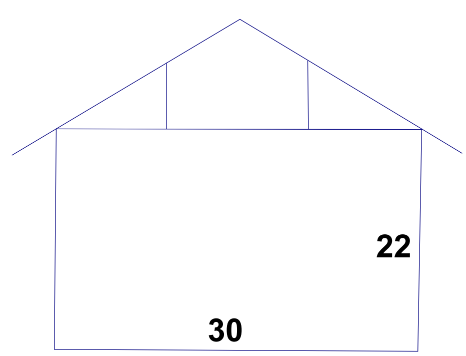
4. (I.18.6) A small pool is 20 feet long, 12 feet wide and 4 feet deep. There are 7.5 gallons of water in every cubic foot. At the rate of 5 gallons per minute, how long will it take to fill this pool?

5. (I.18.1) To do a college visit, Wes must make a 240-mile trip by car. The time required to complete the trip depends on the speed at which Wes drives, of course, as the table below shows. Fill in the missing entries, and plot points on the grid provided. Do the quantities time and speed vary directly? It makes sense to connect your plotted points with a continuous graph. Explain why.

speed	15	20	25			48		60		*r*
time		12		8	6		4.8		3	



6. (I.18.3) A small town's building code does not permit building a house that is more than 35 feet tall. An architect working on the design shown at right would like the roof to be sloped so that it rises 10 inches for each foot of horizontal run.
(a) Given the other dimensions in the diagram, will the builder be allowed to carry out this plan?
(b) Two vertical supports are to be placed 9 feet from the edges of the building. How long should they be?

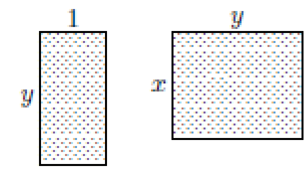


1. (I.18.5) Combine over a common denominator without using a calculator.

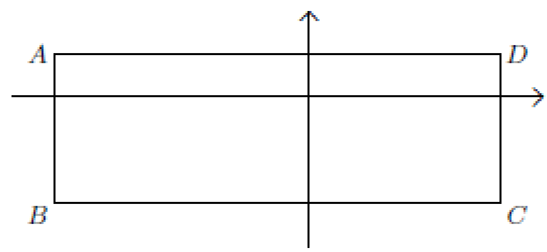
(a) $\frac{1}{4}+\frac{1}{5}$ (b) $\frac{1}{10}+\frac{1}{11}$ (c) $\frac{1}{x}+\frac{1}{x+1}$

Evaluate your answer to (c) with x = 4 and then with x = 10. How do these answers compare to your answers to (a) and (b)?

2. (I.18.7) Shown at right, the y-block and xy-block are two more members of the algebra-block family. Draw an algebra-block diagram that illustrates the equation $y+xy=y(1+x)$.



3. (I.19.1) The rectangle $ABCD$ shown at right has sides that are parallel to the coordinate axes. It is three times as wide as it is tall, and its perimeter is 56 units.
(a) Find the length and width of $ABCD$.
(b) Given the information D = (9, 2), find the coordinates for points A, B, and C.



4. (I.19.3) Each step of the stairs leading from the first floor to the second floor of Rowe Hall has a vertical *rise* of 5.5 inches and a horizontal *run* of 13 inches. Each step of the stairs leading to the Pit under the pool has a vertical rise of 7 inches and a horizontal run of 12 inches.
(a) Which flight of stairs do you think is steeper? Why?
(b) Calculate the ratio *rise/run* for each flight of stairs, and verify that the greater ratio belongs to the flight you thought to be steeper.

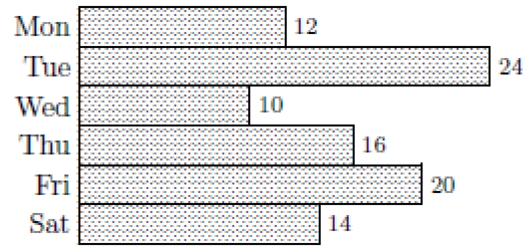
5. (I.19.4) (Continuation) The *slope* of a line is a measure of how steep the line is. It is calculated by dividing the change in y-coordinates by the corresponding change in x-coordinates between two points on the line: slope = $\frac{\text{change in } y}{\text{change in } x}$. Calculate the slope of the line that goes through the two points (1, 3) and (7, 6). Calculate the slope of the line that goes through the two points (0, 0) and (9, 6). Which line is steeper?

6. (I.19.5) Explain why the descriptions “right 5 up 2”, “right 10 up 4”, “left 5 down 2”, “right 5/2 up 1”, and “left 1 down 2/5” all describe the same inclination for a straight line.

7. (I.19.6) At noon one day, the Allegheny River peaked at 11 feet above flood stage. It then began to recede, its depth dropping at 4 inches per hour.
(a) At 3:30 that afternoon, how many inches above flood stage was the river?
(b) Let t stand for the number of hours since noon, and h stand for the corresponding number of inches that the river was above flood stage. Make a table of values, and write an equation that expresses h in terms of t.
(c) Plot h versus t, with t on the horizontal axis.
(d) For how many hours past noon was the river at least 36 inches above flood stage?

1. (I.19.7) Solve the following for x: (a) $\frac{x}{2}+\frac{x}{5}=6$ (b) $\frac{x}{3}+\frac{x+1}{6}=4$

2. (I.20.1) Jess and Taylor go into the cookie-making business. The chart shows how many dozens of cookies were baked and sold (at \$3.50 per dozen) during the first six days of business.



(a) What was their total income during those six days?
(b) Which was more profitable, the first three days or the last three days?
(c) What was the percentage decrease in sales from Tuesday to Wednesday? What was the percentage increase in sales from Wednesday to Thursday?
(d) Thursday's sales were what percent of the total sales?
(e) On average, how many dozens of cookies did Jess and Taylor bake and sell each day?

3. (I.20.2) The perimeter of a rectangle is 100 and its length is x. What expression represents the width of the rectangle?

4. (I.20.5) Pat and Kim are solving the inequality $132-4x \le 36$. Each begins by subtracting 132 from both sides to get $-4x \le -96$, and then each divides both sides by −4. Pat gets $x \le 24$ and Kim gets $x \ge 24$, however. Always happy to offer advice, Alex now suggests to Pat and Kim that answers to inequalities can often be checked by substituting $x = 0$ into both the original inequality and the answer. What do you think of this advice? Graph each of these answers on a number line. How do the results of this question relate to the flooding of the Allegheny River?

5. (I.20.6) (Continuation) After hearing Alex's suggestion about using a test value to check an inequality, Wes suggests that the problem could have been done by solving the equation $132-4x=36$ first. Complete the reasoning behind this strategy.

6. (I.20.7) (Continuation) Deniz, who has been keeping quiet during the discussion, remarks, "The only really tricky thing about inequalities is when you try to multiply them or divide them by negative numbers, but this kind of step can be avoided altogether. Wes just told us one way to avoid it, and there is another way, too." Explain this remark by Deniz.

7. (I.20.3) When a third of a number is subtracted from a half of the same number, 60 is the result. Find the number.

8. (I.20.4) Suppose that n represents an integer. What expression represents the next larger integer? the previous integer? the sum of these three consecutive integers?

9. (I.20.8) Draw the segment from (3, 1) to (5, 6), and the segment from (0, 5) to (2, 0). Calculate their slopes. You should notice that the segments are equally steep, and yet they differ in a significant way. Do your slope calculations reflect this difference?

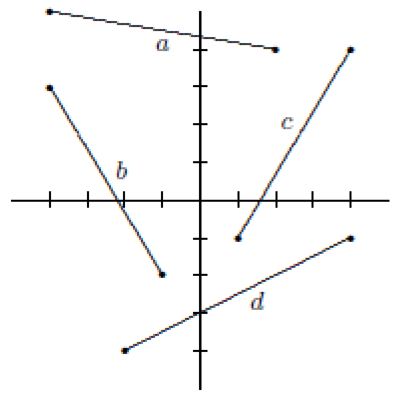
10. (I.20.9) Solve the following inequality for x: $2(1-3x)-(x-5)>1$.

1. (I.21.1) The volume of blood pumped by your heart each minute varies directly with your pulse rate. Each heartbeat pumps approximately 0.006 liter of blood.

(a) If your heart beats 50 times a minute, how much blood is pumped in one minute?
(b) If your heart pumps 0.45 liters per minute, what is your pulse rate in beats per minute?

2. (I.21.2) (Continuation) Direct-variation equations can be written in the form $y = kx$. In this equation, we say *y depends on x*. Is it more natural to say that the volume of blood being pumped depends on your heart rate, or that your heart rate depends on the volume of blood being pumped? Find an equation that relates the volume v and the rate r. Graph this equation, using an appropriate scale, and calculate its slope. What does the slope represent in this context?

3. (I.21.3) Estimate the slopes of all the segments in the diagram. Identify those whose slopes are negative. Find words to characterize lines that have negative slopes.



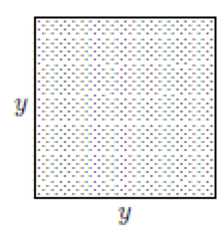
4. (I.21.4) Find the slope of the line containing the points $(4, 7)$ and $(6, 11)$. Find coordinates for another point that lies on the same line and be prepared to discuss the method you used to find them.

5. (I.21.6) To win a meet, a girl on the cross-country team must run the 5-km course in less than 20 minutes. What is the average speed of a 20-minute runner, in km per hour? in meters per second? Express your answers to two decimal places.

6. (I.21.7) (Continuation) The *proportion* $\frac{5}{20} = \frac{x}{60}$ is helpful for the first question. Explain this proportion, and assign units to all four of its members.

7. (I.21.10) Corey deposits \$300 in a bank that pays 4% annual interest. How much interest does Corey earn in one year? What would the interest be if the rate were 6%?

8. (I.21.8) The diagram shows the last member of the algebra-block family, the y^2-block. Show how an xy-block and a y^2-block can be combined to illustrate the equation $xy + y^2 = y(x + y)$.



9. (I.21.11) Alex was hired to unpack and clean 580 very small items of glassware, at ten cents per piece successfully unpacked. For every item broken during the process, however, Alex had to pay \$2. At the end of the job, Alex received \$49.60. How many items did Alex break?

10. (I.24.5) If 6% of x is the same as 5% of 120, then what is x?

1. (I.22.1) Each of the data sets at right represents points on a line. In which table is one variable directly related to the other? Why does the other table not represent a direct variation? Fill in the missing entry in each table.

x	y
0	4
4	10
10	19
16	

x	y
0	0
4	6
10	15
16	

2. (I.22.2) (Continuation) Plot the data from the tables in the previous question on the same set of axes and use a ruler to draw a line through each set of points. By looking at the graph, how could you recognize the direct variation? What similarities and differences are there between the two lines drawn?

3. (I.22.4) A car and a small truck started out from Shady Side Academy at 8:00 am. Their distances in km from SSA, recorded at hourly intervals, are recorded in the tables at right. Plot this information on the same set of axes and draw two lines connecting the points in each set of data. What is the slope of each line? What is the meaning of these slopes in the context of this problem?

time	*car*	*truck*
8 : 00	0	0
9 : 00	52	46
10 : 00	104	92
11 : 00	156	138
12 : 00	208	184

4. (I.22.5) (Continuation) Let t be the number of hours each vehicle has been traveling since 8:00 am (thus $t = 0$ means 8:00 am), and let d be the number of km traveled after t hours. For each vehicle, write an equation relating d and t.

5. (I.22.6) Day student Chris does a lot of babysitting. When parents drop off their children and Chris can supervise at home, the hourly rate is \$3. If Chris has to travel to the child's home, there is a fixed charge of \$5 for transportation in addition to the \$3 hourly rate.
(a) Graph $y = 3x$ and $y = 3x + 5$. What do these lines have to do with the babysitting context? What feature do they have in common? How do they differ?
(b) Predict what the graph of $y = 3x + 6$ will look like. What change in the babysitting context does this line suggest?

6. (I.22.7) If k stands for an integer, then is it possible for $k^2 + k$ to stand for an odd integer? Explain.

7. (I.22.8) Can you think of a number k for which $k^2 < k$ is true? Graph all such numbers on a number line. Also describe them using words, and using algebraic notation.

8. (I.22.10) Solve $\frac{x}{4} + \frac{x+1}{3} \le \frac{1}{2}$ and shade the solution interval on a number line.

9. (I.23.4) Find coordinates for the points where the line $3x + 2y = 12$ intersects the x-axis and the y-axis. These points are called the *x-intercept* and *y-intercept*, respectively.

1. (I.23.7) Driving from Boston to New York one day, Sasha covered the 250 miles in five hours. Because of heavy traffic, the 250-mile return took six hours and fifteen minutes. Calculate average speeds for the trip *to* New York, the trip *from* New York, and the round trip. Explain why the terminology *average speed* is a bit misleading.

2. (I.23.5) Drivers in distress near Fox Chapel have two towing services to choose from: Brook's Body Shop charges \$3 per mile for the towing, and a fixed \$25 charge regardless of the length of the tow. Morgan Motors charges a flat \$5 per mile. On the same system of axes, represent each of these choices by a linear graph that plots the cost of the tow versus the length of the tow. If you needed to be towed, which service would you call, and why?

3. (I.23.6) Predict what a graph of (a) $y = 2x + 5$ and (b) $y = 3x + 5$ will look like. In each case, confirm your prediction on the calculator, and describe a context from which the equation might emerge. (c) Linear equations that look like $y = mx + b$ are said to be in *slope-intercept form*. Explain. The terminology refers to which of the two intercepts?

4. (I.23.8) Find the value of x that makes $0.1x + 0.25(102 - x) = 17.10$ true.

5. (I.23.9) (Continuation) So that it will be handy for paying tolls and parking meters, Lee puts pocket change (dimes and quarters only) into a cup attached to the dashboard. There are 102 coins in the cup, and their monetary value is \$17.10. How many of the coins are dimes?

6. (I.23.10) (Continuation) Find all the values of x that make $0.1x + 0.25(102 - x) < 17.10$ true.

7. (I.24.1) Day student Morgan left home at 7:00 one morning, determined to make the ten-mile trip to SSA on bicycle for a change. Soon thereafter, a parent noticed forgotten math homework on the kitchen table, got into the family car, and tried to catch up with the forgetful child. Morgan had a fifteen-minute head start, and was pedaling at 12 mph, while the parent pursued at 30 mph. Was Morgan reunited with the homework before reaching SSA that day? If so, where? If not, at what time during first period (math, which starts at 8:15) was the homework delivered?

8. (I.24.2) Farmer MacGregor needs to put a fence around a rectangular carrot patch that is one and a half times as long as it is wide. The project uses 110 feet of fencing. How wide is the garden?

9. (I.24.3) Combine over a common denominator: $\frac{1}{a} + \frac{2}{3a} + 3$.

x	y
-3	7
-2	5
-1	3
0	1
1	-1

10. (I.24.4) Confirm that the five points in the table all lie on a single line. Write an equation for that line. Use your calculator to make a *scatter plot*, and graph the line on the same system of axes.

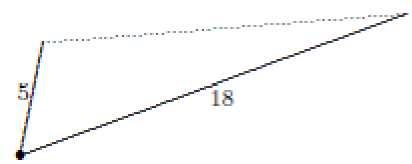
11. (I.24.8) The population of a small town increased 25% two years ago and then decreased by 25% last year. The population is now 4500 persons. What was the population before the two changes?

1. (I.24.10) The volume of a circular cylinder is given by the formula $V = \pi r^2 h$.
(a) To the nearest tenth of a cm^3, find the volume of a cylinder that has a 15-cm radius and is 12 cm high.
(b) Solve the volume formula for *h*, and then find *h* to the nearest tenth of a cm if the volume is 1000 cc and the radius is 10 cm.

2. (I.25.1) Which of the following pairs of quantities vary directly?
(a) the circumference of a circle and the diameter of the circle;
(b) the distance traveled in two hours and the (average) rate of travel;
(c) the number of gallons of gasoline bought and the cost of the purchase;
(d) the area of a circle and the radius of the circle.

3. (I.25.2) A jet, cruising at 26400 feet, begins its descent into Pittsburgh International Airport, which is 48 miles away. Another jet, cruising at 31680 feet, is 60 miles from the airport when it begins its descent. Which of these two paths of descent is steeper? Explain.

4. (I.25.4) The diagram shows two steel rods hinged at one end. The other end is connected by a bungee cord (the dotted segment), whose unstretched length is 10 inches. The rods are 5 inches and 18 inches long. Use inequality symbols to describe all the possible lengths for the bungee cord, which stays attached at both ends while it is being stretched.



5. (I.25.6) Multiply $2 + x$ by $2x$. You may use an algebra-block diagram.

6. (I.25.8) Solve the following inequalities and shade their solution intervals on a number line.

(a) $\dfrac{2x}{3} + \dfrac{3x+5}{2} \le 5$ (b) $\dfrac{1}{2}(x-1) + 3 > \dfrac{1}{3}(2x+1) - 1$

7. (I.25.7) For each of the following situations, draw a plausible graph that shows the relationship between the time elapsed (horizontal axis) and the indicated speed (vertical axis). In other words, graph speed versus time for each of the following:
(a) A car in a bumper test travels at a steady speed until it crashes into a wall.
(b) Your workout consists of some jogging, some hard running, some more jogging, some more hard running, and finally some walking.
(c) A roller coaster slowly climbs up a steep ramp and then zooms down the other side. (Plot the car's speed just to the bottom of the first hill.)
(d) A car speeds at a steady rate along a highway until an officer pulls it over and gives the driver a ticket. The car then resumes its journey at a more responsible speed.

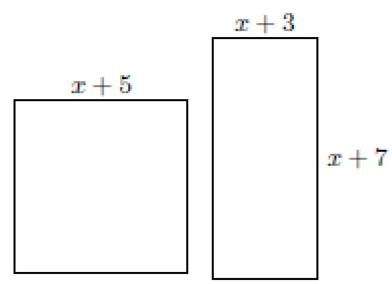
8. (I.9.1) You may recall that Fahrenheit and Celsius temperatures are related by the equation $F = \frac{9}{5}C + 32$. A quick way to get an approximate Fahrenheit temperature from a Celsius temperature is to double the Celsius temperature and add 30. Explain why this is a good approximation. Convert 23° Celsius the quick way. What is the difference between your answer and the correct value? For what Celsius temperature does the quick way give the correct value?

1. (I.26.2) At noon, my odometer read 6852 miles. At 3:30 pm, it read 7034 miles.
(a) What was my average rate of change during these three and a half hours?
(b) Let t represent the number of hours I have been driving since noon and y represent my odometer reading. Write an equation that relates y and t. Assume constant speed.
(c) Graph your equation.
(d) Show that the point (5, 7112) is on your line, and then interpret this point in the context of this problem.

2. (I.26.4) Avery's long-distance phone company charges a connect fee in addition to a per-minute charge for each call. Avery's most recent bill included a \$4.24 charge for a 12-minute call and a \$6.00 charge for a 20-minute call.
(a) What is the per-minute charge?
(b) What is the connect fee?
(c) How much would Avery be billed for a five-minute call?
(d) How much would Avery be charged for a call m minutes long?
(e) Avery was able to figure out answers to (a) and (b) without doing any algebra. How?

3. (I.26.5) (Continuation) I was recently called by a telemarketer asking me to change my long-distance phone company. This new company charges \$0.33 per minute with no connect fee. Is this a better deal than the company described in the previous problem? Show evidence.

4. (I.26.6) For what values of x will the square and the rectangle shown at right have the same perimeter?



5. (I.26.7) The point (3, 2) is on the line $y = 2x + b$. Find the value of b. Graph the line.

6. (I.26.8) Are (2, 9) and (−3, −6) both on the line $y = 4x + 6$? If not, find an equation for the line that does pass through both points.

7. (I.26.9) After you graph the line $y = 4x + 6$, find (a) the y-coordinate of the point on the line whose x-coordinate is 2; (b) the x-coordinate of the point on the line whose y-coordinate is 2.

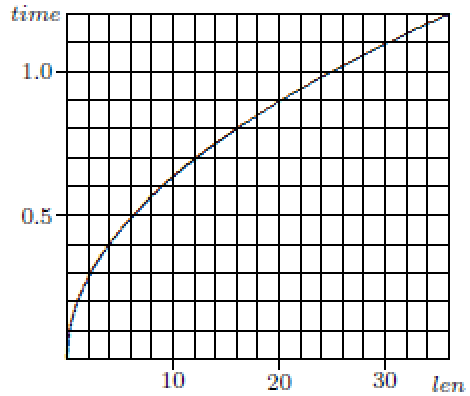
8. (I.26.10) In each of the following, describe the rate of change between the first pair and the second, assuming that the first coordinate is measured in minutes and the second coordinate is measured in feet. What are the units of your answer?

(a) (2, 5) and (5, 17) (b) (3.4, 6.8) and (7.2, 8.7) (c) $\left(\frac{3}{2}, -\frac{3}{4}\right)$ and $\left(\frac{1}{4}, 2\right)$

9. (I.27.1) If you double all the sides of a square, a larger square results. By what percentage has the perimeter increased? By what percentage has the area increased?

10. (I.27.3) Find the x-intercept and the y-intercept of the equation $y = -\frac{3}{2}x + 6$. Graph.

1. (I.27.4) The graph shows how the length (measured in cm) of a *pendulum* is related to the time (measured in sec) needed for the pendulum to make one complete back-and-forth movement (which is called the *pendulum period*). Find the length of a pendulum that swings twice as often as a 30-cm pendulum.



2. (I.27.8) A cyclist rides 30 km at an average speed of 9 km/hr. At what rate must the cyclist cover the next 10 km in order to bring the overall average speed up to 10 km/hr?

3. (I.27.6) A toy manufacturer is going to produce a new toy car. Each one costs \$3 to make, and the company will also have to spend \$200 to set up the machinery to make them.
(a) What will it cost to produce the first hundred cars? the first n cars?
(b) The company sells the cars for \$4 each. Thus the company takes in \$400 by selling one hundred cars. How much money does the company take in by selling n cars?
(c) How many cars does the company need to make and sell in order to make a profit?

4. (I.27.7) What is the distance between 6 and −6? between 24 and 17? between 17 and 24? between t and 4? The distance between two points is always positive. If a and b are two points on a number line, the distance is therefore either $a - b$ or $b - a$, whichever is nonnegative. This is an example of an *absolute-value* calculation, and the result is written $|a-b|$. What is the meaning of $|b-a|$?

5. (I.28.1) Given that $48 \le n \le 1296$ and $24 \le d \le 36$, what are the largest and smallest values that the expression $\frac{n}{d}$ can possibly have? Write your answer $smallest \le \frac{n}{d} \le largest$.

6. (I.28.2) Jess has 60 ounces of an alloy that is 40% gold. How many ounces of pure 100% gold must be added to this alloy to create a new alloy that is 75% gold?

7. (I.27.9) Let $P = (x, y)$ and $Q = (1, 5)$. Write an equation that states that the slope of line PQ is 3. Show how this slope equation can be rewritten in the form $y-5=3(x-1)$. This linear equation is said to be in *point-slope form*. Explain the terminology. Find coordinates for three different points P that fit this equation.

8. (I.27.10) (Continuation) What do the lines $y=3(x-1)+5$, $y=2(x-1)+5$, and $y=-\frac{1}{2}(x-1)+5$ all have in common? How do they differ from each other?

9. (I.28.4) Write an equation for the line that goes through the point (1, 5) and that has slope $\frac{2}{3}$.

Problem-Based Mathematics I

1. (I.28.5) The equation $5x-8y=20$ expresses a linear relationship between x and y. The point (15, 7) is either on the graph of this line, above it, or below it. Which? How do you know?

2. (I.28.6) Write an equation for the line that contains the points in the table, and make up a context for it.

x	0	15	30	45	60
y	100	160	220	280	340

3. (I.28.8) Solve (a) $A=\frac{1}{2}bh$ for b; (b) $A=2\pi rh+2\pi r^2$ for h.

4. (I.28.9) On a number line, how far is each of the following numbers from 5?
(a) 17 (b) -4 (c) x (d) $x+3$ (e) $x-1$

5. (I.28.10) Interpret each of the following as the distance between two numbers on a number line. (a) $|x-7|$ (b) $|3-x|$ (c) $|x+5|$ (d) $|x|$

6. (I.28.11) To graph linear equations such as $3x+5y=30$, one can put the equation into slope-intercept form, but (unless the slope is needed) it is easier to find the x-and y-intercepts and use them to sketch the graph. Find the axis intercepts of each of the following and use them to draw the given line. An equation $ax+by=c$ is said to be in *standard form*.
(a) $20x+50y=1000$ (b) $4x-3y=72$

7. (I.28.12) Find an equation for the line containing the points (−3, 0) and (0, 4).

8. (I.28.13) Multiply $x+2y$ by $3y$. Draw an algebra-block diagram to illustrate this calculation.

9. (I.29.1) Write an equation in point-slope form for
(a) the line that goes through (2, 5) and (6, −3);
(b) the line that goes through point (h, k) and that has slope m.

10. (I.29.2) Casey goes for a bike ride from SSA to FCHS, while an odometer keeps a cumulative record of the number of miles traveled. The equation $m=12t+37$ describes the odometer reading m after t hours of riding. What is the meaning of 12 and 37 in the context of this trip?

11. (I.29.4) Find coordinates for all the points on a number line that are
(a) six units from 0; (b) six units from four; (c) six units from −7; (d) six units from x.

12. (I.29.8) Translate the sentence "the distance between x and 12 is 20" into an equation using algebraic symbols. What are the values of x being described?

13. (I.29.10) Translate "x is 12 units from 20" into an equation. What are the values of x being described?

1. (I.29.6) As you know, temperatures can be measured by either Celsius or Fahrenheit units; $30°C$ is equivalent to $86°F$, $5°C$ is equivalent to $41°F$, and $-10°C$ is equivalent to $14°F$.
(a) Plot this data with C on the horizontal axis and F on the vertical axis.
(b) Verify that these three data points are *collinear*.
(c) Find a linear equation that relates C and F.
(d) Graph F versus C. In other words, graph the linear equation you just found.
(e) Graph C versus F. You will need to re-plot the data, with C on the vertical axis.
(f) On New Year's Day, I heard a weather report that said the temperature was a balmy $24°C$. Could this have happened? What is the corresponding Fahrenheit temperature?
(g) Water boils at $100°C$ and freezes at $0°C$ at sea level. Find the corresponding Fahrenheit temperatures.
(h) Is it ever the case that the temperature in degrees Fahrenheit is the same as the temperature in degrees Celsius?

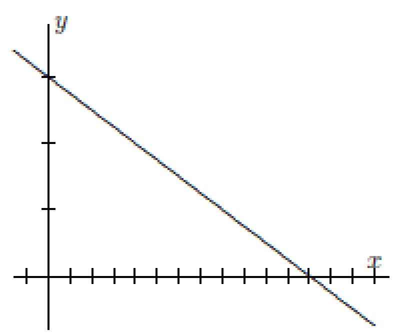
2. (I.30.3) The equation $|x-7|=2$ is a translation of "the distance from x to 7 is 2."
(a) Translate $|x-7|\le 2$ into English, and graph its solutions on a number line.
(b) Convert "the distance from -5 to x is at most 3" into symbolic form, and solve it.

3. (I.30.5) Verify that (0, 4) is on the line $3x+2y=8$. Find another point on this line. Use these points to calculate the slope of the line. Is there another way to find the slope of the line?

4. (I.30.6) Graph a horizontal line through the point (3, 5). Choose another point on this line. What is the slope of this line? What is the y-intercept of this line? What is an equation for this line? Describe a context that could be modeled by this line.

5. (I.30.7) Graph a vertical line through the point (3, 5). Does this line have a slope or y-intercept? What is an equation for this line? Describe a context that could be modeled by this line.

6. (I.30.9) The figure shows the graph of $20x+40y=1200$. Find the x-and y-intercepts, the slope of the line, and the distances between tick marks on the axes. Duplicate this figure on your calculator. What window settings did you use?



7. (I.30.11) A handicapped-access ramp starts at ground level and rises 27 inches over a distance of 30 feet. What is the slope of this ramp?

8. (I.30.12) Jay thinks that the inequality $k<3$ implies the inequality $k^2<9$ but Val thinks otherwise. Who is right, and why?

1. (I.31.2) A movie theater charges \$6 for each adult and \$3 for each child. If the total amount in ticket revenue one evening was \$1428 and if there were 56 more children than adults, then how many children attended?

2. (I.31.7) If $|x+1|=5$, then $x+1$ can have two possible values, 5 and −5. This leads to two equations, $x+1=5$ and $x+1=-5$. If $|2x-7|=5$, what possible values could the expression $2x-7$ have? Write two equations using the expression $2x-7$ and solve them.

3. (I.31.8) Write two equations without absolute value symbols that, in combination, are equivalent to $|3x+5|=12$. Solve each of these two equations.

4. (I.31.10) A *lattice point* is defined as a point whose coordinates are integers. If (−3, 5) and (2, 1) are two points on a line, find three other lattice points on the same line.

5. (I.31.11) The equation $13x+8y=128$ expresses a linear relationship between *x* and *y*. The point (5, 8) is on, or above, or below the linear graph. Which is it? How do you know?

6. (I.32.3) Find the value for *h* for which the slope of the line through (−5, 6) and (*h*, 12) is ¾.

7. (I.32.5) The data in each table fits a direct variation. Complete each table, write an equation to model its data, and sketch a graph.

(a)

x	2	4	6	
y	3	6		18

(b)

x	2	3		8
y	−8	−12	−20	

8. (I.32.6) For each of the following equations, find the *x*-intercept and *y*-intercept. Then use them to calculate the slope of the line.
(a) $3x+y=6$ (b) $x-2y=10$ (c) $4x-5y=20$ (d) $ax+by=c$

9. (I.32.7) Blair's average on the first five in-class tests is 67. If this is not pulled up to at least a 70, Blair will not be allowed to watch any more *Law and Order* reruns. To avoid losing those TV privileges, what is the lowest score Blair can afford to make on the last in-class test? Assume that all tests carry equal weight.

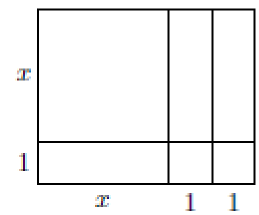
10. (I.32.8) Sketch the graphs of $y=2x$, $y=2x+1$, and $y=2x-2$ all on the same coordinate-axis system. Find the slope of each line. How are the lines related to one another?

Problem-Based Mathematics I

1. (I.32.10) Describe the relationship between the following pairs of numbers:
(a) $24 - 11$ and $11 - 24$ (b) $x - 7$ and $7 - x$ (c) $|x-7|$ and $|7-x|$

2. (I.32.11) In each case, decide whether the three points given are collinear:
(a) (-4, 8), (0, 2), and (2, -1) (b) (350, 125), (500, 300), and (650, 550)

3. (I.33.2) Write $(x+1)(x+2)$ without parentheses. Explain how the diagram to the right explains this product.



4. (I.33.3) Solve the equation $C = \frac{5}{9}(F-32)$ for F.

5. (I.33.7) The manager at Jen and Berry's Ice Cream Company estimates that the cost C (in dollars) of producing n quarts of ice cream in a given week is given by the equation $C = 560 + 1.20n$.
(a) During one week, the total cost of making ice cream was \$1070. How many quarts were made that week?
(b) Explain the meanings of the "560" and the "1.20" in the cost equation.

6. (I.33.8) As anyone knows who has hiked up a mountain, the higher you go, the cooler the temperature gets. At noon on July 4th last summer, the temperature at the top of Mt. Washington—elevation 6288 feet—was 56°F. The temperature at base camp in Pinkham Notch—elevation 2041 feet—was 87°F. It was a clear, still day. At that moment, a group of hikers reached Tuckerman Junction—elevation 5376 feet. To the nearest degree, calculate the temperature the hikers were experiencing at that time and place. When you decided how to model this situation, what assumptions did you make?

7. (I.33.9) Draw a line through the origin with a slope of 0.4. Draw a line through the point (1, 2) with a slope of 0.4. How are these two lines related? What is the vertical distance between the two lines? Find an equation for each line.

8. (I.34.1) Solve $\frac{3m}{4} + \frac{3}{8} = \frac{m}{3} - \frac{5}{6}$ for m, expressing your answer as a fraction in lowest terms.

9. (I.34.2) Find two different ways of determining the slope of the line $11x + 8y = 176$.

10. (I.34.3) When weights are placed on the end of a spring, the spring stretches. If a three-pound weight stretches the spring to a length of 4.25 inches, a five-pound weight stretches the spring to a length of 5.75 inches, and a nine-pound weight stretches the spring to a length of 8.75 inches, what was the initial length of the spring?

11. (I.34.5) Draw rectangles that are composed of x^2-blocks, x-blocks, and 1-blocks to illustrate the results when the following *binomials* are expanded:
(a) $(x+2)(x+3)$ (b) $(2x+1)(x+1)$ (c) $(x+2)(x+2)$

1. (I.34.11) A cube measures x cm on each edge.
(a) Find a formula in terms of x for the volume of this cube in cubic centimeters (cc).
(b) Evaluate this formula when $x = 1.5\text{cm};$ when $x = 10\text{cm}.$
(c) Write an expression for the area of one of the faces of the cube. Write a formula for the total surface area of all six faces.
(d) Evaluate this formula when $x = 1.5\text{cm};$ when $x = 10\text{cm}.$
(e) Although area is measured in square units and volume in cubic units, is there any cube for which the number of square units in the area of its faces would equal the number of cubic units in the volume?

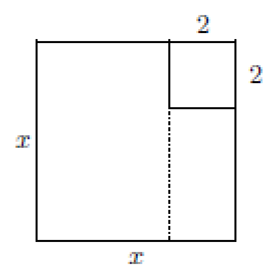
2. (I.34.12) Apply the distributive property to write without parentheses and collect like terms:
(a) $x(x-3)+2(x-3)$ (b) $2x(x-4)-3(x-4)$ (c) $x(x-2)+2(x-2)$

3. (I.35.3) Asked to solve the inequality $3 < |x-5|$ at the board, Corey wrote "$8 < x < 2$," Sasha wrote "$x < 2$ or $8 < x$," and Pat wrote "$x < 2$ and $8 < x$." What do you think of these answers? Do any of them agree with your answer?

4. (I.31.1) The specifications for machining a piece of metal state that it must be 12 cm long, within a 0.01-cm tolerance. What is the longest the piece is allowed to be? What is the shortest? Using l to represent the length of the finished piece of metal, write an absolute-value inequality that states these conditions.

5. (I.35.4) Apply the distributive property to write without parentheses and collect like terms:
(a) $(x+2)(x-3)$ (b) $(2x-3)(x-4)$ (c) $(x+2)(x-2)$

6. (I.35.6) By rearranging the pieces of the puzzle shown at right, you can demonstrate that x^2-4 is equivalent to $(x+2)(x-2)$ *without* using the distributive property. Show how to do it.

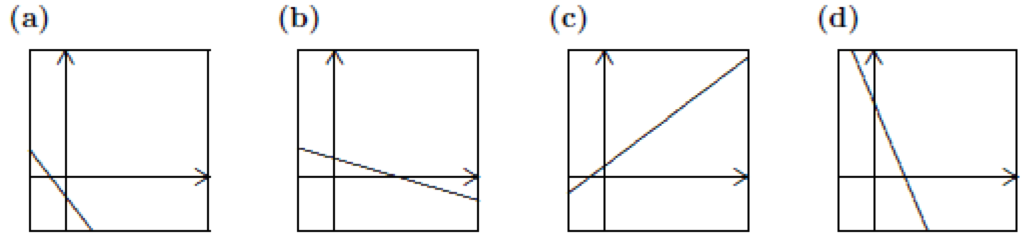


7. (I.36.1) Sandy was told by a friend that "absolute value makes everything positive." So Sandy rewrote the equation $|x-6| = 5$ as $x+6=5$. Do you agree with the statement, or with what Sandy did to the equation? Explain your answer.

8. (I.36.4) Given the line $y = \frac{1}{2}x + 6$, write an equation for the line through the origin that has the same slope. Write an equation for the line through (2, −4) that has the same slope.

9. (I.36.8) Hearing Yuri say "This line has no slope," Tyler responds "Well, 'no slope' actually means slope 0." What are they talking about? Do you agree with either of them?

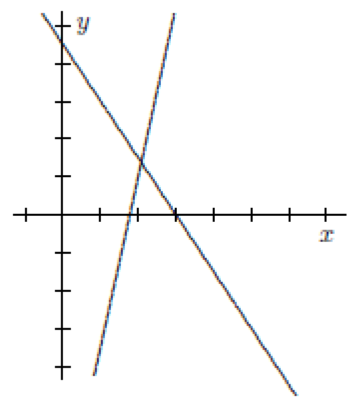
1. (I.36.6) Which of the following calculator screens could represent the graph of $9x + 5y = 40$?



2. (I.36.7) For each of these absolute-value equations, write two equations without absolute-value symbols that are equivalent to the original. Solve each of the equations.
(a) $2|x+7| = 12$ (b) $3 + |2x+5| = 17$ (c) $6 - |x+2| = 3$ (d) $-2|4-3x| = -14$

3. (I.37.1) The edges of a solid cube are $3p$ cm long. At one corner of the cube, a small cube is cut away. All its edges are p cm long. In terms of p, what is the total surface area of the remaining solid? What is the volume of the remaining solid? Make a sketch.

4. (I.37.10) Most linear equations can be rewritten in slope-intercept form $y = mx + b$. Give an example that shows that not all linear equations can be so rewritten.



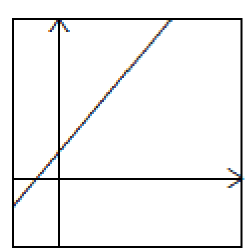
5. (I.37.5) The figure shows the graphs of two lines. Use the graphs (the axis markings are one unit apart) to estimate the coordinates of the point that belongs to both lines.

6. (I.37.6) (Continuation) The *system of equations* that has been graphed is $\begin{cases} 9x - 2y = 16 \\ 3x + 2y = 9 \end{cases}$

Jess took one look at these equations and knew right away what to do. "Just add the equations and you will find out quickly what x is." Follow this advice, and explain why it works.

7. (I.37.7) (Continuation) Find the missing y-value by inserting the x-value you found into either of the two original equations. Do the coordinates of the *point of intersection* agree with your estimate? These coordinates are called a *simultaneous solution* of the original system of equations. Explain the terminology.

8. (I.38.1) Which of the following could be the equation that is graphed on the calculator screen shown at right?
(a) $3y - 7x = 28$ (b) $x + 2y = 5$
(c) $12x = y + 13$ (d) $y - 0.01x = 2000$



1. (I.38.2) Draw a rectangle using two x^2-blocks and two x-blocks. Write the dimensions of your rectangle. What is the area of the rectangle?

2. (I.38.3) (Continuation) Using the same two x^2-blocks and same two x-blocks, draw a different rectangle. What is the area of the rectangle?

3. (I.38.4) (Continuation) One of your diagrams illustrates the equation $x(2x+2)=2x^2+2x$. Explain. Write an equation that is illustrated by the other diagram.

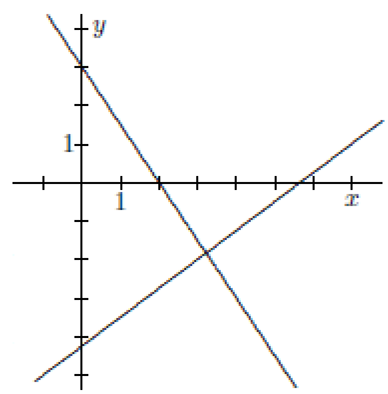
4. (I.38.5) Find values for x and y that fit both of the equations $2x-3y=8$ and $4x+3y=-2$.

5. (I.38.10) Find the value of x that fits the equation $\frac{1}{2}x+\frac{1}{3}x+\frac{1}{4}x=26$.

6. (I.39.1) A movie club has a membership fee of $50, after which individual movies can be bought for $5.50. Non-members pay $8.00 per movie. If you are going to buy a lot of movies, it is better to pay the membership fee. How many movies is "a lot"?

7. (I.38.6) The figure at right shows the graphs of two lines. Use the figure to estimate the coordinates of the point that belongs to both lines. The system of equations is

$$\begin{cases} 3x+2y=6 \\ 3x-4y=17 \end{cases}$$



Randy took one look at these equations and knew right away what to do. "Just subtract the equations and you will find out quickly what y is." Follow this advice.

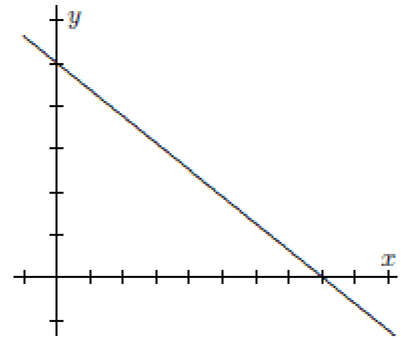
8. (I.38.7) (Continuation) Find the missing x-value by inserting the y-value you found into one of the two original equations. Does it matter which one? Compare the intersection coordinates with your estimate.

9. (I.38.8) (Continuation) If you add the two given equations, you obtain the equation of yet another line. Add its graph to the figure. You should notice something. Was it expected?

10. (I.39.6) Some questions about the line that passes through the points (−3, −2) and (5, 6):
(a) Find the slope of the line.
(b) Is the point (10, 12) on the line? Justify your answer.
(c) Find y so that the point (7, y) is on the line.

11. (I.39.7) Find values for x and y that fit both of the equations $5x+3y=8$ and $4x+3y=-2$.

1. (I.39.2) What is the slope of the line graphed at the right, if
(a) the distance between the x-tick marks is 2 units and the distance between the y-tick marks is 1 unit?
(b) the distance between the x-tick marks is 100 units and the distance between the y-tick marks is 5 units?



2. (I.39.4) Draw a rectangle using one x^2-block, three x-blocks and two 1-blocks to illustrate the equation $x^2+3x+2=(x+1)(x+2)$. What are the dimensions of the rectangle? This equation is called an *identity* because it is true no matter what value is assigned to x.

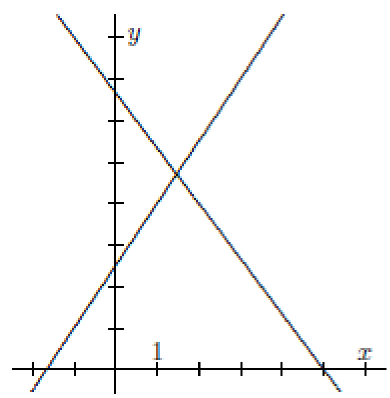
3. (I.39.8) A 100-liter barrel of vinegar is 8% acetic acid. Before it can be bottled and used in cooking, the acidity must be reduced to 5% by diluting it with pure water. In order to produce 64 liters of usable vinegar, how many liters of vinegar from the barrel and how many liters of pure water should be combined?

4. (I.39.10) Express each as a single fraction: (a) $\dfrac{1}{a}+\dfrac{2}{b}+\dfrac{3}{c}$ (b) $\dfrac{1}{a}+\dfrac{1}{b+c}$ (c) $1+\dfrac{2}{a+b}$

5. (I.40.1) The figure at right shows the graphs of two lines. Use the figure to estimate the coordinates of the point that belongs to both lines. The system of equations is

$$\begin{cases} 4x+3y=20 \\ 3x-2y=-5 \end{cases}$$

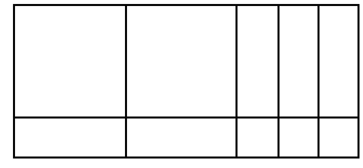
Lee took one look at these equations and announced a plan: "Just multiply the first equation by 2 and the second equation by 3." What does changing the equations in this way do to their graphs?



6. (I.40.2) (Continuation) Lee's plan has now created a familiar situation. Do you recognize it? Complete the solution to the system of equations. Do the coordinates of the *point of intersection* agree with your initial estimate?

7. (I.40.6) Shade the points in the plane whose x-coordinates are greater than their y-coordinates. Write an inequality that describes these points.

8. (I.40.3) The diagram consists of two x^2-blocks, five x-blocks and three 1-blocks. Use this diagram to write a statement that says that the product of the length and width of this particular rectangle is the same as its area. Can you draw another rectangle with the same area but different dimensions?



1. (I.40.4) Sandy's first four test scores this term are 73, 87, 81 and 76. To have at least a B test grade, Sandy needs to average at least 80 on the five term tests (which count equally). Let t represent Sandy's score on the fifth test, and write an inequality that describes the range of t-values that will meet Sandy's goal.

2. (I.41.2) A large telephone company sent out an offer for pre-paid phone cards. The table below accompanied the ad and summarized their offer. Does this data form a linear relationship? Explain your answer. Which offer has the best rate per minute?

75-minute card	150-minute card	300-minute card	500-minute card	1000-minute card	1500-minute card
\$4.95	\$9.90	\$19.80	\$30.00	\$56.00	\$75.00

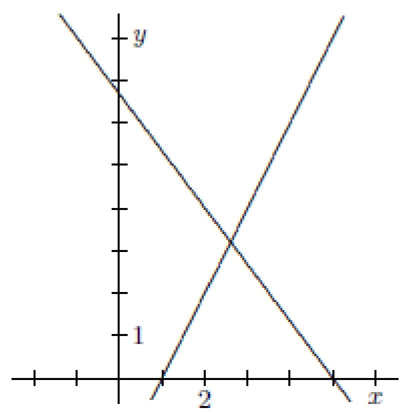
3. (I.41.3) Find an equation for each of the following lines. When possible, express your answer in both point-slope form and slope-intercept form.
(a) The line passes through (3, 5), and has −1.5 as its slope.
(b) The line is *parallel* to the line through (−8, 7) and (−3, 1), and has 6 as its x-intercept.
(c) The line is *parallel* to the line $x = -4$, and it passes through (4, 7).

4. (I.41.5) Use a different color for the regions described in parts (a) and (b):
(a) Shade all points whose x-and y-coordinates sum to less than 10.
(b) Shade all points whose x-and y-coordinates are both greater than zero.
(c) Write a system of three inequalities that describe where the two regions overlap.

5. (I.41.6) The figure at right shows the graphs of two lines. Use the figure to estimate the coordinates of the point that belongs to both lines. The system of equations is

$$\begin{cases} 4x+3y=20 \\ y=2x-2 \end{cases}$$

Dale took one look at these equations and offered a plan: "The second equation says you can *substitute* $2x-2$ for y in the first equation. Then you have only one equation to solve." Explain the logic behind Dale's substitution strategy. Carry out the plan, and compare the exact coordinates of the intersection point with your estimates.



6. (I.41.7) Farmer MacGregor wants to know how many cows and ducks are in the meadow. After counting 56 legs and 17 heads, the farmer knows. How many cows and ducks are there?

7. (I.42.4) Express in dollars the combined value of d dimes and n nickels.

8. Sam has \$2.45 in dimes and nickels. If 30 total coins are in Sam's pocket, how many of them are dimes?

Problem-Based Mathematics I

1. (I.42.1) Create a rectangle by combining two x^2-blocks, three x-blocks and a single 1-block. Write expressions for the length and width of your rectangle. Using these expressions, write a statement that says that the product of the length and width equals the area.

2. (I.42.2) (Continuation) Instead of saying, "Find the dimensions of a rectangle made with two x^2-blocks, three x-blocks and one 1-block", mathematicians say "*Factor* $2x^2+3x+1$" It is also customary to write the answer $2x^2+3x+1=(2x+1)(x+1)$. Explain why the statement about the blocks is the same as the algebraic equation.

3. (I.42.8) If the price of a stock goes from \$4.25 per share to \$6.50 per share, by what percent has the value of the stock increased?

4. (I.42.9) Your company makes spindles for the space shuttle. NASA specifies that the length of a spindle must be 12.45 ± 0.01 cm. What does this mean? What are the smallest and largest acceptable lengths for these spindles? Write this range of values as an inequality, letting L stand for the length of the spindle. Write another inequality using absolute values that models these constraints.

5. (I.42.10). Factor each expression. You may also draw an algebra-block diagram for each:
(a) $3x^2+12x$ (b) x^2+5x+6 (c) $4xy+2y^2$

6. (I.42.12) What is unusual about the graphs of the equations $9x-12y=27$ and $-3x+4y=-9$?

7. (I.43.2) With parental assistance, Corey buys some snowboarding equipment for \$500, promising to pay \$12 a week from part-time earnings until the 500-dollar debt is retired. How many weeks will it take until the outstanding debt is under \$100? Write an inequality that models this situation and then solve it algebraically.

8. (I.43.5) On 3 January 2004, after a journey of 300 million miles, the rover Spirit landed on Mars and began sending back information to Earth. It landed only six miles from its target. This accuracy is comparable to shooting an arrow at a target fifty feet away and missing the exact center by what distance?

9. (I.44.1) Find an equation for the line that passes through the point (−3, 6), parallel to the line through the points (0, −7) and (4, −15). Write your answer in point-slope form.

10. (I.44.4) Find the value of x that fits the equation $1.24x-(3-0.06x)=4(0.7+6)$.

11. (I.44.5) At the SSA Candy Shop, Jess bought 5.5 pounds of candy—a mixture of candy priced at \$4 per pound and candy priced at \$3.50 per pound. Given that the bill came to \$20.75, figure out how many pounds of each type of candy Jess bought.

Problem-Based Mathematics I

1. (I.44.2) Sid has a job at Morgan Motors. The salary is \$1200 a month, plus 3% of the sales price of every car or truck Sid sells (this is called a *commission*).
(a) The total of the sales prices of all the vehicles Sid sold during the first month on the job was \$72000. What was Sid's income (salary plus commission)?
(b) In order to make \$6000 in a single month, how much selling must Sid do?
(c) Write a linear equation that expresses Sid's monthly income y in terms of the value x of the vehicles Sid sold.
(d) Graph this equation. What is the meaning of its y-intercept, and the meaning of its slope?

2. (I.44.6) Explain how to evaluate 4^3 without a calculator. The small raised number is called an *exponent*, and 4^3 is *a power of 4* .Write $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$ as a power of 4. Write the product $4^3 \cdot 4^5$ as a power of 4.

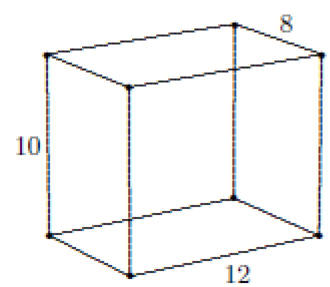
3. (I.44.8) Sketch the region that is common to the graphs of $x \geq 2, y \geq 0,$ and $x + y \leq 6.$Find the area of this region.

4. (I.44.10) You have one x^2-block, six x-blocks (all of which you must use), and a supply of 1-blocks. How many different rectangles can you make? Draw an algebra-block diagram for each.

5. (I.45.2) Given that s varies directly with t, and that s = 4.56 meters when t = 3 seconds, find s when t is 4.2 seconds.

6. (I.45.3) I recently paid \$85.28 for 12.2 pounds of coffee beans. What was the price per pound of the coffee? How many pounds did I buy per dollar?

7. (I.45.5) The figure at the right shows a rectangular box whose dimensions are 8 cm by 10 cm by 12 cm.
(a) Find the volume of the solid.
(b) What is the combined area of the six faces?
(c) If you were to outline the twelve edges of this box with decorative cord, how much would you need?



8. (I.45.7) A rectangle is four times as long as it is wide. If its length were diminished by 6 meters and its width were increased by 6 meters, it would be a square. What are its dimensions?

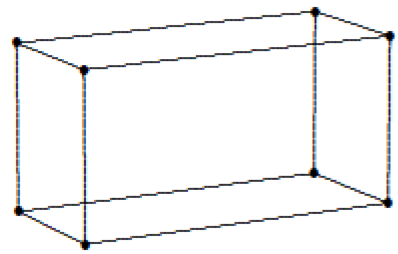
9. (I.45.8) What percent decrease occurs when a stock goes from \$6.50 per share to \$4.25 per share?

10. (I.45.11) You have one x^2-block, twelve 1-blocks (all of which you must use), and an ample supply of x-blocks. How many rectangles with different dimensions can you make? Draw an algebra-block diagram for each.

11. (I.42.6) Find the point (x, y) that fits both of the equations $y = 1.5x + 2$ and $9x + 4y = 41$.

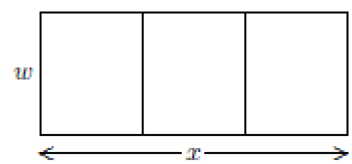
1. (I.46.1) After a weekend of rock-climbing in the White Mountains, Dylan is climbing down a 400-foot cliff. It takes 20 minutes to descend the first 60 feet. Assuming that Dylan makes progress at a steady rate, write an equation that expresses Dylan's height h above level ground in terms of t, the number of minutes of descending. Use your equation to find how much time it will take Dylan to reach level ground.

2. (I.46.4) The diagram at the right shows the wire framework for a rectangular box. The length of this box is 8 cm greater than the width and the height is half the length. A total of 108 cm of wire was used to make this framework.
(a) What are the dimensions of the box?
(b) The faces of the box will be panes of glass. What is the total area needed for the six panes?
(c) What is the volume of the box?



3. (I.46.7) You might not have seen an algebra-block diagram yet for a factorization that contains a minus sign. Try drawing a diagram to illustrate the identity $2x^2 - x - 1 = (2x+1)(x-1)$.

4. (I.46.9) A farmer has 90 meters of fencing material with which to construct three rectangular pens side-by-side as shown at right. If w were 10 meters, what would the length x be? Find a general formula that expresses x in terms of w.



5. (I.47.1) Find how many pairs (x, y) satisfy the equation $x + y = 25$, assuming that
(a) there is no restriction on the values of x and y;
(b) both x and y must be positive integers;
(c) the values of x and y must be equal.

6. (I.47.4) The table at the right shows the value of a car as it depreciates over time. Does this data satisfy a linear relationship? Explain.

year	*value*
1992	24000
1993	20400
1994	16800
1995	13200

7. (I.48.5) Solve each of the systems of equations below.

(a) $\begin{cases} 3x + 4y = 1 \\ 4x + 8y = 12 \end{cases}$ (b) $\begin{cases} 2x + 3y = -1 \\ 6x - 5y = -7 \end{cases}$

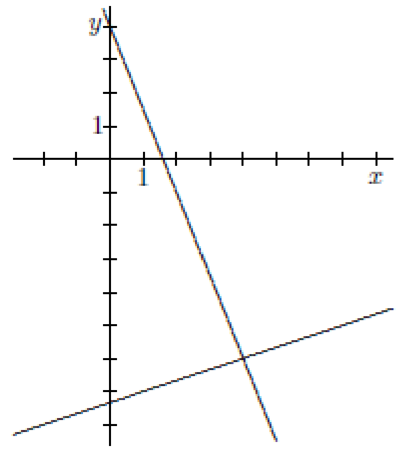
8. (I.47.7) Write and graph an equation that states
(a) that the *perimeter* of an $l \times w$ rectangle is 768 cm;
(b) that the width of an $l \times w$ rectangle is half its length.

9. (I.47.8) (Continuation) Explain how the two graphs show that there is a unique rectangle whose perimeter is 768 cm, and whose length is twice its width. Find the dimensions of this rectangle.

1. (I.47.9) When asked to solve the system of equations

$$\begin{cases} 5x + 2y = 8 \\ x - 3y = 22 \end{cases}$$

Kelly said "Oh that's easy—you just set them equal to each other." Looking puzzled, Wes replied "Well, I know the method of *linear combinations*, and I know the method of *substitution*, but I do not know what method you are talking about." First, explain each of the methods to which Wes is referring, and show how they can be used to solve the system. Second, explain why Wes did not find sense in Kelly's comment. Third, check that your answer agrees with the diagram.



2. (I.48.2) What algebra blocks would you need to order from the Math Warehouse so that you could build a square whose edges are all $x + 4$ units long?

3. (I.48.3) Giant Eagle carries two types of apple juice. One is 100% fruit juice, while the other is only 40% juice. Yesterday there was only one 48-ounce bottle of the 100% juice left. I bought it, along with a 32-ounce bottle of the 40% juice. I am about to mix the contents of the two bottles together. What percent of the mixture will be actual fruit juice?

4. (I.48.4) (Continuation) On second thought, I want the mixture to be at least 80% real fruit juice. How much of the 32-ounce bottle can I add to the mixture and be satisfied?

5. (I.48.7) If you have one x^2-block and two x-blocks, how many 1-blocks do you need to form a square? What are the dimensions of the square? Draw a diagram of the finished arrangement. Fill in the blanks in the equation $x^2 + 2x + \_\_\_\_ = (\quad)(\quad) = (\quad)^2$.

6. (I.49.1) Solve each of the systems of equations below.

(a) $\begin{cases} 3r + 5s = 6 \\ 9r = 13s + 4 \end{cases}$ (b) $\begin{cases} 3a = 1 + \frac{1}{3}b \\ 5a + b = 11 \end{cases}$

7. (I.49.2) Use the distributive property to write each of the following in *factored form*:

(a) $ab^2 + ac^2$ (b) $3x^2 - 6x$ (c) $wx + wy + wz + w$

8. (I.51.9) The SSA Tree Company charges a certain amount per cord for firewood and a fixed amount for each delivery, no matter how many cords are delivered. My bill from SSATC last winter was \$155 for one cord of wood, and my neighbor's was \$215 for one and one-half cords. What is the charge for each cord of wood and what is the delivery charge?

1. (I.49.5) Faced with the problem of multiplying 5^6 times 5^3, Brook is having trouble deciding which of these four answers is correct: 5^{18}, 5^9, 25^{18}, or 25^9. Your help is needed. Once you have answered Brook's question, experiment with other examples of this type until you are able to formulate the *common-base principle for multiplication* of expressions $b^m \cdot b^n$.

2. (I.49.9) A large family went to a restaurant for a buffet dinner. The price of the dinner was \$12 for adults and \$8 for children. If the total bill for a group of 13 persons came to \$136, how many children were in the group?

3. (I.50.1) Write each of the following in factored form:
(a) $2x^2+3x^3+4x^4$ (b) $5xp+5x$ (c) $2\pi r^2+2\pi rh$

4. (I.50.4) Exponents are routinely encountered in scientific work, where they help investigators deal with large numbers:
(a) The human population of Earth is roughly 7000000000, which is usually expressed in *scientific notation* as 7×10^9. The average number of hairs on a human head is 5×10^5. Use scientific notation to estimate the total number of human head hairs on Earth.
(b) Light moves very fast—approximately 3×10^8 meters every second. At that rate, how many meters does light travel in one year, which is about 3×10^7 seconds long? This so-called *light year* is used in astronomy as a yardstick for measuring even greater distances.

5. (I.50.6) Solve the equation $1.2x+0.8(20-x)=17.9$ for *x*. Make up a word problem that could use this equation in its solution. In other words, the equation needs a context.

6. (I.50.8) Write the following sentence using mathematical symbols: "The absolute value of the sum of two numbers *a* and *b* is equal to the sum of the absolute values of each of the numbers *a* and *b*." Is this a true statement? Explain.

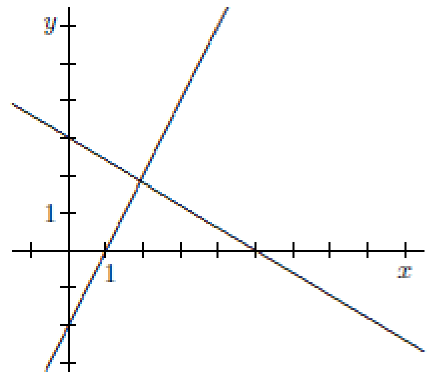
7. (I.50.9) The perimeter of a square is *p* inches. Write expressions, in terms of *p*, for the length of the side of the square and the area of the square.

8. (I.51.1) You have one x^2-block, eight *x*-blocks, and an ample supply of 1-blocks. How many 1-blocks do you need to form a square? What are the dimensions of the square? Fill in the blanks in the identity x^2+8x+ ____ = (   ) (   ) = (   )^2.

9. (I.51.8) A laser beam is shot from the point (0, 2.35) along the line whose slope is 3.1. Will it hit a very thin pin stuck in this coordinate plane at the point (10040, 31126)?

10. Evgeni Malkin's assists per year so far are: 52, 59, 78, 49, 22, 59, 42, and 24. What is the mean of this data? The median? The mode?

Problem-Based Mathematics I



1. (I.51.2) The figure shows the graphs of two lines, whose axis intercepts are integers. Use the graphs to estimate the coordinates of the point that belongs to both lines, then calculate the exact value. You will of course have to find equations for the lines.

2. (I.51.4) Find three lattice points on the line $x + 3y = 10$. How many others are there?

3. (I.51.5) In a coordinate plane, shade the region that consists of all points that have positive x-and y-coordinates whose sum is less than 5. Write a system of three inequalities that describes this region.

4. (I.51.11) A *monomial* is a product of variables—no additions, subtractions, or divisions—times an optional constant. If some factors occur more than once, it is customary to use exponents to consolidate them. Thus $3ax^2$ and x^5 are monomials, but $3xy^4 + 3x^4y$ is not. Rewrite each of these as a monomial:
(a) $x \cdot x^2 \cdot x^3 \cdot x^4$ (b) $3a^4 \cdot 5a^6$ (c) $(2x)^7$ (d) $\left(\frac{2}{3}w\right)^3 \cdot 5w^3$

5. (I.52.1) The point (2, 3) lies on the line $2x + ky = 19$. Find the value of k.

6. (I.52.3) Do the three lines $5x - y = 7, x + 3y = 11,$ and $2x + 3y = 13$ have a common point of intersection? If so, find it. If not, explain why not.

7. (I.52.7) A *polynomial* is obtained by adding (or subtracting) monomials. Use the distributive property to rewrite each of the following polynomials in factored form. In each example, you will be *finding a common monomial factor*.
(a) $x^2 - 2x$ (b) $6x^2 + 21x$ (c) $80t - 16t^2$ (d) $9x^4 - 3x^3 + 12x^2 - x$
A *binomial* is the sum of two unlike monomials, and a *trinomial* is the sum of three unlike monomials. The monomials that make up a polynomial are often called its *terms*.

8. (I.52.9) In $7^4 \cdot 7^4 \cdot 7^4 = \left(7^4\right)^{\Delta}$ and $b^9 \cdot b^9 \cdot b^9 \cdot b^9 = \left(b^9\right)^{\nabla}$, replace the triangles by correct exponents. The expression $\left(p^5\right)^6$ means to write p^5 as a factor how many times? To rewrite this expression without exponents as $p \cdot p \cdot p \ldots$, how many factors would you need?

9. (I.52.10) Graph the system of equations shown at right. What special relationship exists between the two lines? Confirm this by solving the equations algebraically.

$$\begin{cases} 3x - y = 10 \\ 6x = 20 + 2y \end{cases}$$

Problem-Based Mathematics I

1. (I.53.3) Faced with the problem of calculating $\left(5^4\right)^3$, Brook is having trouble deciding which of these three answers is correct: 5^{64}, 5^{12}, or 5^7. Once you have answered Brook's question, experiment with other examples of this type until you are ready to formulate the principle that tells how to write $\left(b^m\right)^n$ as a power of b.

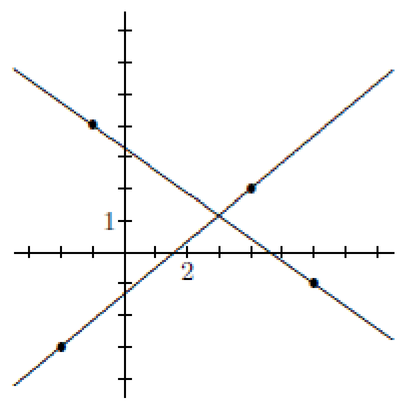
2. (I.53.4) The diameter of an atom is so small that it would take about 10^8 of them, arranged in a line, to span one centimeter. It is thus a plausible estimate that a cubic centimeter contains about $10^8 \times 10^8 \times 10^8 = \left(10^8\right)^3$ atoms. Write this huge number as a power of 10.

3. (I.53.2) During a phone call about the system of equations $\{5x + 2y = 8,\ 8x + 4y = 8\}$, Dylan told Max, "It's easy, just set them equal to each other." But Max replied, "That doesn't help—I get $-2y = 3x$. What good is that?" Help these two students solve the problem.

4. (I.54.1) A math teacher is designing a test, and wants (3, −4) to be the solution to the system of equations $\{3x - 5y = a,\ 7x + y = b\}$. What values should the teacher use for a and b?

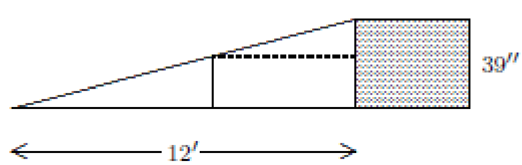
5. (I.54.8) Find k so that the three equations $3x - y = 2$, $2x + 8 = 3y$, and $y = kx$ have a common solution.

6. (I.54.2) The figure at right shows the graphs of two lines. Use the figure to estimate the coordinates of the point that belongs to both lines, then calculate the exact value. You will of course have to find equations for the lines, which both go through designated lattice points.



7. (I.54.4) A square can be formed from one x^2-block, a hundred x-blocks and a certain number of 1-blocks. How many 1-blocks? Show how to do it. What are the dimensions of the square? Fill in the blanks in the equation $x^2 + 100x + \_\_\_\_ = (\quad)(\quad) = (\quad)^2$.

8. (I.54.5) The figure shows a loading dock and a side view of an attached ramp, whose run is 12 feet and whose rise is 39 inches. Alex is wondering whether a long rectangular box can be stored underneath the ramp, as suggested by the dotted lines. The box is 2 feet tall and 5 feet long. Answer Alex's question.



9. (I.55.3) You have one x^2-block and $2n$ x-blocks, where n is a positive whole number. How many 1-blocks do you need to make a square? What are the dimensions of the square? Fill in the blanks in the equation $x^2 + 2nx + \_\_\_\_ = (\quad)(\quad) = (\quad)^2$.

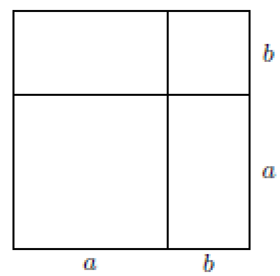
1. (I.55.5) Jan had the same summer job for the years 2008 through 2011, earning \$250 in 2008, \$325 in 2009, \$400 in 2010, and \$475 in 2011.

(a) Plot the four data points.
(b) If the points are connected, what is the slope of the line? What does the slope represent?
(c) Should the points be connected? Why or why not?
(d) Guess what Jan's earnings were for 2007 and 2013, assuming the same summer job.
(e) Write an inequality that states that Jan's earnings in 2013 were within 10% of the amount you guessed.

2. (I.55.7) Replace the triangles in $\dfrac{x\cdot x\cdot x\cdot x\cdot x\cdot x\cdot x}{x\cdot x\cdot x\cdot x} = x^{\Delta}$ and $\dfrac{6^9}{6^4} = 6^{\nabla}$ by correct exponents.

3. (I.56.1) Rewrite each of the following polynomials as a product of two factors. One of the factors should be the greatest common monomial factor.
(a) $24x^2 + 48x + 72$ (b) $\pi r^2 + \pi re$ (c) $7m - 14m^2 + 21m^3$

4. (I.56.6) Refer to the diagram at right, which shows a large square that has been subdivided into two squares and two rectangles. Write formulas for the areas of these four pieces, using the dimensions *a* and *b* marked on the diagram. Then write an equation that states that the area of the large square is equal to the combined area of its four pieces. Do you recognize this equation?



5. (I.56.8) Find the equations of at least three lines that intersect each other at the point $(6, -2)$.

6. (I.56.9) Driving along Route 8 one day, a math teacher reached the railroad crossing in Shaler at exactly the same time as a long freight train. While waiting patiently for the caboose to finally arrive and pass, the teacher decided to estimate the length of the train, which seemed to be moving at about 10 miles per hour. Given that it was a five-minute wait, how many feet did the teacher estimate the length of the train to be?

7. (I.57.5) Factor the following: (a) $2x^2 - 4x$ (b) $x^2 + 24x + 144$ (c) $x^2 + 3x$

8. (I.57.7) Faced with the problem of dividing 5^{24} by 5^8, Brook is having trouble deciding which of these four answers is correct: 5^{16}, 5^3, 1^{16}, and 1^3. Your help is needed. Once you have answered Brook's question, experiment with other examples of this type until you are ready to formulate the *common-base principle for division* that tells how to divide b^m by b^n to get another power of *b*. Then apply this principle to the following situations:
(a) Earth's human population is roughly 7×10^9, and its total land area, excluding the polar caps, is roughly 5×10^7 square miles. If the human population were distributed uniformly over all available land, approximately how many persons would be found per square mile?
(b) At the speed of light, which is 3×10^8 meters per second, how many seconds does it take for the Sun's light to travel the 1.5×10^{11} meters to Earth?

Problem-Based Mathematics I

1. (I.58.6) In each of the following, find the correct value for ∇:

(a) $y^4y^7 = y^\nabla$ (b) $y^{12}y^\nabla = y^{36}$ (c) $y^4y^4y^4y^4 = y^\nabla$ (d) $\left(y^\nabla\right)^3 = y^{27}$

2. (I.58.2) Given the equation $3x + y = 6$, write a second equation that, together with the first, will create a system of equations that
(a) has one solution;
(b) has an infinite number of solutions;
(c) has no solution;
(d) has the ordered pair (4, −6) as its only solution.

3. (I.58.7) There are 55 ways to make $x^\heartsuit x^\diamondsuit x^\clubsuit = x^{12}$ an identity, by assigning positive integers to the heart, diamond, and club. Find four of them.

4. (I.58.9) Find three consecutive odd numbers whose sum is 627.

5. (I.58.4) Factor the following *perfect-square trinomials*:

(a) $x^2 - 12x + 36$ (b) $x^2 + 14x + 49$ (c) $x^2 - 20x + 100$

As suggested, these should all look like either $\left(x-r\right)^2$ or $\left(x+r\right)^2$. State the important connection between the *coefficients* of the given trinomials and the values you found for *r*.

6. (I.58.5) (Continuation) In the following, choose *k* to create a perfect-square trinomial:

(a) $x^2 - 16x + k$ (b) $x^2 + 10x + k$ (c) $x^2 - 5x + k$

7. (I.59.1) The distance from here to downtown Pittsburgh is 10 miles. If you walked there at 4 mph and returned jogging at 8 mph, how much time would the round trip take? What would your overall average speed be?

8. (I.59.3) Given that three shirts cost *d* dollars,
(a) How many dollars does one shirt cost?
(b) How many dollars do *k* shirts cost?

9. (I.59.6) Ten cc of a solution of acid and water is 30% acid. I wish to dilute the acid in the mixture by adding water to make a mixture that is only 6% acid. How much pure water must I add to accomplish this?

10. (I.59.8) What are the dimensions of a square that encloses the same area as a rectangle that is two kilometers long and one kilometer wide? Answer to the nearest millimeter.

1. (I.60.1) When I ask my calculator for a decimal value of $\sqrt{2}$, it displays 1.41421356237. What is the meaning of this number? To check whether this square root is correct, what needs to be done? Can the square root of 2 be expressed as a ratio of whole numbers—for example as $\frac{17}{12}$? Before you say "impossible", consider the ratio $\frac{665857}{470832}$. By the way, a number expressible as a ratio of whole numbers is called *rational*. All other numbers are called *irrational*.

2. (I.60.2) What happens if you try to find an intersection point for the linear graphs $3x-2y=10$ and $3x-2y=-6$? What does this mean?

3. (I.60.3) A jeweler has 10 ounces of an alloy that is 50% gold. How much more pure gold does the jeweler need to add to this alloy to increase the percentage of gold to 60%?

4. (I.60.4) Evaluate $6-4/2+2\cdot 5$ and then check using your calculator. Show how the insertion of parentheses can make the value of the expression equal to (a) 1 (b) −14 (c) 25.

5. (I.60.8) Pat and Kim are having an algebra argument. Kim is sure that $-x^2$ is equivalent to $(-x)^2$, but Pat thinks otherwise. How would you resolve this disagreement? What evidence does your calculator offer?

6. (I.61.1) What is the value of $\frac{5^7}{5^7}$? of $\frac{8^3}{8^3}$? of $\frac{c^{12}}{c^{12}}$? What is the value of any number divided by itself? If you apply the common-base rule dealing with exponents and division, $\frac{5^7}{5^7}$ should equal 5 raised to what power? and $\frac{c^{12}}{c^{12}}$ should equal c raised to what power? It therefore makes sense to define c^0 to be what?

7. (I.62.1) Write the following monomials without using parentheses:
(a) $(ab)^2(ab^2)$ (b) $(-2xy^4)(4x^2y^3)$ (c) $(-w^3x^2)(-3w)$ (d) $(7p^2q^3r)(7pqr^4)^2$

8. (I.62.2) Complete the table at right. Then graph by hand on separate axes $y=|x|$ and $y=x^2$. Check your graphs with your calculator. In what respects are the two graphs similar? In what respects do the two graphs differ?

x	$\lvert x\rvert$	x^2
-2		
-1		
$-1/2$		
0		
$1/2$		
1		
2		

9. (I.62.5) A box with a square base and rectangular sides is to be 2 feet and 6 inches high, and to contain 25.6 cubic feet. What is the length of one edge of the square base?

Problem-Based Mathematics I

1. (I.62.9) Water pressure varies directly with the depth of submersion. Given that a diver experiences 64 pounds per square inch of pressure at a depth of 100 feet, what pressure will a submarine encounter when it is one mile below the surface of the Atlantic Ocean?

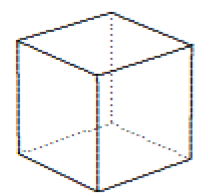
2. (I.64.2) Evaluate each of the following expressions by *substituting* $s = 30$ and $t = -4$

(a) $t^2 + 5t + s$ (b) $2t^2 s$ (c) $3t^2 - 6t - 2s$ (d) $s - 0.5t^2$

3. (I.63.5) When two rational numbers are multiplied together, their product is also a rational number. Explain. Is it necessarily true that the product of two irrational numbers is irrational? Explain. In some cases, the product of two irrationals has a predictable value. For each of the following, guess what the product will be, then confirm your guess with the help of your calculator.

(a) $\sqrt{3} \cdot \sqrt{27}$ (b) $\sqrt{2} \cdot \sqrt{6} \cdot \sqrt{3}$ (c) $\sqrt{6} \cdot \sqrt{12}$ (d) $\left(\sqrt{6}\right)^3$ (e) $\sqrt{3} \cdot \left(\sqrt{3}\right)^2$

4. (I.65.7) The total area of six faces of a cube is 1000 sq cm. What is the length of one edge of the cube? Round your answer to three decimal places.



5. (I.65.9) Solve each of the following equations. Give decimal answers, accurate to three decimal places.

(a) $x^2 = 11$ (b) $5s^2 - 101 = 144$ (c) $x^2 = 0$ (d) $30 = 0.4m^2 + 12$

6. (I.64.6) Perform the indicated operations, and record your observations:

(a) $\sqrt{2} \cdot \sqrt{18}$ (b) $\sqrt{8} \cdot \sqrt{8}$ (c) $2\sqrt{5} \cdot 3\sqrt{20}$

Suggest a rule for multiplying numbers in the form $\sqrt{a} \cdot \sqrt{b}$. Extend your rule to problems in the form $p\sqrt{a} \cdot q\sqrt{b}$.

7. (I.64.7) (Continuation) Use what you have just seen to explain why $\sqrt{20} = \sqrt{4} \cdot \sqrt{5} = 2\sqrt{5}$. Rewrite the following square roots in the same way—as the product of a whole number and a square root of an integer *that has no perfect square factors*. The resulting expression is said to be in *simplest radical form*.

(a) $\sqrt{50}$ (b) $\sqrt{108}$ (c) $\sqrt{125}$ (d) $\sqrt{128}$

8. (I.65.1) At Sam's Warehouse, a member pays \$25 a year for membership, and buys at the regular store prices. A non-member does not pay the membership fee, but does pay an additional 5% above the store prices. Under what conditions would it make sense to buy a membership?

Problem-Based Mathematics I

1. (I.66.4) When asked to solve the equation $(x-3)^2 = 11$, Jess said, "That's easy—just take the square root of both sides." Perhaps Jess also remembered that 11 has two square roots, one positive and the other negative. What are the two values for *x*, in exact form? (In this situation, "exact" means no decimals.)

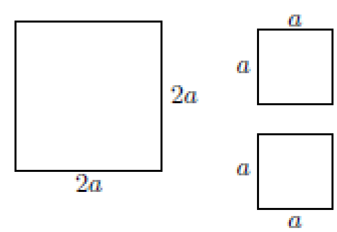
2. (I.66.5) (Continuation) When asked to solve the equation $x^2 - 6x = 2$, Deniz said, "Hmm ... not so easy, but I think that adding something to both sides of the equation is the thing to do." This is indeed a good idea, but what number should Deniz add to both sides? How is this equation related to the previous one?

3. (I.66.7) Suppose that *m* and *n* stand for positive numbers, with $n < m$. Which of the following expressions has the largest value? Which one has the smallest value?

(a) $\dfrac{m+1}{n+1}$ (b) $\dfrac{m+1}{n}$ (c) $\dfrac{m}{n}$ (d) $\dfrac{m}{n+2}$ (e) $\dfrac{m}{n+1}$

4. (I.67.6) Solving a *quadratic equation* by rewriting the left side as a perfect-square trinomial is called solving by *completing the square*. Use this method to solve each of the following equations. Leave your answers in exact form.
(a) $x^2 - 8x = 3$ (b) $x^2 + 10x = 11$ (c) $x^2 - 5x - 2 = 0$ (d) $x^2 + 1.2x = 0.28$

5. (I.68.9) Write $(2a)^2$ without parentheses. Is $(2a)^2$ larger than, smaller than, or the same as $2a^2$? Make reference to the diagram at right in writing your answer. Draw a similar diagram to illustrate the non-equivalence of $(3a)^2$ and $3a^2$.



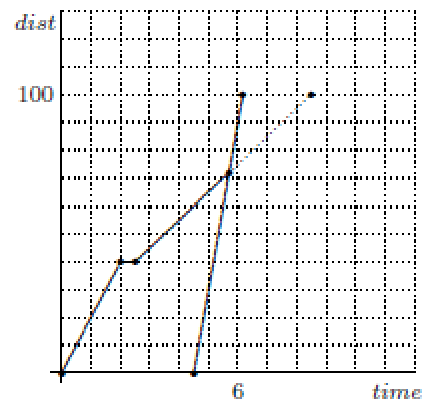
6. (I.67.1) Use your calculator to evaluate the following: (a) $\dfrac{\sqrt{50}}{\sqrt{2}}$ (b) $\dfrac{\sqrt{28}}{\sqrt{7}}$ (c) $\dfrac{\sqrt{294}}{\sqrt{6}}$

Explain why your results make it reasonable to write $\dfrac{\sqrt{a}}{\sqrt{b}} = \sqrt{\dfrac{a}{b}}$. Check that this rule also works for: (d) $\dfrac{\sqrt{48}}{\sqrt{6}}$ (e) $\dfrac{\sqrt{84}}{\sqrt{12}}$ (f) $\dfrac{\sqrt{180}}{\sqrt{15}}$.

7. (I.67.2) *Rationalizing denominators*. How are the decimal approximations for $\dfrac{6}{\sqrt{6}}$ and $\sqrt{6}$ related? Was this predictable? Verify that the decimal approximations for $\dfrac{1}{\sqrt{8}}$ and $\dfrac{\sqrt{2}}{4}$ are equal. Was this predictable? What is the effect of multiplying $\dfrac{1}{\sqrt{8}}$ and $\dfrac{\sqrt{2}}{\sqrt{2}}$? To show equivalence of expressions, you might have to transform one *radical expression* to make it look like another.

1. (I.67.3) Without a calculator, decide whether the first expression is equivalent to the second:
(a) $\sqrt{75}$ and $5\sqrt{3}$ (b) $\frac{\sqrt{800}}{2}$ and $10\sqrt{2}$ (c) $\frac{2}{\sqrt{8}}$ and $\frac{\sqrt{2}}{2}$ (d) $\sqrt{\frac{1000}{6}}$ and $\frac{10\sqrt{15}}{3}$

2. (I.67.4) At noon one day, Allie left home to make a long bike ride to the family camp on Mud Lake, a distance of 100 km. Later in the day, the rest of the family packed some things into their van and drove to the lake along Allie's bike route. They overtook Allie after driving for 1.2 hrs, stopped long enough to put Allie and the bicycle in the van, and continued to the camp. Refer to the graph as you answer the following questions about the day's events:

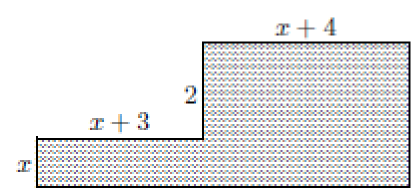


(a) Allie pedaled at two different rates during the biking part of the trip. What were they?
(b) After biking for a while, Allie stopped to take a rest. How far from home was Allie then? How long was the rest?
(c) How far from home was Allie when the family caught up?
(d) At what time did the family arrive at the camp?
(e) At what time would Allie have arrived, if left to bicycle all the way?
(f) What distance separated Allie and the rest of the family at 5 pm?

3. (I.67.5) Use the distributive property to factor each of the following
(a) $x^2 + x^3 + x^4$ (b) $\pi r^2 + 2\pi rh$ (c) $25x - 75x^2$ (d) $px + qx^2$

4. (I.71.1) Given $P = (1, 4)$, $Q = (4, 5)$, and $R = (10, 7)$, decide whether or not PQR is a straight line, and give your reasons.

5. (I.69.3) In the diagram, the dimensions of a piece of carpeting have been marked in terms of x. All lines meet at *right angles*. Express the area and the perimeter of the carpeting in terms of x.

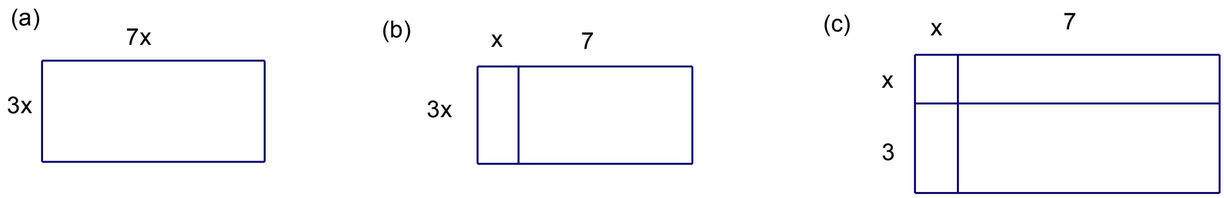


6. (I.69.6) The work at right shows the step-by-step process used by a student to solve $x^2 + 6x - 5 = 0$ by the method of completing the square. Explain why the steps in this process are reversible. Apply this understanding to find a quadratic equation $ax^2 + bx + c = 0$ whose solutions are $x = 7 + \sqrt{6}$ and $x = 7 - \sqrt{6}$.

$$x^2 + 6x - 5 = 0$$
$$x^2 + 6x + 9 = 5 + 9$$
$$(x+3)^2 = 14$$
$$x + 3 = \pm\sqrt{14}$$
$$x = -3 \pm \sqrt{14}$$

Problem-Based Mathematics I

1. (I.69.9) Express the areas of the following large rectangles in two ways. First, find the area of each small rectangle and add the expressions. Second, multiply the total length by the total width.



2. (I.70.8) (Continuation) Multiply: (a) $(3x)(7x)$ (b) $(3x)(7+x)$ (c) $(3+x)(7+x)$

3. (I.71.7) Given the equation $s = \pi r + \pi r e$, solve the formula for: (a) *e* (b) *r*.

4. (I.76.1) Perform the indicated operations and combine like terms where possible:
(a) $(x+7)(x-6)$ (b) $(x-5)^2$ (c) $(x+9)(x-9)$

5. (I.77.1) Use the distributive property to multiply $(x+p)(x+q)$. The result of this multiplication can be expressed in the form $x^2 + \nabla x + \Delta$; what do ∇ and Δ stand for?

6. (I.77.2) (Continuation) When attempting to factor $x^2 + 5x + 4$ into a product of two binomials of the form $(x+p)(x+q)$, Dylan set up the identity $x^2 + 5x + 4 = (x+\quad)(x+\quad)$. Using a trial-and-error process, try to figure out what numbers go in the blank spaces. What is the connection between the numbers in the blank spaces and the coefficients 5 and 4 in the quadratic expression being factored?

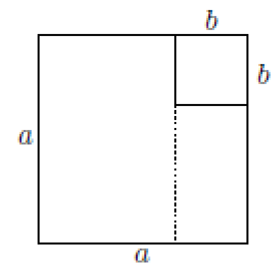
7. (I.77.3) (Continuation) Use the same trial-and-error process to express each of the following trinomials as a product of two binomials.
(a) $x^2 + 6x + 5$ (b) $x^2 - 7x + 12$ (c) $x^2 + 3x - 4$ (d) $x^2 - x - 6$

8. (I.76.3) Jess bought a can of paint, whose label stated that the contents of the can were sufficient to cover 150 square feet. To the nearest inch, what are the dimensions of the largest square that Jess could cover using this paint?

9. (I.77.4) Solve the following quadratic equations:
(a) $x^2 + 6x + 5 = 0$ (b) $x^2 - 7x + 12 = 0$ (c) $x^2 + 3x - 4 = 0$ (d) $x^2 - x - 6 = 0$

10. (I.78.2) Write in as compact form as possible:
(a) $x^4 \cdot \frac{1}{x^3}$ (b) $\left(\frac{2}{x^3}\right)^4$ (c) $(2x + x + 2x)^3$ (d) $\frac{x^6}{x^2}$

11. (I.78.5) By using square roots, express the solutions to $(x-5)^2 - 7 = 0$ exactly (no decimals).



1. (I.78.6) By rearranging the pieces of the puzzle shown at right, show that $a^2 - b^2$ is equivalent to $(a+b)(a-b)$.

2. (I.78.7) Expand the following products:
(a) $(x-4)(x+4)$ (b) $(7+x)(7-x)$ (c) $(3x-2)(3x+2)$
Use the pattern to predict the factors of $x^2 - 64$ and $4x^2 - 25$. Explain why this pattern is called *the difference of two squares*.

3. (I.79.1) The *degree* of a monomial counts how many variable factors would appear if it were written without using exponents. For example, the degree of $6ab$ is 2, and the degree of $25x^3$ is 3, since $25x^3 = 25xxx$. The degree of a polynomial is the largest degree found among its monomial terms. Find the degree of the following polynomials:
(a) $x^2 - 6x$ (b) $5x^3 - 6x^2$ (c) $x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$ (d) $4\pi r^2 h$

4. (I.79.9) The product of two polynomials is also a polynomial. Explain. When a polynomial of degree 3 is multiplied by a polynomial of degree 2, what is the degree of the result?

5. (I.82.3) The *period* of a pendulum is the time T it takes for it to swing back and forth once. This time (measured in seconds) can be expressed as a function of the pendulum length L, measured in feet, by the physics formula $T = \frac{1}{4}\pi\sqrt{2L}$.
(a) To the nearest tenth of a second, what is the period for a 2-foot pendulum?
(b) To the nearest inch, how long is a pendulum whose period is 2.26 seconds?

6. (I.83.9) Factor:
(a) $x^2 - 81$ (b) $4x^2 - 81$ (c) $81 - x^2$ (d) $0.04x^2 - 81$

7. (I.79.5) Plot a point near the upper right corner of a sheet of graph paper. Move your pencil 15 graph-paper units (squares) to the left and 20 units down, then plot another point. Use your ruler to measure the distance between the points. Because the squares on your graph paper are probably larger or smaller than the squares on your classmates' graph paper, it would not be meaningful to compare ruler measurements with anyone else in class. You should therefore finish by converting your measurement to graph-paper units.

8. (I.79.6) (Continuation) Square your answer (in graph-paper units), and compare the result with the calculation $15^2 + 20^2$.

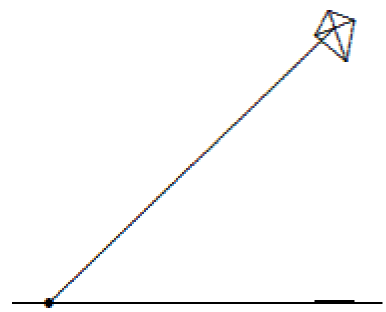
9. (I.79.7) (Continuation) Repeat the entire process, starting with a point near the upper left corner, and use the instructions "20 squares to the right and 21 squares down." You should find that the numbers in this problem again fit the equation. These are instances of the *Pythagorean Theorem*, which is a statement about right-angled triangles. Write a clear statement of this useful result. You will need to refer to the longest side of a right triangle, which is called the *hypotenuse*.

1. (I.80.3) Starting at school, you and a friend ride your bikes in different directions—you ride 4 blocks north and your friend rides 3 blocks west. At the end of this adventure, how far apart are you and your friend?

2. (I.80.4) From the library, you ride your bike east at a rate of 10 mph for half an hour while your friend rides south at a rate of 15 mph for 20 minutes. How far apart are you? How is this problem similar to the preceding problem? How do the problems differ?

3. (I.80.6) Imagine a circle of rope, which has twelve evenly spaced knots tied in it. Suppose that this rope has been pulled into a taut, triangular shape, with stakes anchoring the rope at knots numbered 1, 4, and 8. Make a conjecture about the angles of the triangle.

4. (I.81.4) While flying a kite at the beach, you notice that you are 100 yards from the kite's shadow, which is directly beneath the kite. You also know that you have let out 150 yards of string. How high is the kite?

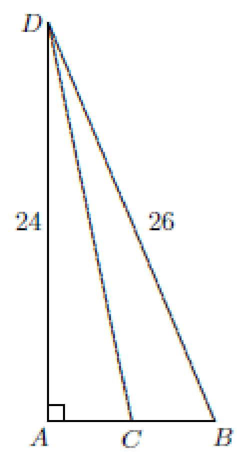


5. (I.81.5) Starting from home, Jamie haphazardly walks 2 blocks north, 3 blocks east, 1 block north, 3 blocks east, 1 block north, 5 blocks east, and 1 block north. How far is Jamie from home if each block is 150 meters long?

6. (I.81.6) The sides of Fran's square are 5 cm longer than the sides of Tate's square. Fran's square has 225 sq cm more area. What is the area of Tate's square?

7. (I.82.4) A football field is a rectangle, 300 feet long (from goal to goal) and 160 feet wide (from sideline to sideline). To the nearest foot, how far is it from one corner of the field (on one of the goal lines) to the furthest corner of the field (on the other goal line)?

8. (I.81.7) In the figure at right, BAD is a right angle, and C is the midpoint of *segment* AB. Given the dimensions marked in the figure, find the length of CD.



9. (I.83.11) The expression $4x+3x$ can be combined into one term, but $4x+3y$ cannot. Explain. Can $4\sqrt{5}+3\sqrt{5}$ be combined into one term? Can $\sqrt{2}+\sqrt{2}$ be combined into one term? Can $\sqrt{2}+\sqrt{3}$ be combined into one term? At first glance, it may seem that $\sqrt{2}+\sqrt{8}$ cannot be combined into one term. Take a close look at $\sqrt{8}$ and show that $\sqrt{2}+\sqrt{8}$ can in fact be combined.

10. (I.84.3) Because $\sqrt{8}$ can be rewritten as $2\sqrt{2}$, the expression $\sqrt{8}+5\sqrt{2}$ can be combined into a single term $7\sqrt{2}$. Combine each of the following into one term, without using a calculator:
(a) $\sqrt{12}+\sqrt{27}$ (b) $\sqrt{63}-\sqrt{28}$ (c) $\sqrt{6}+\sqrt{54}+\sqrt{150}$ (d) $2\sqrt{20}-3\sqrt{45}$

1. (I.84.5) Solve each of the following for x. Leave your answers in exact form.
(a) $x\sqrt{2} = \sqrt{18}$ (b) $x\sqrt{6} = -\sqrt{30}$ (c) $\sqrt{2x} = 5$ (d) $2\sqrt{5x} = \sqrt{30}$

2. (I.84.6) Show by finding examples that it is hardly ever true that $\sqrt{a+b}$ is the same as $\sqrt{a} + \sqrt{b}$.

3. (I.84.8) Given that $\sqrt{72} + \sqrt{50} - \sqrt{18} = \sqrt{h}$, find h without using a calculator.

4. (I.85.3) A sign going down a hill on Route 910 says "8% grade. Trucks use lower gear." The hill is a quarter of a mile long. How many vertical feet will a truck descend while going from the top of the hill to the bottom?

5. (I.85.6) Given that $\sqrt{k} = \sqrt{2} + \sqrt{2} + \sqrt{2} + \sqrt{2} + \sqrt{2}$, find the value of k without using a calculator.

6. (I.86.2) Find $\sqrt{4 + \frac{1}{16}}$ on your calculator. Is the result equivalent to $\sqrt{4} + \sqrt{\frac{1}{16}}$? Explain.

7. (I.86.10) What is the distance from the point (4, 2) to the point (−3, −2)? Be prepared to explain your method.

8. (I.87.1) Calculate the following distances, and briefly explain your method:
(a) from (2, 1) to (10, 10) (b) from (-2, 3) to (7, -5)
(c) from (0, 0) to (9, 8) (d) from (4, -3) to (-4, 6)

9. (I.87.4) Pat and Kim are having an algebra argument. Pat is sure that $\sqrt{x^2}$ is equivalent to x, but Kim thinks otherwise. How would you resolve this disagreement?

10. (I.87.7) The distance from (0, 0) to (8, 6) is exactly 10.
(a) Find coordinates for all the lattice points that are exactly 10 units from (0, 0).
(b) Find coordinates for all the lattice points that are exactly 10 units from (−2, 3).

11. (I.87.8) Given four numbers a, b, c, and d, one can ask for the distance from (a, b) to (c, d). Write a procedure for computing this distance, using the four numbers.

12. (I.88.3) Find the distance from $P = (3,\ 1)$ to $Q = (x,\ 1)$; from $P = (3,\ 1)$ to $Q = (x,\ y)$.

13. (I.88.4) Complete the following, without using any *variable* names: Given two points in a coordinate plane, you find the distance between them by ____________________.

1. (I.88.5) Both legs of a right triangle are 8 cm long. In simplest radical form, how long is the hypotenuse? How long would the hypotenuse be if both legs were k cm long?

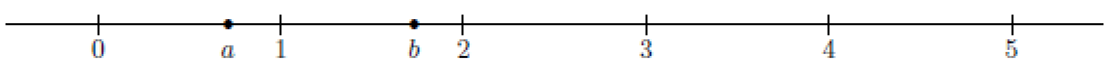
2. (I.88.6) The hypotenuse of a right triangle is twice as long as the shortest side, whose length is m. In terms of m, what is the length of the intermediate side?

3. (I.88.7) Can you find integer lengths for the legs of a right triangle whose hypotenuse has length $\sqrt{5}$? What about $\sqrt{7}$? Explain your reasoning.

4. (II.13.9) The diagonal of a rectangle is 15 cm, and the perimeter is 38 cm. What is the area? It is possible to find the answer without finding the dimensions of the rectangle—try it.

5. (I.88.8) Find as many points as you can that are exactly 25 units from (0, 0). How many of them are lattice points?

6. (I.88.9) On the number line shown below, a is a number between 0 and 1, and b is a number between 1 and 2. Mark possible positions on this line for $\sqrt{a}, \sqrt{b}, a^2, b^2$, and $\sqrt{\dfrac{b}{a}}$.



7. (I.88.11) A triangle has $K = (3,1)$, $L = (-5,-3)$, and $M = (-8,3)$ for its vertices. Verify that the lengths of the sides of triangle KLM fit the Pythagorean equation.

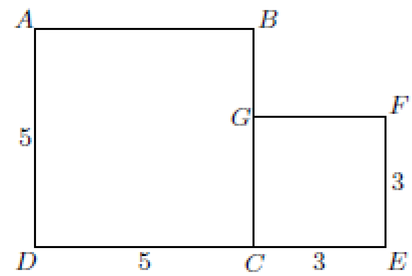
8. (I.88.13) How far is the point (5, 5) from the origin? Find two other first-quadrant lattice points that are exactly the same distance from the origin as (5, 5) is.

9. (I.89.6) What is the y-intercept of the line $ax + by = c$? What is the x-intercept?

10. (I.89.1) At noon one day, AJ decided to follow a straight course in a motor boat. After one hour of making no turns and traveling at a steady rate, the boat was 5 miles east and 12 miles north of its point of departure. What was AJ's position at two o'clock? How far had AJ traveled? What was AJ's speed?

11. (I.89.2) (Continuation) Assume that the gas tank initially held 12 gallons of fuel, and that the boat gets 4 miles to the gallon. How far did AJ get before running out of fuel? When did this happen? How did AJ describe the boat's position to the Coast Guard when radioing for help?

12. (I.90.7) Give an example of a line that is parallel to $2x + 5y = 17$. Describe your line by means of an equation. Which form for your equation is most convenient? Now find an equation for a line that is equidistant from your line and the line $2x + 5y = 41$.



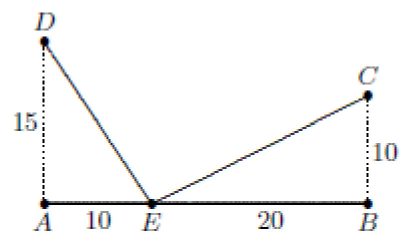
1. (II.1.1) A 5×5 square and a 3×3 square can be cut into pieces that will fit together to form a third square.
(a) Find the length of a side of the third square.
(b) In the diagram at right, mark *P* on segment *DC* so that *PD* = 3, then draw segments *PA* and *PF*. Calculate the lengths of these segments.
(c) Segments *PA* and *PF* divide the squares into pieces. Arrange the pieces to form the third square.

2. (II.1.2) (Continuation) Change the sizes of the squares to *AD* = 8 and *EF* = 4, and redraw the diagram. Where should point P be marked this time? Form the third square again.

3. (II.1.3) (Continuation) Will the preceding method *always* produce pieces that form a new square? If your answer is *yes*, prepare a written explanation. If your answer is *no*, provide a counterexample—two specific squares that *cannot* be converted to a single square.

4. (II.1.5) Let *A* = (0, 0), *B* = (7, 1), *C* = (12, 6), and *D* = (5, 5). Plot these points and connect the dots to form the *quadrilateral ABCD*. Verify that all four sides have the same length. Such a figure is called *equilateral*.

5. (II.1.6) The main use of the Pythagorean Theorem is to find distances. Originally (6th century BC), however, it was regarded as a statement about *areas*. Explain this interpretation.



6. (II.1.8) In the diagram, *AEB* is straight and angles *A* and *B* are right. Calculate the total distance *DE* + *EC*.

7. (II.1.9) (Continuation) If *AE* = 15 and *EB* = 15 instead, would *DE* + *EC* be the same?

8. (I.77.7) When taking an algebra quiz, Dale was asked to factor the trinomial $x^2 + x + 6$. Dale responded that this particular trinomial was not factorable. Decide whether Dale was correct, and justify your response.

9. (II.2.1) Two different points on the line *y* = 2 are each exactly 13 units from the point (7, 14). Draw a picture of this situation, and then find the coordinates of these points.

10. (II.2.2) Give an example of a point that is the same distance from (3, 0) as it is from (7, 0). Find lots of examples. Describe the configuration of all such points. In particular, how does this configuration relate to the two given points?

11. (II.2.3) Verify that the hexagon formed by *A* = (0, 0), *B* = (2, 1), *C* = (3, 3), *D* = (2, 5), *E* = (0, 4), and *F* = (−1, 2) is equilateral. Is it also *equiangular*?

1. (II.2.4) Draw a 20-by-20 square $ABCD$. Mark P on AB so that AP =8, Q on BC so that BQ = 5, R on CD so that CR =8, and S on DA so that DS = 5. Find the lengths of the sides of quadrilateral $PQRS$. Is there anything special about this quadrilateral? Explain.

2. (II.2.5) Verify that $P = (1, -1)$ is the same distance from $A = (5, 1)$ as it is from $B = (-1, 3)$. It is customary to say that P is *equidistant* from A and B. Find three more points that are equidistant from A and B. By the way, to "find" a point means to find its *coordinates*. Can points equidistant from A and B be found in every *quadrant*?

3. (II.2.8) If you were writing a geometry book, and you had to define a mathematical figure called a *kite*, how would you word your definition?

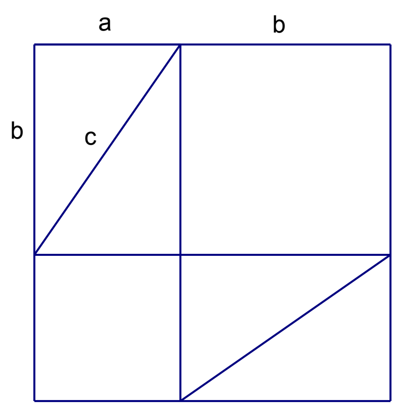
4. (II.2.9) Find both points on the line $y = 3$ that are 10 units from $(2, -3)$.

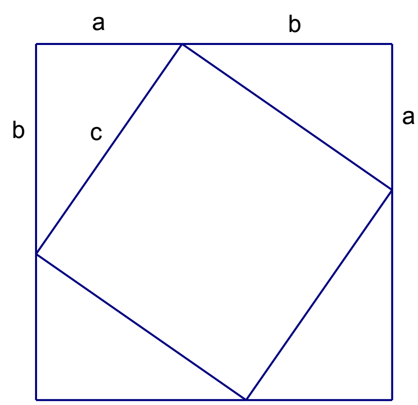
5. (II.2.10) On a number line, where is $\frac{1}{2}(p+q)$ in relation to p and q? What is a good name for this point? Experiment with various values of p and q to see this relationship.

6. (II.3.6) Find two points on the y-axis that are 9 units from (7, 5).

7. (II.4.11) What is the relation between the lines described by the equations $-20x+12y=36$ and $-35x+21y=63$? Find a third equation in the form $ax+by=90$ that fits this pattern.

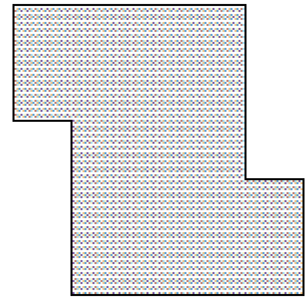
8. (II.2.6) The two-part diagram below, which shows two different dissections of the same square, was designed to help *prove* the Pythagorean Theorem. Provide the missing details.





9. (II.2.7) Inside a 5-by-5 square, it is possible to place four 3-4-5 triangles so that they do not overlap. Show how. Then explain why you can be sure that it is impossible to squeeze in a fifth triangle of the same size.

Problem-Based Mathematics I



1. (II.3.1) Some terminology: Figures that have exactly the same shape and size are called *congruent*. Dissect the region shown at right into two congruent parts. How many different ways of doing this can you find?

2. (II.3.2) Let $A = (2, 4)$, $B = (4, 5)$, $C = (6, 1)$, $T = (7, 3)$, $U = (9, 4)$, and $V = (11, 0)$. Triangles ABC and TUV are specially related to each other. Make calculations to clarify this statement, and write a few words to describe what you discover.

3. (II.3.3) A triangle that has two sides of equal length is called *isosceles*. Make up an example of an isosceles triangle, one of whose vertices is (3, 5). If you can, find a triangle that does not have any horizontal or vertical sides.

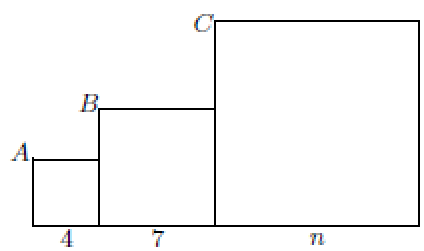
4. (II.3.5) Let $A = (1, 5)$ and $B = (3, -1)$. Verify that $P = (8, 4)$ is equidistant from A and B. Find at least two more points that are equidistant from A and B. Describe all such points.

5. (II.3.7) A lattice point is a point whose coordinates are integers. Find two lattice points that are exactly $\sqrt{13}$ units apart. Is it possible to find lattice points that are $\sqrt{15}$ units apart? Is it possible to form a square whose area is 18 by connecting four lattice points? Explain.

6. (II.5.8) Given that $2x - 3y = 17$ and $4x + 3y = 7$, and without using paper, pencil, or calculator, find the value of x.

7. (II.5.9) A slope can be considered to be a *rate*. Explain this interpretation.

8. (II.6.1) Three squares are placed next to each other as shown. The vertices A, B, and C are collinear. Find the dimension n. When geometric figures have the same shape and corresponding lengths are in proportion they are called *similar*.



9. (II.5.2) Consider the linear equation
$y = 3.62(x - 1.35) + 2.74.$
(a) What is the slope of this line?
(b) What is the value of y when $x = 1.35$?
(c) This equation is written in *point-slope* form. Explain the terminology.
(d) Use your calculator to graph this line.
(e) Find an equation for the line through $(4.23, -2.58)$ that is parallel to this line.

10. (II.6.3) A five-foot freshman casts a shadow that is 40 feet long while standing 200 feet from a streetlight. How high above the ground is the lamp?

11. (II.6.4) (Continuation) How far from the streetlight should the freshman stand, in order to cast a shadow that is exactly as long as the freshman is tall?

Some terminology: When two angles fit together to form a straight angle (a 180-degree angle, in other words), they are called *supplementary angles*, and either angle is the *supplement* of the other. When an angle is the same size as its supplement (a 90-degree angle), it is called a *right angle*. When two angles fit together to form a right angle, they are called *complementary angles*, and either angle is the *complement* of the other. Two lines that form a right angle are said to be *perpendicular*.

1. (II.3.8) The three angles of a triangle fit together to form a straight angle. In one form or another, this statement is a fundamental *postulate* of *Euclidean geometry*—accepted as true, without proof. Taking this for granted, then, what can be said about the two non-right angles in a right triangle?

2. (II.3.9) Let $P = (a, b)$, $Q = (0, 0)$, and $R = (-b, a)$, where a and b are positive numbers. Prove that angle PQR is right, by introducing two congruent right triangles into your diagram. Verify that the slope of *segment QP* is the *negative reciprocal* of the slope of segment QR.

3. (II.4.3) The point on segment AB that is equidistant from A and B is called the *midpoint* of AB. For each of the following, find coordinates for the midpoint of AB:

(a) $A = (-1, 5)$ and $B = (5, -7)$ (b) $A = (m, n)$ and $B = (k, l)$

4. (II.4.4) Write a formula for the distance from $A = (-1, 5)$ to $P = (x, y)$, and another formula for the distance from $P = (x, y)$ to $B = (5, 2)$. Then write an equation that says that P is equidistant from A and B. Simplify your equation to linear form. This line is called the *perpendicular bisector of AB*. Verify this by calculating two slopes and one midpoint.

5. (II.5.11) Given the points $A = (-2, 7)$ and $B = (3, 3)$, find two points P that are on the perpendicular bisector of AB. In each case, what can be said about the triangle PAB?

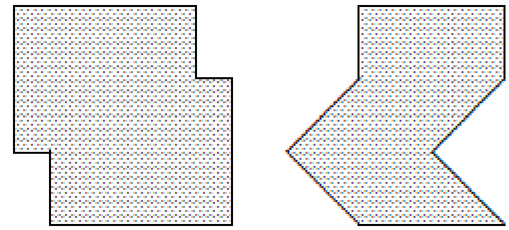
6. (II.5.12) Explain the difference between a line that has no slope and a line whose slope is zero.

7. (II.6.5) An airplane 27000 feet above the ground begins descending at the rate of 1500 feet per minute. Assuming the plane continues at the same rate of descent, how long will it be before it is on the ground?

8. (II.6.6) (Continuation) Graph the line $y = 27000 - 1500x$, using an appropriate window on your calculator. With the preceding problem in mind, explain the significance of the slope of this line and its two intercepts.

9. (II.6.7) An airplane is flying at 36000 feet directly above Lincoln, Nebraska. A little later the plane is flying at 28000 feet directly above Des Moines, Iowa, which is 160 miles from Lincoln. Assuming a constant rate of descent, predict how far from Des Moines the airplane will be when it lands.

Problem-Based Mathematics I



1. (II.6.9) Find a way to show that points $A = (-4, -1)$, $B = (4, 3)$, and $C = (8, 5)$ are collinear.

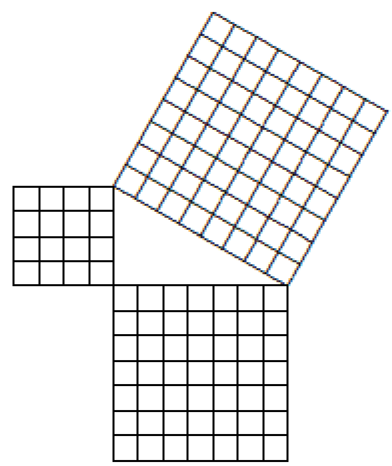
2. (II.6.10) Find as many ways as you can to dissect each figure at right into two congruent parts.

3. (II.7.6) Suppose that numbers a, b, and c fit the equation $a^2 + b^2 = c^2$, with $a = b$. Express c in terms of a. Draw a good picture of such a triangle. What can be said about its angles?

4. (II.7.8) Golf balls cost \$0.90 each at Jerzy's Club, which has an annual \$25 membership fee. At Rick & Tom's sporting-goods store, the price is \$1.35 per ball for the same brand. Where you buy your golf balls depends on how many you wish to buy. Explain, and illustrate your reasoning by drawing a graph.

5. (II.6.11) Let $A = (4, 2)$, $B = (11, 6)$, $C = (7, 13)$, and $D = (0, 9)$. Show that $ABCD$ is a square. Calculate the slopes of the sides of the square, and notice any significant patterns.

6. (II.8.1) Is it possible for a positive number to exceed its reciprocal by exactly 1? One number that comes close is $\frac{8}{5}$, because $\frac{8}{5} - \frac{5}{8}$ is $\frac{39}{40}$. Can you find a fraction that comes closer?



7. (II.7.2) Is there anything wrong with the figure shown at right?

8. (II.7.4) At noon one day, Corey decided to follow a straight course in a motor boat. After one hour of making no turns and traveling at a steady rate, the boat was 6 miles east and 8 miles north of its point of departure. What was Corey's position at two o'clock? How far had Corey traveled? What was Corey's speed?

9. (II.7.5) (Continuation) Assume that the fuel tank initially held 12 gallons, and that the boat gets 4 miles to the gallon. How far did Corey get before running out of fuel? When did this happen? When radioing the Coast Guard for help, how should Corey describe the boat's position?

10. (II.9.7) Let $A = (-5, 0)$, $B = (5, 0)$, and $C = (2, 6)$; let $K = (5, -2)$, $L = (13, 4)$, and $M = (7, 7)$. Verify that the length of each side of triangle ABC matches the length of a side of triangle KLM. Because of this data, it is natural to regard the triangles as being in some sense equivalent. It is customary to call the triangles *congruent*. The basis used for this judgment is called the *side-side-side* criterion. What can you say about the sizes of angles ACB and KML? What is your reasoning? What about the other angles?

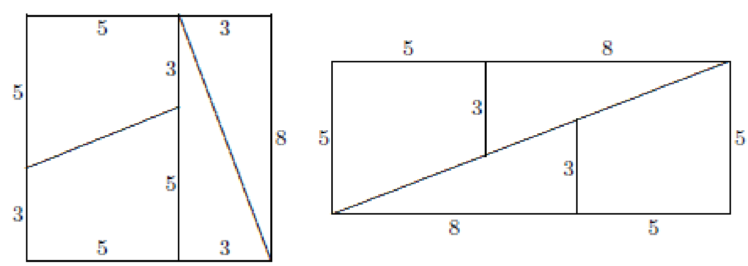
1. (II.10.1) In baseball, the bases are placed at the corners of a square whose sides are 90 feet long. Home plate and second base are at opposite corners. To the nearest eighth of an inch, how far is it from home plate to second base?

2. (II.10.6) Let $A = (0, 0)$, $B = (2, -1)$, $C = (-1, 3)$, $P = (8, 2)$, $Q = (10, 3)$, and $R = (5, 3)$. Plot these points. Angles BAC and QPR should look like they are the same size. Find evidence to support this conclusion.

3. (II.10.7) An equilateral quadrilateral is called a *rhombus*. A square is a simple example of a rhombus. Find a non-square rhombus whose *diagonals* and sides are *not* parallel to the rulings on your graph paper. Use coordinates to describe its vertices. Write a brief description of the process you used to find your example.

4. (II.10.8) Using a ruler and protractor, draw a triangle that has an 8-cm side and a 6-cm side, which make a 30-degree angle. This is a *side-angle-side* description. Cut out the figure so that you can compare triangles with your classmates. Will your triangles be congruent?

5. (II.10.9) Compare the two figures shown below. Is there anything wrong with what you see?



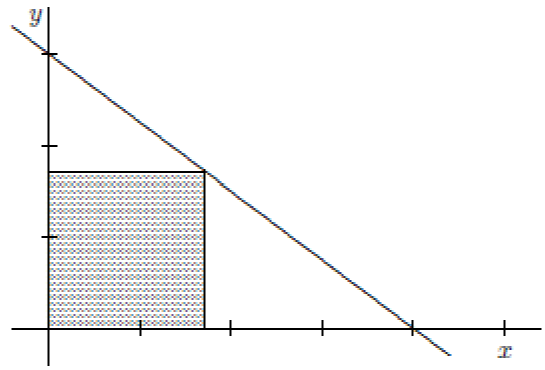
6. (II.11.4) With the aid of a ruler and protractor, draw a triangle that has an 8-cm side, a 6-cm side, and a 45-degree angle that is not formed by the two given sides. This is a *side-side-angle* description (SSA?). Cut out the figure so that you can compare triangles with your classmates. Do you expect your triangles to be congruent?

7. (II.11.7) What is the slope of the line $ax + by = c$? Find an *equation* for the line through the origin that is perpendicular to the line $ax + by = c$.

8. (II.12.5) With the aid of a ruler and protractor, draw and cut out three non-congruent triangles, each of which has a 40-degree angle, a 60-degree angle, and an 8-cm side. One of your triangles should have an *angle-side-angle* description, while the other two have *angle-angle-side* descriptions. What happens when you compare your triangles with those of your classmates?

9. (II.12.6) A triangle has six principal parts—three sides and three angles. The SSS criterion states that three of these items (the sides) determine the other three (the angles). Are there other combinations of three parts that determine the remaining three? In other words, if the class is given three measurements with which to draw and cut out a triangle, which three measurements will guarantee that everyone's triangles will be congruent?

1. (II.13.1) Let $A = (1, 4)$, $B = (0, -9)$, $C = (7, 2)$, and $D = (6, 9)$. Prove that angles *DAB* and *DCB* are the same size. Can anything be said about the angles *ABC* and *ADC*?



2. (II.13.4) The diagram at right shows the graph of $3x + 4y = 12$. The shaded figure is a square, three of whose vertices are on the coordinate axes. The fourth vertex is on the line. Find
(a) the *x*-and *y*-intercepts of the line;
(b) the length of the side of the square.

3. (II.13.5) (Continuation) Draw a rectangle that is twice as wide as it is tall, and that fits snugly into the triangular region formed by the line $3x + 4y = 12$ and the positive coordinate axes, with one corner at the origin and the opposite corner on the line. Find the dimensions of this rectangle.

4. (II.13.6) Plot the three points $P = (1, 3)$, $Q = (5, 6)$, and $R = (11.4, 10.8)$. Verify that $PQ = 5$, $QR = 8$, and $PR = 13$. What is special about these points?

5. (II.13.7) Sidney calculated three distances and reported them as $PQ = 29$, $QR = 23$, and $PR = 54$. What do you think of Sidney's data, and why?

6. (II.14.3) Given the points $K = (-2, 1)$ and $M = (3, 4)$, find coordinates for a point *J* that makes angle *JKM* a right angle.

7. (II.14.4) When two lines intersect, four angles are formed. It is not hard to believe that the nonadjacent angles (called *vertical angles*) in this arrangement are congruent. If you had to prove this to a skeptic, what reasons would you offer?

8. (II.14.7) Find a point on the line $y = 2x - 3$ that is 5 units from the *x*-axis.

9. (II.14.13) Given that $P = (-1, -1)$, $Q = (4, 3)$, $A = (1, 2)$, and $B = (7, k)$, find the value of *k* that makes the line *AB* (a) parallel to *PQ*; (b) perpendicular to *PQ*.

10. (II.14.14) Let $A = (-6, -4)$, $B = (1, -1)$, $C = (0, -4)$, and $D = (-7, -7)$. Show that the opposite sides of quadrilateral *ABCD* are parallel. Such a quadrilateral is called a *parallelogram*.

11. (II.15.10) Let $A = (3, 2)$, $B = (1, 5)$, and $P = (x, y)$. Find *x*-and *y*-values that make *ABP* a right angle.

12. (II.15.11) (Continuation) Describe the configuration of all such points *P*.

1. (II.19.1) An *altitude* of a triangle is a segment that joins one of the three vertices to a point on the line that contains the opposite side, the intersection being *perpendicular*. For example, consider the triangle whose vertices are $A = (0, 0)$, $B = (8, 0)$, and $C = (4, 12)$.

(a) Find the length of the altitude from C to side AB.
(b) Find an equation for the line that contains the altitude from A to side BC.
(c) Find an equation for the line BC.
(d) Find coordinates for the point where the altitude from A meets side BC.
(e) Find the length of the altitude from A to side BC.
(f) As a check on your work, calculate BC and multiply it by your answer to part (e). You should be able to predict the result. Hint: What is the formula for the area of a triangle?
(g) It is possible to deduce the length of the altitude from B to side AC from what you have already calculated. Show how.

2. (II.27.3) You have recently seen that there is no generally reliable SSA criterion for congruence. If the angle part of such a correspondence is a *right* angle, however, the criterion *is* reliable. Justify this so-called *hypotenuse-leg* criterion (which is abbreviated HL).

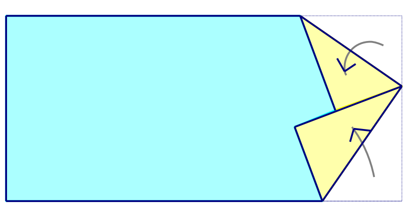
3. (II.19.7) How large a square can be put inside a right triangle whose legs are 5 cm and 12 cm?

4. (II.19.8) You are one mile from the railroad station, and your train is due to leave in ten minutes. You have been walking at a steady rate of 3 mph, and you can run at 8 mph if you have to. For how many more minutes can you continue walking, until it becomes necessary for you to run the rest of the way to the station?

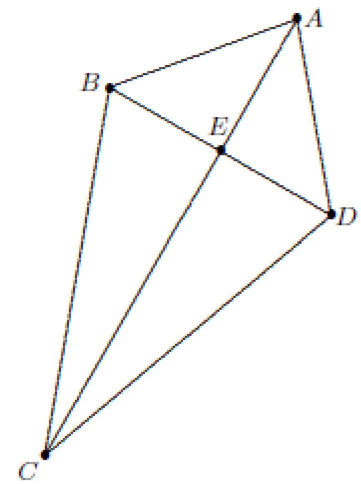
5. (II.20.4) A segment from one of the vertices of a triangle to the midpoint of the opposite side is called a *median*. Consider the triangle defined by $A = (-2, 0)$, $B = (6, 0)$, and $C = (4, 6)$.
(a) Find an equation for the line that contains the median drawn from A to BC.
(b) Find an equation for the line that contains the median drawn from B to AC.
(c) Find coordinates for G, the intersection of the medians from A and B. Do this by solving the system of equations from parts (a) and (b).
(d) Let M be the midpoint of AB. Determine whether or not M, G, and C are collinear.

6. (II.21.10) Prove that one of the diagonals of a kite bisects two of the angles of the kite. What about the other diagonal—must it also be an *angle bisector*? Explain your response.

7. Fold down a corner of a rectangular sheet of paper. Then, fold the next corner so that the edges touch. Measure the angle formed by the folded lines. Repeat with another sheet of paper, folding the corner at a different angle. What do you notice? Explain why the angles formed are congruent.



Here are two examples of proofs that do not use coordinates. Both proofs show how specific *given* information can be used to logically deduce *new* information. Each example concerns a *kite ABCD*, for which $AB = AD$ and $BC = DC$ is the given information. The first proof, which consists of simple text, shows that diagonal AC creates angles *BAC* and *DAC* of the same size.



Proof 1: Because AB = AD and BC = DC, and because the segment AC is shared by the triangles ABC and ADC, it follows from the SSS criterion that these triangles are congruent. Thus it is safe to assume that all the corresponding parts of these triangles are congruent as well (often abbreviated to *CPCTC*, as in proof 2 below.) In particular, angles BAC and DAC are the same size.

Now let E mark the intersection of diagonals AC and BD. The second proof, which is an example of a two-column proof, is written symbolically in outline form. It shows that the diagonals intersect perpendicularly. This proof builds on the first proof, which thus reappears as the first five lines.

Proof 2:

$AB = AD$	Given
$BC = DC$	Given
$AC = AC$	Shared side
$\Delta ABC \cong \Delta ADC$	SSS
$\angle BAC \cong \angle DAC$	CPCTC
E = intersection of AC and BD	
$AB = AD$	Given
$\angle BAE \cong \angle DAE$	Preceding CPCTC
$AE = AE$	Shared side
$\Delta ABE \cong \Delta ADE$	SAS
$\angle BEA \cong \angle DEA$	CPCTC
$\angle BEA$ and $\angle DEA$ supplementary	E is on BD
$\angle BEA$ is right	Definition of right angle

1. (II.18.1) In the fourth line, why would it have been wrong to write $\Delta ABC \cong \Delta ACD$?

2. (II.18.2) Refer to the kite data above and prove that angles *ABC* and *ADC* are also the same size.

3. (II.19.11) Refer to the data above and prove that one of the diagonals of a kite is bisected by the other.

1. (II.19.3) Let $A = (0, 0)$, $B = (8, 1)$, $C = (5, -5)$, $P = (0, 3)$, $Q = (7, 7)$, and $R = (1, 10)$. Prove that angles *ABC* and *PQR* have the same size.

2. (II.19.4) (Continuation) Let *D* be the point on segment *AB* that is exactly 3 units from *B*, and let *T* be the point on segment *PQ* that is exactly 3 units from *Q*. What evidence can you give for the congruence of triangles *BCD* and *QRT*?

3. During a two-week period, the number of cars in the student parking lot each day was: 85, 101, 92, 70, 101, 85, 122, 93, 79, 85. What is the mean number of cars for this period? What is the modal average? What is the median number of cars?

4. Combine over a common denominator:

(a) $\dfrac{3x}{5}+\dfrac{2x}{3}$ (b) $\dfrac{4}{3b}-\dfrac{7}{6b}+2$ (c) $\dfrac{6}{5a}+\dfrac{3}{10b}-1$

5. (II.21.11) Let $A = (2, 9)$, $B = (6, 2)$, and $C = (10, 10)$. Verify that segments *AB* and *AC* have the same length. Measure angles *ABC* and *ACB*. On the basis of your work, propose a general statement that applies to any triangle that has two sides of equal length. Prove your assertion, which might be called the *Isosceles-Triangle Theorem*.

6. (II.23.1) If triangle *ABC* is isosceles, with $AB = AC$, then the medians drawn from vertices *B* and *C* must have the same length. Write a *two-column proof* of this result.

7. Use properties of exponents to remove parentheses and write each expression using positive exponents only.

(a) $(3x^2y^3)^2$ (b) $\dfrac{12a^3b^7}{8ab^{15}}$ (c) $(2d^5e^3)(5d^2e^{-3})$

8. Translate the sentence "the distance between *x* and 12 is at least 30" into an equation using algebraic symbols. What are the values of *x* being described?

9. Translate "*x* is at most 12 units from 30" into an equation. What are the values of *x* being described?

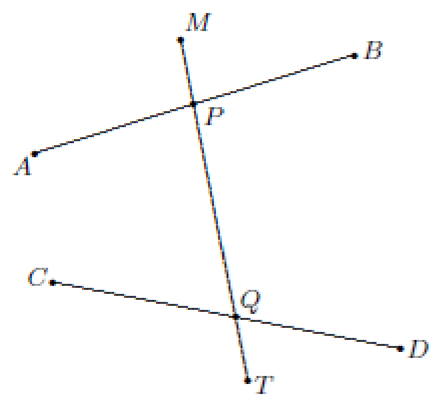
10. Recall the rule of thumb for estimating the conversion of degrees Celsius to degrees Fahrenheit. Approximately how many degrees Fahrenheit is 30° Celsius?

11. Solve the following equation for h: $d = ht^2 + h$

1. Alex starts making a triangular wind chime out of a 12-cm long steel rod by bending the rod 4 cm from one end.
(a) Describe one place Alex could make the second bend to form the triangle. Explain how you know a triangle will be formed.
(b) Describe all the places Alex could make a second bend to form a triangle.

2. (II.27.10) Suppose that triangle *PAB* is isosceles, with $AP = PB$, and that *C* is on side *PB*, between *P* and *B*. Show that $CB < AC$.

3. (II.31.1) The diagram at right shows lines *APB* and *CQD* intersected by line *MPQT*, which is called a *transversal*. There are two groups of angles: one group of four angles with vertex at *P*, and another group with vertex at *Q*. There is special terminology to describe pairs of angles, one from each group. If the angles are on different sides of the transversal, they are called *alternate*, for example *APM* and *PQD*. Angle *BPQ* is an *interior* angle because it is between the lines *AB* and *CD*, and angle *CQT* is *exterior*. Thus angles *APQ* and *PQD* are called *alternate interior*, while angles *MPB* and *CQT* are called *alternate exterior*. On the other hand, the pair of angles *MPB* and *PQD*—which are non-alternate angles, one interior, and the other exterior—is called *corresponding*.



Refer to the diagram and name
(a) the other pair of alternate interior angles;
(b) the other pair of alternate *exterior angles*;
(c) the angles that correspond to *CQT* and to *TQD*.

4. (II.31.2) Mark points $A = (1, 7)$ and $B = (6, 4)$ on your graph paper. Use your protractor to draw two lines of positive slope that make 40-degree angles with line *AB*—one through *A* and one through *B*. What can you say about these two lines, and how can you be sure?

5. (II.31.3) If one pair of alternate interior angles is equal, what can you say about the two lines that are crossed by the transversal? If one pair of corresponding angles is equal, what can you say about the two lines that are crossed by the transversal?

6. (II.31.4) If it is known that one pair of alternate interior angles is equal, what can be said about
(a) the other pair of alternate interior angles?
(b) either pair of alternate exterior angles?
(c) any pair of corresponding angles?
(d) either pair of non-alternate interior angles?

1. (II.31.5) You probably know that the sum of the angles of a triangle is a straight angle. One way to confirm this is to draw a line through one of the vertices, parallel to the opposite side. This creates some alternate interior angles. Finish the demonstration.

2. (II.31.6) Suppose that two of the angles of triangle *ABC* are known to be congruent to two of the angles of triangle *PQR*. What can be said about the third angles?

3. (II.31.7) Suppose that *ABCD* is a square, and that *CDP* is an equilateral triangle, with *P* *outside* the square. What is the size of angle *PAD*?

4. How many liters of a 70% alcohol solution must be mixed with 4 liters of a 25% solution to make a 34% solution?

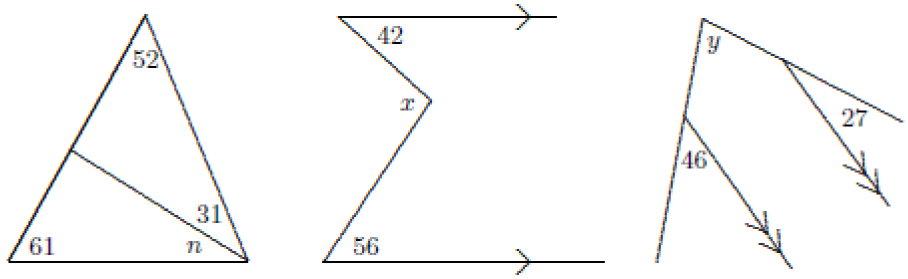
5. (II.32.2) Triangle *ABC* is isosceles, with *AB* congruent to *AC*. *Extend* segment *BA* to a point *T* (in other words, *A* should be between *B* and *T*). Prove that angle *TAC* must be twice the size of angle *ABC*. Angle *TAC* is called one of the *exterior angles* of triangle *ABC*.

6. (II.32.3) If *ABC* is any triangle, and *TAC* is one of its exterior angles, then what can be said about the size of angle *TAC*, in relation to the other angles of the figure?

7. Solve the following equation for h: $S = lw + 2lh + 2wh$

8. (II.32.5) Given an arbitrary triangle, what can you say about the *sum* of the three exterior angles, one for each vertex of the triangle?

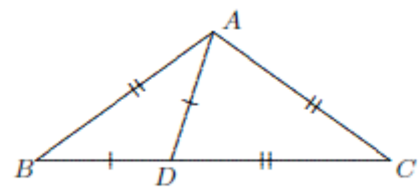
9. (II.32.6) In the diagrams below, the goal is to find the sizes of the angles marked with letters, using the given numerical information. Angles are measured in degrees. Notice the custom of marking arrows on lines to indicate that they are known to be parallel.



10. (II.32.8) Recall that a quadrilateral that has two pairs of parallel opposite sides is called a *parallelogram*. What can be said about the angles of such a figure?

11. (II.32.11) Prove that the sum of the angles of any quadrilateral is 360 degrees. What about the sum of the angles of a pentagon? a hexagon? a 57-sided polygon?

1. (II.33.2) Given parallelogram *PQRS*, let *T* be the intersection of the bisectors of angles *P* and *Q*. Without knowing the sizes of the angles of *PQRS*, calculate the size of angle *PTQ*.



2. (II.33.5) In the figure at right, it is given that *BDC* is straight, $BD = DA$, and $AB = AC = DC$. Find the size of angle *C*.

3. (II.33.6) Mark the point *P* inside square *ABCD* that makes triangle *CDP* equilateral. Calculate the size of angle *PAD*.

4. (II.33.7) The *converse* of a statement of the form "If A then B" is the statement "If B then A."
(a) Write the converse of the statement "If point *P* is equidistant from the coordinate axes, then point *P* is on the line $y = x$."
(b) Give an example of a true statement whose converse is false.
(c) Give an example of a true statement whose converse is also true.

5. (II.33.8) If a quadrilateral is a parallelogram, then both pairs of opposite angles are congruent. What is the converse of this statement? If you think that the converse is true, then prove it; if not, explain why not.

6. (II.34.1) In regular pentagon *ABCDE*, draw diagonal *AC*. What are the sizes of the angles of triangle *ABC*? Prove that segments *AC* and *DE* are parallel.

7. Given $A = (-3,10)$ and $B = (9,-2)$. Find the equation, in point-slope form, of the perpendicular bisector of AB.

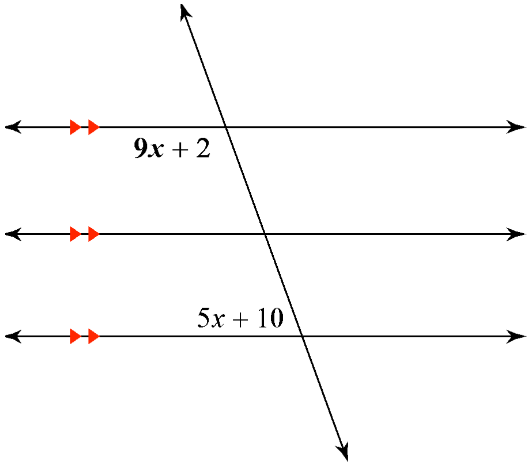
8. (II.34.3) The sides of an equilateral triangle are 12 cm long. How long is an altitude of this triangle? What are the angles of a right triangle created by drawing an altitude? How does the short side of this right triangle compare with the other two sides?

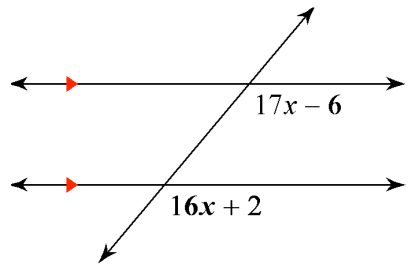
9. (II.34.4) If a quadrilateral is a parallelogram, then both pairs of opposite sides are congruent. Explain. What is the converse of this statement? Is it true?

1. (II.34.6) In triangle *ABC*, it is given that angle *A* is 59 degrees and angle *B* is 53 degrees. The altitude from *B* to line *AC* is extended until it intersects the line through *A* that is parallel to segment *BC*; they meet at *K*. Calculate the size of angle *AKB*.

2. (II.34.10) If the diagonals of a quadrilateral bisect each other, then the figure is a parallelogram. Prove that this is so. What about the converse statement?

3. Solve for x.





4. (II.34.11) Suppose that one of the medians of a triangle happens to be exactly half the length of the side to which it is drawn. What can be said about the angles of this triangle? Justify your response.

5. (II.34.12) (Continuation) Prove that the midpoint of the hypotenuse of a right triangle is equidistant from all three vertices of the triangle. How does this statement relate to the preceding?

6. (II.35.12) A right triangle has a 24-cm perimeter, and its hypotenuse is twice as long as its shorter *leg*. To the nearest tenth of a cm, find the lengths of all three sides of this triangle.

7. (II.34.13) Tate walks along the boundary of a four-sided plot of land, writing down the number of degrees turned at each corner. What is the sum of these four numbers?

8. (II.34.14) How can one tell whether a given quadrilateral is a parallelogram? In other words, how much evidence is needed to be sure of such a conclusion?

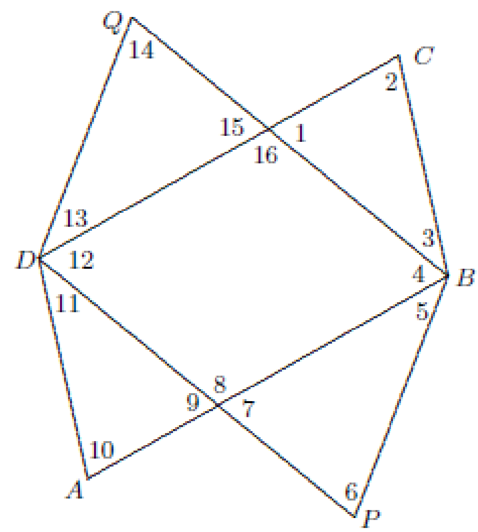
9. (II.38.11) Suppose that quadrilateral *ABCD* has the property that *AB* and *CD* are congruent and parallel. Is this enough information to prove that *ABCD* is a parallelogram? Explain.

1. (II.35.9) Jackie walks along the boundary of a five-sided plot of land, writing down the number of degrees turned at each corner. What is the sum of these five numbers?

2. (II.35.10) Marty walks along the boundary of a seventy-sided plot of land, writing down the number of degrees turned at each corner. What is the sum of these seventy numbers?

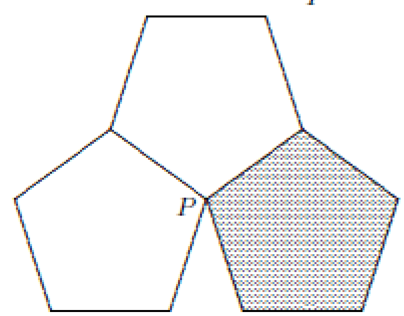
3. (II.35.11) The preceding two questions illustrate the *Sentry Theorem*. What does this theorem say, and why has it been given this name?

4. (II.37.2) In the figure at right, it is given that *ABCD* and *PBQD* are parallelograms. Which of the numbered angles must be the same size as the angle numbered 1?



5. (II.39.3) There are four special types of lines associated with triangles: Medians, perpendicular bisectors, altitudes, and angle bisectors.
(a) Which of these lines *must* go through the vertices of the triangle?
(b) Is it possible for a median to also be an altitude? Explain.
(c) Is it possible for an altitude to also be an angle bisector? Explain.

6. (II.38.1) The diagram at right shows three congruent regular pentagons that share a common vertex *P*. The three polygons do not quite surround *P*. Find the size of the uncovered acute angle at *P*.



7. (II.38.2) (Continuation) If the shaded pentagon were removed, it could be replaced by a regular *n*-sided polygon that would exactly fill the remaining space. Find the value of *n* that makes the three polygons fit perfectly.

8. (II.39.4) The diagonals of a rhombus have lengths 18 and 24. How long are the sides of the rhombus?

9. (II.40.8) Is it possible for the sides of a triangle to be 23, 19, and 44? Explain.

10. (II.42.6) How many diagonals can be drawn inside a pentagon? a hexagon? a *decagon*? A twenty-sided polygon? an *n*-sided polygon?

11. (II.44.7) Given regular hexagon *BAGELS*, show that *SEA* is an equilateral triangle.

1. (II.48.3) Hexagon *ABCDEF* is regular. Prove that segments *AE* and *ED* are perpendicular.

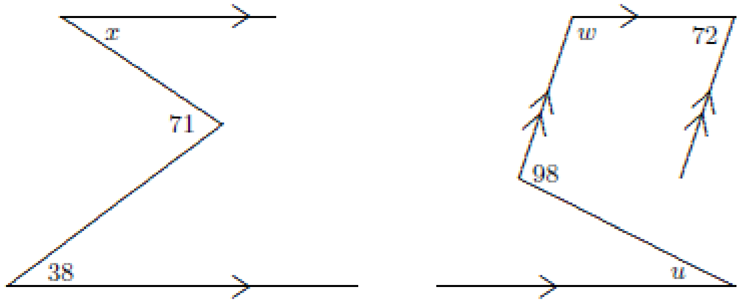
2. (II.48.4) Suppose that *PQRS* is a rhombus, with *PQ* = 12 and a 60-degree angle at *Q*. How long are the diagonals *PR* and *QS*?

3. (II.50.7) The hypotenuse of a right triangle is twice as long as one of the legs. How long is the other leg? What is the size of the smallest angle?

4. (II.53.9) Show that the altitude drawn to the hypotenuse of any right triangle divides the triangle into two triangles that have the same angles as the original.

5. (II.32.4) Given triangle *ABC*, with *AB* = *AC*, extend segment *AB* to a point *P* so that *BP* = *BC*. In the resulting triangle *APC*, show that angle *ACP* is exactly three times the size of angle *APC*. (By the way, notice that extending segment *AB* does *not* mean the same thing as extending segment *BA*.)

6. (II.33.3) In the figures below, find the sizes of the angles indicated by letters:



Problem-Based Mathematics I Extra Problems

1. (I.1.2) How long would it take you to count nonstop to one billion, if you counted by ones? First, write a guess into your notebook. Now try to solve the problem. One approach is to actually do it and have someone time you. There is another way to approach the problem, however. What do you need to know? What assumptions are you making?

2. (I.1.3) It takes 1.25 seconds for light to travel from the Moon to the Earth. How many miles away is the Moon?

3. (I.1.5) You have perhaps heard the saying, “A journey of 1000 miles begins with a single step.” How many steps would you take to finish a journey of 1000 miles? What information do you need in order to answer this question? Find a reasonable answer. What would your answer be if the journey were 1000 kilometers?

4. (I.1.6) In an offshore pipeline, a cylindrical mechanism called a “pig” is run through the pipes periodically to clean them. These pigs travel at 2 feet per second. What is this speed, expressed in miles per hour?

5. (I.2.8) Explain why there are two ways to compute each of the following:

a) $3(2+3+5)$ b) $\frac{1}{3}(9+6-3)$ c) $(9+6-3)\div 3$

6. (I.3.4) To buy a ticket for a weekly state lottery, a person selects 6 integers from 1 to 36, the order not being important. There are 1947792 such combinations of six digits. Alex and nine friends want to win the lottery by buying every possible ticket (all 1947792 combinations), and plan to spend 16 hours a day doing it. Assume that each person buys one ticket every five seconds. What do you think of this plan? Can the project be completed within a week?

7. (I.4.6) The distributive property states that $(-1)x+1x$ is the same as $(-1+1)x$, and this is 0. It follows that $(-1)x$ is the same as $-x$. Explain why, then use similar reasoning to explain why $(-x)y$ is the same as $-(xy)$. By the way, is it correct to say that “$-x$ is a negative number”?

8. (I.4.7) This year there are 497 students at the Academy of whom 45 live in Croft Hall. To the nearest tenth of a percent, what part of the student population lives in Croft?

9. (I.5.5) Here is another number puzzle: Pick a number, add 5 and multiply the result by 4. Add another 5 and multiply the result by 4 again. Subtract 100 from your result and divide your answer by 8. How does your answer compare to the original number? You may need to do a couple of examples like this until you see the pattern. Use a variable for the chosen number and show how the pattern holds for any number.

10. (I.5.6) (Continuation) Make up a number puzzle of your own. Be able to verify the pattern using a variable for the number chosen initially.

11. (I.6.4) Using the four integers 1, 2, 3 and 4 once each — in any order — and three arithmetic operations selected from among addition, subtraction, multiplication, and division, is it possible to write an expression whose value is 1? Using the same numbers and conditions, how many of the integers from 1 to 10 can you form? You will need to use parentheses.

Problem-Based Mathematics I Extra Problems

1. (I.7.7) True or false, with justification: $\frac{7}{12}+\frac{11}{12}+\frac{1}{12}+\frac{19}{12}$ is equivalent to $\frac{1}{12}(7+11+1+19)$.

2. (I.7.10) If m and n stand for integers, then $2m$ and $2n$ stand for even integers. Explain. Use the distributive property to show that the sum of any two even numbers is even.

3. (I.7.11) (Continuation) Show that the sum of any two odd numbers is even.

4. (I.8.1) Jan has a $18''\times 18''\times 12''$ gift box that needs to be placed carefully into a $2'\times 2'\times 2'$ shipping carton, surrounded by packing peanuts.
(a) How many 1-cubic-foot bags of peanuts does Jan need to buy?
(b) Jan opens one bag of peanuts and spreads them evenly on the bottom of the shipping carton. What is the resulting depth of the peanuts?
(c) Jan centers the square base of the gift box on the peanut layer, pours in another bag of peanuts, and spreads them around evenly. Now how deep are the peanuts?
(d) Explain why the third bag of peanuts will cover the gift box.

5. (I.8.5) During a recent episode of Who Wants to Be a Billionaire, your friend Terry called you, needing help with solving the equation $5x+1=2x+7$. Write down the step-by-step instructions you would give Terry over the phone.

6. (I.8.7) Several Form III students were meeting in a room. After 45 of them left, the room was 5/8 as full as it was initially. How many Form III students were in the room at the start of the meeting?

7. (I.9.2) You measure your stride and find it to be 27 inches. If you were to walk to Sharpsburg, a town 4.5 miles west of Fox Chapel, how many steps would you have to take? Remember that there are 12 inches in a foot, 3 feet in a yard, and 5280 feet in a mile.

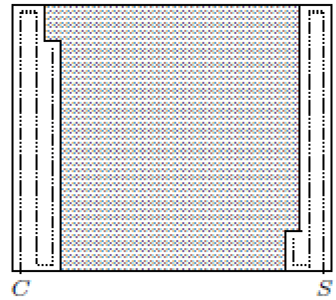
8. (I.10.3) At West Point, the “plebe” (first year cadet) who brings dessert to the table must divide it into pieces that are exactly the size requested by the cadets at the table. One night, the two seniors assigned to the table requested 1/6 of the pie and 1/5 of the pie, respectively. How much of the pie did that leave for the younger cadets?

9. (I.11.2) The sum of four consecutive integers is 2174. What are the integers?

10. (I.11.3) (Continuation) The smallest of four consecutive integers is n. What expression represents the next larger integer? Write an expression for the sum of four consecutive integers, the smallest of which is n. Write an equation that states that the sum of four consecutive integers is s. Solve the equation for n in terms of s. Check that your answer to the previous question satisfies this equation by considering the case $s = 2174$.

11. (I.11.5) There are three feet in a yard. Find the number of feet in 5 yards. Find the number of yards in 12 feet. Find the number of feet in y yards. Find the number of yards in f feet.

Problem-Based Mathematics I Extra Problems



1. (I.11.6) Sam and Cam have a lawn-mowing service. Their first job tomorrow morning is one that usually takes Sam 40 minutes to do alone, or Cam 30 minutes to do alone. This time they are going to team up, Sam starting at one side and Cam at the other side. The problem is to predict how many minutes it will take them to finish the job. What part of the lawn will Sam complete in the first ten minutes? What part of the lawn will Cam complete in the first ten minutes? What part of the lawn will the team complete in ten minutes? Set up a guess-and-check table with columns titled "minutes", "Sam part", "Cam part" and "Team part". What is the target value for the team part? Fill in two rows of the chart by making guesses in the minutes column. Then guess m and complete the solution algebraically.

2. (I.12.4) Find the smallest positive integer divisible by every positive integer less than or equal to 10.

3. (I.12.7) One of the SSA interscholastic teams has started its season badly, winning 1 game, losing 6, and tying none. The team will play a total of 25 games this season.
(a) What percentage of the seven games played so far have been wins?
(b) Starting with its current record of 1 win and 6 losses, what will the cumulative winning percentage be if the team wins the next 4 games in a row?
(c) Starting with its current record of 1 win and 6 losses, how many games in a row must the team win in order for its cumulative winning percentage to reach at least 60%?
(d) Suppose that the team wins ten of its remaining 18 games. What is its final winning percentage?
(e) How many of the remaining 18 games does the team need to win so that its final winning percentage is at least 60%? Is it possible for the team to have a final winning percentage of 80%? Explain.

4. (I.13.5) Last year, three fifths of the Outing Club were girls, but this year the number of boys doubled and six new girls joined. There are now as many boys in the club as there are girls. How many members did the club have last year?

5. (I.13.8) Place a common mathematical symbol between the numerals 2 and 3, so as to produce a number that lies between 2 and 3 on a number line.

6. (I.13.7) The base of a rectangular tank is three feet by two feet, and the tank is three feet tall. The water in the tank is currently just nine inches deep.
(a) The water level will rise when a one-foot metal cube (denser than water) is placed on the bottom of the tank. By how much?
(b) The water level will rise some more when a second one-foot metal cube is placed on the bottom of the tank, next to the first one. By how much?

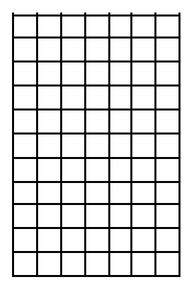
Problem-Based Mathematics I Extra Problems

1. (I.13.4) *Guessing birthdays*. Pat is working a number trick on Kim, whose birthday is the 29th of February. The table below shows the sequence of questions that Pat asks, as well as the calculations that Kim makes in response. Another column is provided for the algebra you are going to do to solve the trick. Use the letters m and d for month and day. After hearing the result of the last calculation, Pat can do a simple mental calculation and then state Kim's birthday. Explain how. To test your understanding of this trick, try it on someone whose birthday is

Instruction	Kim	Algebra
Write the number of your birthmonth	2	m
Multiply by 5	10	
Add 7	17	
Multiply by 4	68	
Add 13	81	
Multiply by 5	405	
Add the day of the month of your birthday	434	

unknown to you.

2. (I.15.1) A flat, rectangular board is built by gluing together a number of square pieces of the same size. The board is m squares wide and n squares long. Using the letters m and n, write expressions for
(a) the total number of 1×1 squares;
(b) the total number of 1×1 squares with free edges (the number of 1×1 squares that are not completely surrounded by other squares);
(c) the number of completely surrounded 1×1 squares;
(d) the perimeter of the figure.



3. (I.15.6) Ryan took 25 minutes to type the final draft of a 1200-word English paper. How much time should Ryan expect to spend typing the final draft of a 4000-word History paper?

4. (I.17.4) Chase began a number puzzle with the words "Pick a number, add 7 to it, and double the result." Chase meant to say, "Pick a number, double it, and add 7 to the result." Are these two instructions equivalent? Explain.

5. (I.17.8) Alden paid to have some programs printed for the football game last weekend. The printing cost per program was 54 cents, and the plan was to sell them for 75 cents each. Poor weather kept many fans away from the game, however, so unlucky Alden was left with 100 unsold copies, and lost \$12 on the venture. How many programs did Alden have printed?

6. (I.17.9) The Mount Major hike starts in Alton Bay, 716 feet above sea level. The summit is 1796 feet above sea level, and it takes about 45 minutes for a typical hiker to make the climb. Find the rate at which this hiker gains altitude, in feet per minute.

7. (I.18.2) Pat bought several pens at Walgreen's for 60 cents each. Spending the same amount of money at the Tuck Shop, Pat then bought a few more pens that cost 80 cents each. In all, 42 pens were bought. How many pens did Pat buy at the Tuck Shop?

Problem-Based Mathematics I Extra Problems

1. (I.19.2) A ladder is leaning against the side of a building. Each time I step from one rung to the next, my foot moves 6 inches closer to the building and 8 inches further from the ground. The base of the ladder is 9 ft from the wall. How far up the wall does the ladder reach?

2. (I.19.8) A sign placed at the top of a hill on Route 910 says "8% grade. Trucks use lower gear." What do you think that "8% grade" might mean?

3. (I.21.5) Find an easy way to do the following calculations mentally: a) $25 \cdot 39 \cdot 4$ b) $\frac{632}{50}$

4. (I.21.9) Which is greater, 73 percent of 87, or 87 percent of 73?

5. (I.22.3) Suppose that n represents a positive even integer. What expression represents the next even integer? the next odd integer? I am thinking of three consecutive even integers, whose sum is 204. What are they?

6. (I.22.9) One year after Robin deposits 400 dollars in a savings account that pays r% annual interest, how much money is in the account? Write an expression using the variable r.

7. (I.23.1) How much time does it take for a jet to go 119 miles, if its speed is 420 mph? Be sure to specify the units for your answer.

8. (I.23.2) Find three consecutive odd numbers whose sum is 117. Think of two ways to do this problem.

9. (I.23.3) *Word chains*. As the ancient alchemists hoped, it is possible to turn *lead* into *gold*. You change one letter at a time, always spelling real words: lead—load—toad—told—gold. Using the same technique, show how to turn *work* into *play*.

10. (I.23.11) Without using parentheses, write an expression equivalent to $3(4(3x-6)-2(2x+1))$.

11. (I.23.12) One year after Robin deposits P dollars in a savings account that pays r% annual interest, how much money is in the account? Write an expression in terms of the variables P and r. If you can, write your answer using just a single P.

12. (I.24.6) Find and graph the solution sets for the related questions:
(a) $46 - 3(x + 10) = 5x + 20$ (b) $46 - 3(x + 10) < 5x + 20$ (c) $46 - 3(x + 10) > 5x + 20$

13. (I.24.7) At 1 pm, you start out on your bike at 12 mph to meet a friend who lives 8 miles away. At the same time, the friend starts walking toward you at 4 mph. How much time will elapse before you meet your friend? How far will your friend have to walk?

14. (I.24.9) Given that it costs \$2.75 less to buy a dozen doughnuts than to buy twelve single doughnuts, and that 65 doughnuts cost \$25.25, and that $65 = 5 \cdot 12 + 5$, what is the price of a single doughnut?

Problem-Based Mathematics I Extra Problems

1. (I.24.11) It takes ten people ten days to paint ten houses. How many houses can five people paint in five days?

2. (I.25.3) When it is eighty miles west of its destination, a jet is flying at 31680 feet. When it is forty miles west of its destination, the jet is at 11560 feet. Using this data, sketch a graph of the jet's descent. Is a linear model reasonable to use in this situation? Explain.

3. (I.25.5) According to the US Census Department, someone born in 1950 has a life expectancy of 68.2 years, while someone born in 1970 has a life expectancy of 70.8 years.
(a) What is a reasonable life expectancy of someone born in 1960?
(b) What is a reasonable life expectancy of someone born in 1980?
(c) What is a reasonable life expectancy of someone born in 2000? Part (a) is an *interpolation* question. Parts (b) and (c) are *extrapolation* questions. Which of your answers are you the most confident about? Explain.

4. (I.26.1) A square game board is divided into smaller squares, which are colored red and black as on a checkerboard. All four corner squares are black. Let r and b stand for the numbers of red and black squares, respectively. What is the value of the expression $b - r$?

5. (I.26.3) What is the slope between (3, 7) and (5, 4)? (5, 4) and (3, 7)? $(a, b)$and (c, d)? (c, d) and (a, b)?

6. (I.27.2) Given the five numbers 8/25, 13/40, 19/60, 33/100, and 59/180, find the two that are closest together on a number line, and find the distance between them.

7. (I.27.5) How far apart on a number line are (a) 12 and 18? (b) 12 and −7? (c) −11 and −4?

8. (I.27.11) Another word chain: Turn *big* into *red* into *win*. Change one letter at a time, always spelling real words.

9. (I.28.3) The table at right shows data that Morgan collected during a 10-mile bike ride that took 50 minutes. The cumulative distance (measured in miles) is tabled at ten-minute intervals.
(a) Make a scatter plot of this data. Why might you expect the data points to line up? Why do they not line up?
(b) Morgan's next bike ride lasted for 90 minutes. Estimate its length (in miles), and explain your method. What if the bike ride had lasted t minutes; what would its length be, in miles?

time	*dist*
0.0	0.0
10.0	2.3
20.0	4.4
30.0	5.7
40.0	8.2
50.0	10.0

10. (I.28.7) On a number line, how far is each of the following numbers from zero?
(a) 45 (b) −7 (c) x (d) $x + 2$ (e) 0

11. (I.29.3) Find an equation for the line that passes through the points (4.1, 3.2) and (2.3, 1.6).

Problem-Based Mathematics I Extra Problems

1. (I.29.5) Rearrange the eight words "between", "4", "the", "17", "is", "and", "*x*", and "distance" to form a sentence that is equivalent to the equation $|x-17|=4$. By working with a number line, find the values of *x* that fit the equation.

2. (I.29.7) A recent CNN poll about crime in schools reported that 67% of the persons polled were in favor of a crime bill currently being debated in Congress. CNN also reported that there was a 3% margin of error in the statistics. Explain why the range of possible percentage points can therefore be written as $|x-0.67| \le 0.03$.

3. (I.29.9) In 2004 a company had a profit of \$420000. In 2009 it reported a profit of \$1400000. Find the average rate of change of its profit for that period, expressed in dollars per year.

4. (I.30.1) Twelve flags are evenly spaced around a running track. Ryan started running at the first flag and took 30 seconds to reach the sixth flag. How many seconds did it take Ryan, running at a constant rate, to reach (a) the 10^{th} flag for the first time? (b) the 8^{th} flag for the 2^{nd} time? (c) the n^{th} flag for the m^{th} time?

5. (I.30.2) Translate the sentence "*x* and *y* are twelve units apart" into algebraic code. Find a pair (x, y) that fits this description. How many pairs are there?

6. (I.30.4) It is common practice to read $-75 < 2$ as "-75 is less than 2." Yet, in a significant sense, it is really -75 that is the larger of the two numbers! Discuss the two meanings of "less than."

7. (I.30.8) After successfully solving an absolute-value problem, Ariel spilled Heath Bar Crunch® all over the problem. All that can be read now is, "The distance between *x* and (mess of ice cream) is (another mess of ice cream)." Given that Ariel's answers are $x = -3$ and $x = 7$, reconstruct the missing parts of the problem.

8. (I.30.10) The *average* of three different positive integers is 8. What is the largest integer that could be one of them?

9. (I.31.9) Given that $0.0001 \le n \le 0.01$ and $0.001 \le d \le 0.1$, what are the largest and smallest values that the expression $\dfrac{n}{d}$ can possibly have? Write your answer $\textit{smallest} \le \dfrac{n}{d} \le \textit{largest}$.

10. (I.32.1) Show that the equation $y = \frac{7}{3}x - \frac{11}{8}$ can be rewritten in the standard form $ax + by = c$, in which *a*, *b*, and *c* are all integers.

11. (I.32.2) Fill in the blanks:
(a) The inequality $|x-1.96| < 1.04$ is equivalent to "*x* is between _________ and _________ ."
(b) The inequality $|x-2.45| \ge 4.5$ is equivalent to "*x* is not between _________ and _________."

12. (I.32.4) Solve the equation $0.05x + 0.25(30-x) = 4.90$. Invent a context for the equation.

Problem-Based Mathematics I Extra Problems

1. (I.32.9) I have 120 cm of framing material to make a picture frame, which will be most pleasing to the eye if its height is 2/3 of its width. What dimensions should I use?

2. (I.33.1) A horse thief riding at 8 mph has a 32-mile head start. The posse in pursuit is riding at 10 mph. In how many hours will the thief be overtaken? [From *The New Arithmetic*, Seymour Eaton, 1885]

3. (I.33.4) Draw the line through the point $(0, 6)$ whose slope is 2/3. If you move 24 units to the right of $(0, 6)$, and then move up to the line, what is the y-coordinate of the point you reach?

4. (I.33.5) (Continuation) Find an equation for the line. What is the x-intercept of the line?

5. (I.34.4) Given that y varies directly with x and that $y = 60$ when $x = 20$, find y when $x = 12$.

6. (I.34.7) Solve for x: $\frac{1}{2}(x-2)+\frac{1}{3}(x-3)+\frac{1}{4}(x-4)=10$.

7. (I.34.10) A chemist would like to dilute a 20-cc solution that is 5% acid to one that is 3% acid. How much water must be added to accomplish this task?

8. (I.35.5) If the width and length of a rectangle are both increased by 10%, by what percent does the area of the rectangle increase? By what percent does the perimeter of the rectangle increase?

9. (I.35.8) Morgan's way to solve the equation $|2x-7|=5$ is to first write $|x-3.5|=2.5$. Explain this approach, then finish the job.

10. (I.35.12) A train is leaving in 11 minutes and you are one mile from the station. Assuming you can walk at 4 mph and run at 8 mph, how much time can you afford to walk before you must begin to run in order to catch the train?

11. (I.36.2) For each of the following points, find the distance to the y-axis:
(a) $(11, 7)$ (b) $(-5, 9)$ (c) $(4, y)$ (d) $(x, -8)$

12. (I.36.3) To mail a first-class letter in 2006, the rate was 39 cents for the first ounce or fraction thereof, and 24 cents for each additional ounce or fraction thereof. Let p be the number of cents needed to mail a first-class letter that weighed w ounces. Graph p versus w, after first making a table that includes some non-integer values for w.

13. (I.36.9) A flat, rectangular board is built by gluing together a number of square pieces of the same size. The board is m squares wide and n squares long. In terms of m and n, write two different expressions for the number of completely surrounded squares.

14. (I.41.4) Jess and Wes used to race each other when they were younger. Jess could cover 8 meters per second, but Wes could cover only 5 meters per second, so Jess would sportingly let Wes start 60 meters ahead. They would both start at the same time and continue running until Jess caught up with Wes. How far did Jess run in those races?

Problem-Based Mathematics I Extra Problems

1. (I.41.1) Raisins make up two thirds of a well-mixed bowl of peanuts and raisins. If half the mixture is removed and replaced with just peanuts, what fraction of the bowl will be raisins?

2. (I.36.5) The table shows census data for New Hampshire since 1950.
(a) Write an equation for the line that contains the data points for 1950 and 2000.
(b) Write an equation for the line that contains the data points for 1990 and 2000.
(c) Make a scatter plot of the data. Graph both lines on it.
(d) Use each of these equations to predict the population of New Hampshire in 2020.
(e) For each prediction, explain why you could expect it to provide an accurate forecast.

year	*pop*
1950	533242
1960	606921
1970	746284
1980	920610
1990	1113915
2000	1238415

3. (I.37.2) Lee's pocket change consists of x quarters and y dimes. Put a dot on every lattice point (x, y) that signifies that Lee has exactly one dollar of pocket change. What equation describes the line that passes through these points? Notice that it does not make sense to connect the dots in this context, because x and y are *discrete* variables, whose values are limited to integers.

4. (I.37.3) (Continuation) Put a dot on every lattice point (x, y) that signifies that Lee has at most one dollar in pocket change. How many such dots are there? What is the relationship between Lee's change situation and the inequality $0.25x + 0.10y \leq 1.00$?

5. (I.37.4) (Continuation) Write two inequalities that stipulate that Lee cannot have fewer than zero quarters or fewer than zero dimes.

6. (I.37.8) Using four x-blocks:
(a) Draw a rectangle. Write the dimensions of your rectangle. What is its area?
(b) Draw a rectangle with dimensions different from those you used in part (a).

7. (I.37.9) The solution of $|x| = 6$ consists of the points 6 and −6. Show how to use a test point on the number line to solve and graph the inequality $|x| \leq 6$. Do the same for $|x| \geq 6$.

8. (I.38.9) Brett is holding three quarters and five dimes. Does Brett have more than one dollar or less than one dollar? Does the point (3, 5) lie above or below the line $0.25x+0.10y = 1.00$?

9. (I.39.5) Graph the equation $2x + 3y = 6$. Now graph the inequality $2x + 3y \leq 6$ by shading all points (x, y) that fit it. Notice that this means shading all the points on one side of the line you drew. Which side? Use a test point like (0, 0) to decide.

10. (I.39.9) Casey can peel k apples in 10 minutes.
(a) In terms of k, how many apples can Casey peel in one minute?
(b) How many apples can Casey peel in m minutes?
(c) In terms of k, how many minutes does it take Casey to peel one apple?
(d) How many minutes does it take Casey to peel p apples?

Problem-Based Mathematics I Extra Problems

1. (I.42.3) Three gears are connected so that two turns of the first wheel turn the second wheel nine times and three turns of the second wheel turn the third wheel five times.
(a) If you turn the first wheel once, how many times does the third wheel turn?
(b) How many times must you turn the first wheel so that the third wheel turns 30 times?

2. (I.42.7) Sam boards a ski lift, and rides up the mountain at 6 miles per hour. Once at the top, Sam immediately begins skiing down the same slope, averaging 54 miles per hour, and does not stop until reaching the entrance to the lift. The whole trip, up and down, takes 40 minutes. How many miles long is the trip down the mountain?

3. (I.42.11) Pat and Kim are walking in the same direction along Freeport Road at a rate of 4 mph. Pat started walking from the Giant Eagle at 8 am, and Kim left from the same spot 15 minutes later. On the same coordinate-axis system, draw two graphs — one for Pat and one for Kim — that plot the distance from walker to Giant Eagle versus time.

4. (I.43.4) In attempting to calculate the carrying capacity of a cylindrical pipe, Avery measured the outer diameter to be 2 inches, neglecting to notice that the pipe was one eighth of an inch thick. By what percent did Avery overestimate the carrying capacity of the pipe?

5. (I.44.9) Does every system of equations $px + qy = r$ and $mx + ny = k$ have a simultaneous solution (x, y)? Explain.

6. (I.45.6) The population of Fox Chapel is about 5000 people. The population of the United States is about 300 million persons. What percent of the US population lives in Fox Chapel?

7. (I.45.9) Sketch the region common to the graphs of $y \ge -1, y - 2x \le 3,$ and $x + y \le 6$. Find the area of this region.

8. (I.46.2) Start with the equations $2x - y = 3$ and $3x + 4y = 1$. Create a third equation by adding *any* multiple of the first equation to *any* multiple of the second equation. When you compare equations with your classmates, you will probably not agree. What is certain to be true about the graphs of *all* these third equations, however?

9. (I.46.3) The Appalachian Trail is a hiking trail that stretches 2158 miles from Georgia to Maine. The record for completing this hike in the shortest time is 52 days. The record-setting hikers averaged 3 miles per hour. How many hours a day did they average?

10. (I.46.5) A slow clock loses 25 minutes a day. At noon on the first of October, it is set to show the correct time. When will this clock next show the correct time?

11. (I.46.8) Cameron bought twelve pounds of candy corn for 79 cents a pound, and eighteen pounds of M&Ms for $1.09 a pound, planning to make packages of candy for the SSA football game. The two types of candy will be mixed and sold in one-pound bags. At what price must Cameron sell these thirty bags in order to make at least a 25% profit?

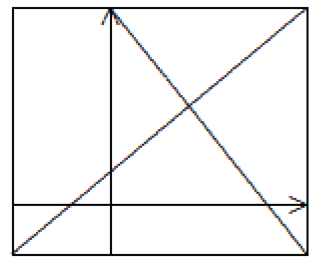
12. (I.47.6) If x varies directly with y, and if x =5 when y = 27, find x when y = 30.

Problem-Based Mathematics I Extra Problems

1. (I.47.10) The owner's manual for my computer printer states that it will print a page in 12 seconds. Re-express this speed in pages per minute, and in minutes per page.

2. (I.48.6) To run 10000 meters in a world-class time of 27:30 (27 minutes and 30 seconds), approximately what time should a competitor expect to hear at the 1600-meter mark?

3. (I.49.6) The diagram at right shows a calculator screen, on which the lines $5x+4y=32$ and $-5x+6y=8$ have been graphed. What are the window specifications for this picture? In other words, the window can be described by the two inequalities $a \le x \le b$ and $c \le y \le d$, using what numbers in place of a, b, c, and d?



4. (I.49.7) For the final in-class test this term I am thinking of giving a 100-question true-false test! Right answers will count one point, wrong answers will deduct half a point, and questions left unanswered will have no effect. One way to get a 94 using this scoring system is to answer 96 correctly and 4 incorrectly (and leave 0 blank). Find another way of obtaining a score of 94.

5. (I.49.8) (Continuation) Let r equal the number of right answers and w equal the number of wrong answers. Write an equation relating r and w that states that the test grade is 94. Write an inequality that states that the grade is at least 94, and graph it. Also graph the inequalities $0 \le r, 0 \le w,$ and $r+w \le 100,$ and explain why they are relevant here. Shade the region that solves all four inequalities. How many lattice points does this region contain? Why is this a lattice-point problem? What is the maximum number of wrong answers one could get and still obtain a grade at least as good as 94?

6. (I.51.6) Suppose that h is 40% of p. What percent of h is p?

7. (I.51.7) Pat is the CEO of Pat's Pickle-Packing Plant, but can still pack 18 jars of pickles per hour. Kim, a rising star in the industry, packs 24 jars per hour. Kim arrived at work at 9:00 am one day, to find that Pat had been packing pickles since 7:30 am. Later that day, Kim had packed exactly the same number of jars as Pat. At what time, and how many jars had each packed?

8. (I.51.10) A long-distance telephone call costs \$2.40 plus \$0.23 per minute. Write an inequality that states that an x-minute call costs at most \$5.00. Solve the inequality to find the maximum number of minutes that it is possible to talk without spending more than \$5.00.

9. (I.52.2) Taylor works after school in a health-food store, where one of the more challenging tasks is to add cranberry juice to apple juice to make a cranapple drink. A liter of apple juice costs \$0.85 and a liter of cranberry juice costs \$1.25. The mixture is to be sold for exactly the cost of the ingredients, at \$1.09 per liter. How many liters of each juice should Taylor use to make 20 liters of the cranapple mixture?

10. (I.42.5) How many nickels have the same combined value as q quarters and d dimes?

Problem-Based Mathematics I Extra Problems

1. (I.54.9) A catering company offers three monthly meal contracts:

Contract A costs a flat fee of \$480 per month for 90 meals;
Contract B costs \$200 per month plus \$4 per meal;
Contract C costs a straight \$8 per meal.
If you expect to eat only 56 of the available meals in a month, which contract would be best for you? When might someone prefer contract A? contract B? contract C?

2. (I.55.4) You are buying some cans of juice and some cans of soda for the dorm. The juice is \$0.60 per can while the soda is \$0.75. You have \$24 of dorm funds, all to be spent.
(a) Write an equation that represents all the different combinations you can buy for \$24.
(b) Is it possible to buy exactly 24 cans of juice and spend the remainder on soda? Explain.
(c) How many different combinations of drinks *are* possible?

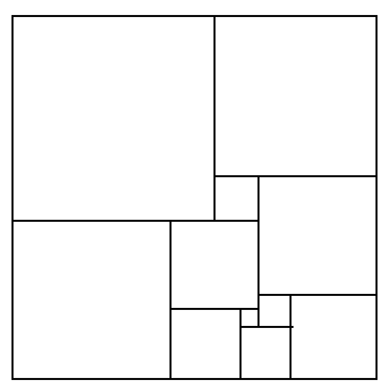
3. (I.56.3) *A number trick.* Arrange the nonnegative integers into seven infinite columns, as shown in the table at right. Without telling you what they are, someone selects two numbers, one from the 2-column (the column that contains 2) and one from the 5 column, and multiplies them. You predict the column in which the answer will be found. How?

0	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31	32	33	34
35	36	37	38	39	40	41
⋮	⋮	⋮	⋮	⋮	⋮	⋮

4. (I.56.7) Find coordinates for the point where the line $3x-2y=3001$ intersects the line $4x-3y=4001$. First solve the problem without using your calculator, then confirm your answer using your calculator.

5. (I.58.3) At noon, a team bus left Shady Side Academy for WRA. Soon thereafter, SSA's first-line player Brett Starr arrived at the gym. A loyal day student volunteered to overtake the bus and deliver Brett. The two left at 12:15 pm. The day student drove at 54 mph, while ahead of them the ancient yellow bus poked along at 48 mph. Did the car catch the bus before it reached WRA, which is 110 miles from SSA? If so, where and when?

6. (I.58.8) According to the US Census Bureau, the population of the USA has a net gain of 1 person every 14 seconds. How many additional persons does that amount to in one year?



7. (I.59.2) The diagram at right shows a rectangle that has been cut into eleven square pieces, no two being the same size. Given that the smallest piece is 9 cm by 9 cm, figure out the sizes of the other ten pieces. The original rectangle also looks like it could be square. Is it?

8. (I.59.7) Corey is out on the roads doing a long run, and also doing some mental calculations at the same time. Corey's pace is 3 strides per second, and each stride covers 5 feet.
(a) How much time does it take Corey to cover a mile?
(b) If Corey's stride increased to 5.5 feet per step, how much time would be needed to cover a mile? to run a marathon which is 26 miles and 385 yards?

1. (I.61.2) If $\sqrt{2}$ can be expressed as a ratio $\frac{m}{n}$ of two whole numbers, then this fraction can be put in lowest terms. Assume that this has been done.
(a) Square both sides of the equation $\sqrt{2} = \frac{m}{n}$.

(b) Multiply both sides of the new equation by n^2. The resulting equation tells you that m must be an even number. Explain.
(c) Because m is even, its square is divisible by 4. Explain.
(d) It follows that n^2 is even, hence so is n. Explain.
(e) Thus both m and n are even. Explain why is this a contradictory situation. A number expressible as a ratio of whole numbers is called *rational*. All other numbers, such as $\sqrt{2}$ are called *irrational*.

2. (I.61.4) One morning, Ryan remembered lending a friend a bicycle. After breakfast, Ryan walked over to the friend's house at 3 miles per hour, and rode the bike back home at 7 miles per hour, using the same route both ways. The round trip took 1.75 hours. What distance did Ryan walk?

3. (I.62.3) Taylor starts a trip to the mall with \$160 cash. After 20% of it is spent, seven-eighths of the remainder is lost to a pickpocket. This leaves Taylor with how much money?

4. (I.63.1) From the tombstone of Diophantus, a famous Greek mathematician: "God granted him to be a boy for a sixth part of his life, and, adding a twelfth part to this, He clothed his cheeks with down. He lit him the light of wedlock after a seventh part, and five years after this marriage He granted him a son. Alas! late-born wretched child—after attaining the measure of half his father's life, chill Fate took him. After consoling his grief by his science of numbers for four more years, then did Diophantus end his life." Calculate how old Diophantus lived to be.

5. (I.63.6) You have seen a demonstration that $\sqrt{2}$ is irrational. Give a similar demonstration that $\sqrt{3}$ is irrational.

6. (I.64.3) There are several positive integers that leave a remainder of 12 when they are divided into 192. Find the smallest and the largest of those integers.

7. (I.64.4) A student set out to bicycle from SSA to downtown, a distance of 10 miles. After going a short while at 15 miles per hour, the bike developed a flat tire, and the trip had to be given up. The walk back to SSA was made at a dejected 3 miles per hour. The whole episode took 48 minutes. How many miles from SSA did the flat occur?

8. (I.64.5) A car traveling at 60 miles per hour is covering how many feet in one second? A football field is 100 yards long. At 60 mph, how many seconds does it take to cover this distance? State your answer to the nearest tenth of a second.

Problem-Based Mathematics I Extra Problems

1. (I.64.8) Taylor has enough money to buy either 90 granola bars or 78 pop-tarts. After returning from the store, Taylor has no money, 75 granola bars, and p pop-tarts. Assuming that Taylor has not yet eaten anything, figure out what p is.

2. (I.66.6) Some coffee roasters dealers mix beans with different flavor profiles to customize their product. Selling prices are adjusted appropriately. For example, suppose that a roaster mixed some coffee worth \$6.49 a pound with some coffee worth \$10.89 a pound, thus obtaining 100 pounds of a mixture worth \$9.24 a pound. How many pounds of each type of bean was used for this mixture?

3. (I.68.1) The hot-water faucet takes four minutes to fill the tub, and the cold-water faucet takes three minutes for the same job. How long to fill the tub if both faucets are used.

4. (I.68.3) The speed of sound in air is 1100 feet per second. The speed of sound in steel is 16500 feet per second. Robin, one ear pressed against the railroad track, hears a sound through the rail six seconds before hearing the same sound through the air. To the nearest foot, how far away is the source of that sound?

5. (I.69.1) The cost of a ham-and-bean supper at a local church was \$6 for adults and \$4 for children. At the end of the evening, the organizers of the supper found they had taken in a total of \$452 and that 86 persons had attended. How many of these persons were adults?

6. (I.69.4) Evaluate the expression 397(2.598) + 845(2.598) − 242(2.598) mentally.

7. (I.69.7) If n stands for a perfect square, what formula stands for the next largest perfect square?

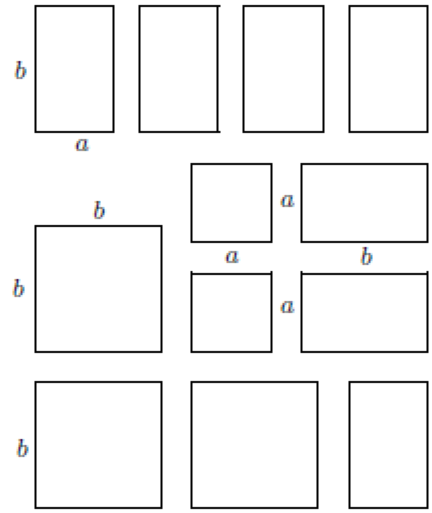
8. (I.69.8) Dale hikes up a mountain trail at 2 mph. Because Dale hikes at 4 mph downhill, the trip down the mountain takes 30 minutes less time than the trip up, even though the downward trail is three miles longer. How many miles did Dale hike in all?

9. (I.70.7) Without using a calculator, simplify $\left|3-\sqrt{5}\right|+4$ by writing an equivalent expression without absolute-value signs. Do the same for $\left|3-\sqrt{10}\right|+4$. Does your calculator agree?

10. (I.71.5) The hands of a clock point in the same direction at noon, and also at midnight. How many times between noon and midnight does this happen?

11. (I.72.3) Lee finds the identity $\left(a+b\right)^2 = a^2+2ab+b^2$ useful for doing mental arithmetic. For example, just ask Lee for the value of 75^2 and you will get the answer 5625 almost immediately—with no calculator assistance. The trick is to use algebra by letting $10k+5$ represent a typical integer that ends with 5. Show that the square of this number is represented by $100k(k+1)+25$. This should enable you to explain how Lee is able to calculate $75^2 = 5625$ so quickly. Try the trick yourself: Evaluate 35^2, 95^2, and 205^2 without using calculator, paper, or pencil.

Problem-Based Mathematics I Extra Problems

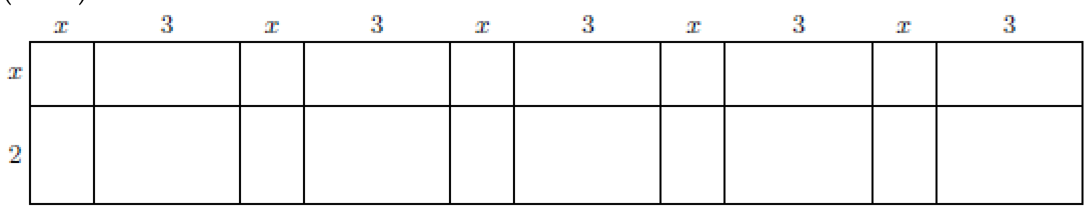


1. (I.71.2) All the dimensions of the twelve rectangles in the figure are either a or b. Write an expression for the sum of the areas of the twelve pieces. This should help you to show how these twelve pieces can be fit together to form one large rectangle.

2. (I.75.4) If a hen and a half can lay an egg and a half in a day and a half, then how much time is needed for three hens to lay three eggs?

3. (I.76.6) The area of a rectangle is 12 square feet, and each of its dimensions is an integral number of feet. What are the dimensions of all the possible rectangles that could have this area? What are all the integral factors of 12?

4. (I.73.3) The expression $5(x+2)(x+3)$ can be pictured as five rectangles, each $(x+2)$ by $(x+3)$, as shown below:



(a) Write out the product $5(x+2)(x+3)$, and show that it also corresponds to the diagram.
(b) Explain why $5(x+2)(x+3)$ is equivalent to $(5x+10)(x+3)$, using algebraic code as well as a labeled diagram to support your answer.
(c) Explain why $5(x+2)(x+3)$ is equivalent to $(x+2)(5x+15)$, using algebraic code as well as a labeled diagram to support your answer.

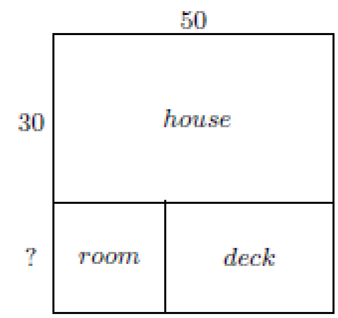
5. (I.77.10) Find the value for c that forces the graph of $3x+4y=c$ to go through (2, −3).

6. (I.83.2) The final digit of 3^6 is 9. What is the final digit of 3^{2001}?

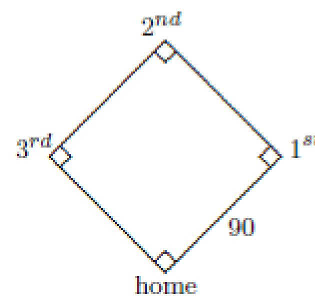
7. (I.84.2) Last year, I spent $72 to buy a lot of ping-pong balls to use in geometry class. This year, the price of a ping-pong ball is 6 cents higher, and $72 buys 60 fewer balls. Figure out how many ping-pong balls I bought last year.

Problem-Based Mathematics I Extra Problems

1. (I.78.1) Pat and Kim own a small, pleasant house that is rectangular in shape, 50 feet by 30 feet. They want to add on a family room that will be square, and then fill in the space adjoining the new room with a deck. A plan of the setup is at the right. They have not decided how large a family room to build, but they do have 400 square feet of decking. If they use it all, and keep to the plan, how large will the family room be? Is there more than one solution to this problem?



2. (I.81.1) In baseball, the infield is a square that is 90 feet on a side, with bases located at three of the corners, and home plate at the fourth. If the catcher at home plate can throw a baseball at 70 mph, how many seconds does it take for the thrown ball to travel from home plate to 2nd base?



3. (I.83.3) The mathematician Augustus de Morgan enjoyed telling his friends that he was x years old in the year x^2. Figure out the year of de Morgan's birth, given that he died in 1871.

4. (I.83.4) (Continuation) Are there persons alive today who can truthfully make the same statement that de Morgan did?

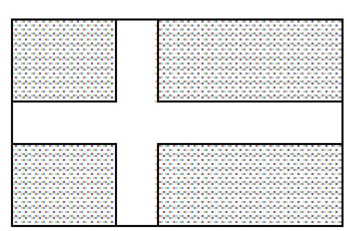
5. (I.84.9) My car averages 35 miles to a gallon of gas. When the price of gasoline was \$3.09 per gallon, what was the cost per mile for gasoline for this car? What was the average distance I could travel per dollar?

6. (I.84.10) What is the exact value of the expression $x^2 - 5$ when $x = 2 + \sqrt{5}$?

7. (I.85.2) *Eureka*! A museum acquires an ancient crown that was supposed to be pure gold. Because of suspicions that the crown also contains silver, the crown is measured. Its weight is 42 ounces and its volume is 4 cubic inches. Given that gold weighs 11 ounces per cubic inch and silver weighs 6 ounces per cubic inch, and assuming that the crown really *is* an alloy of silver and gold, figure out how many ounces of silver are mixed with the gold.

8. (I.85.8) Show that $x = 3 + \sqrt{2}$ is a solution to the equation $x^2 - 6x + 7 = 0$.

9. (I.86.11) The diagram at right shows the flag of Sweden, which consists of a gold cross of uniform width against a solid blue background. The flag measures 3 feet 4 inches by 5 feet 4 inches, and the area of the gold cross is 30% of the area of the whole flag. Use this information to find the width of the gold cross.



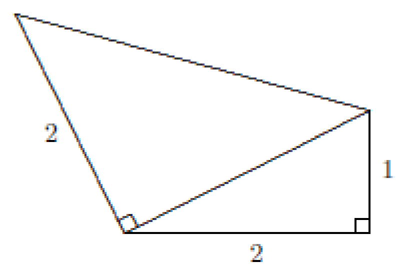
Problem-Based Mathematics I Extra Problems

1. (I.87.2) Halfway through the basketball season, Fran Tastik has made 24 free throws out of 40 shots attempted.
(a) What is Fran's average, expressed as a percent?
(b) Fran anticipates getting 30 more free throw tries by the end of the season. How many of these must Fran make, in order to have a season average that is at least 70%?

2. (I.88.10) What is the meaning of the number k when you graph the equation $y = mx + k$? What is the meaning of the number k when you graph the equation $x = my + k$?

3. (I.89.7) Wes and Kelly decide to test their new walkie-talkies, which have a range of six miles. Leaving from the spot where Kelly is standing, Wes rides three miles east, then four miles north. Can Wes and Kelly communicate with each other? What if Wes rides another mile north? How far can Wes ride on this northerly course before communication breaks down?

4. (I.90.2) Draw a right triangle whose legs are 2 cm and 1 cm long, as shown at right. Find the length of its hypotenuse.



(a) Use this hypotenuse as one of the legs of a second right triangle, and construct the other leg so that it is 2 cm long and adjacent to the previous 2-cm leg, as shown. Find the length of the hypotenuse of this right triangle.
(b) Use this hypotenuse as one of the legs of a third right triangle, and construct the other leg so that it is 2 cm long and adjacent to the previous 2-cm leg. Find the hypotenuse of this right triangle.
(c) This process can be continued. What are the lengths of the legs of the next triangle that has a rational hypotenuse? Are there more triangles like this?

5. (I.91.2) Most positive integers can be expressed as a sum of two or more consecutive positive integers. For example, 24 = 7+8+9, 36 = 1+2+3+4+5+6+7+8, and 51 = 25+26. A positive integer that cannot be expressed as a sum of two or more consecutive positive integers is therefore *interesting*. The simplest example of an interesting number is 1.
(a) Show that no other odd number is interesting.
(b) Show that 14 is not an interesting number.
(c) Show that 82 is not an interesting number.
(d) Find three ways to show that 190 is not an interesting number.
(e) Find three ways to show that 2004 is not an interesting number.
(f) How many interesting numbers precede 2004?

6. (II.1.4) Instead of walking along two sides of a rectangular field, Fran took a shortcut along the diagonal, thus saving distance equal to half the length of the longer side. Find the length of the long side of the field, given that the length of the short side is 156 meters.

7. (II.4.1) Find an example of an equilateral hexagon whose sides are all $\sqrt{13}$ units long. Give coordinates for all six points.

Problem-Based Mathematics I Extra Problems

1. (II.4.5) Find the slope of the line through
(a) (3, 1) and $(3 + 4t, 1 + 3t)$ (b) $(m - 5, n)$ and $(5 + m, n^2)$

2. (II.4.6) Is it possible for a line $ax + by = c$ to lack a y-intercept? To lack an x-intercept? Explain.

3. (II.4.8) Pat races at 10 miles per hour, while Kim races at 9 miles per hour. When they both ran in the same long-distance race last week, Pat finished 8 minutes ahead of Kim. What was the length of the race, in miles? Briefly describe your reasoning.

4. (II.4.9) (Continuation) Assume that Pat and Kim run at p and k miles per hour, respectively, and that Pat finishes m minutes before Kim. Find the length of the race, in miles.

5. (II.5.1) Rewrite the equation $3x - 5y = 30$ in the form $ax + by = 1$. Are there lines whose equations cannot be rewritten in this form?

6. (II.5.10) Find a and b so that $ax + by = 1$ has x-intercept 5 and y-intercept 8.

7. (II.6.12) Lynn takes a step, measures its length and obtains 3 feet. Lynn uses this measurement in attempting to pace off a 1-mile course, but the result is 98 feet too long. What is the actual length of Lynn's stride, and how could Lynn have done a more accurate job?

8. (II.7.1) One of the legs of a right triangle is 12 units long. The other leg is b units long and the hypotenuse c units long, where b and c are both integers. Find b and c. Hint: both sides of the equation $c^2 - b^2 = 144$ can be factored.

9. (II.7.3) Show that a 9-by-16 rectangle can be transformed into a square by dissection. In other words, the rectangle can be cut into pieces that can be reassembled to form the square. Do it with as few pieces as possible.

10. (II.7.7) The Krakow airport is 3 km west and 5 km north of the city center. At 1 pm, Zuza took off in a Cessna 730. Every six minutes, the plane's position changed by 9 km east and 7 km north. At 2:30 pm, Zuza was flying over the town of Jozefow. In relation to the center of Krakow, (a) where is Jozefow? (b) where was Zuza after t hours of flying?

11. (II.8.3) The perimeter of an isosceles right triangle is 24 cm. How long are its sides?

12. (II.8.7) Leaving home on a recent business trip, Kyle drove 10 miles south to reach the airport, then boarded a plane that flew a straight course—6 miles east and 3 miles north each minute. What was the airspeed of the plane? After two minutes of flight, Kyle was directly above the town of Greenup. How far is Greenup from Kyle's home? A little later, the plane flew over Kyle's birthplace, which is 50 miles from home. When did this occur?

13. (I.52.5) Calculate the area of the region defined by the simultaneous inequalities $y \ge x - 4$, $y \le 10$, and $5 \le x + y$.

Problem-Based Mathematics I Extra Problems

1. (II.10.10) Tracy and Kelly are running laps on the indoor track—at steady speeds, but in opposite directions. They meet every 20 seconds. It takes Tracy 45 seconds to complete each lap. How many seconds does it take for each of Kelly's laps? Check your answer.

2. (II.13.2) A puzzle: Cut out four copies of the quadrilateral $ABCD$ formed by points $A = (0, 0)$, $B = (5, 0)$, $C = (6, 2)$, and $D = (0, 5)$. Show that it is possible to arrange these four pieces to form a square. Explain why you are sure that the pieces fit exactly.

3. (II.13.3) Two of the sides of a right triangle have lengths $360\sqrt{1994}$ and $480\sqrt{1994}$. Find the possible lengths for the third side.

4. (II.13.11) Dissect a 1-by-3 rectangle into three pieces that can be reassembled into a square.

5. (II.14.5) One of the legs of a right triangle is twice as long as the other, and the perimeter of the triangle is 28. Find the lengths of all three sides, to three decimal places.

6. (II.14.6) A car traveling east at 45 miles per hour passes a certain intersection at 3 pm. Another car traveling north at 60 miles per hour passes the same intersection 25 minutes later. To the nearest minute, figure out when the cars are exactly 40 miles apart.

7. (II.14.10) If I were to increase the length of my stride by one inch, it would take me 60 fewer strides to cover a mile. What was the length of my original stride?

8. (II.15.1) Ashley saved a distance equal to 80% of the length of the shortest side of a rectangular field by cutting across the diagonal of the field instead of along two of the sides. Find the ratio of the length of the shortest side of the field to the length of its longest side.

9. (II.15.3) If a line intersects the x-axis at $(a, 0)$ and intersects the y-axis at $(0, b)$, at what point does it intersect the line $y = x$?

10. (II.15.6) The sides of a right triangle are $x-y$, x, and $x+y$, where x and y are positive numbers, and $y < x$. Find the ratio of x to y.

11. (II.16.4) Choose positive integers m and n, with $m < n$. Let $x = 2mn$, $y = n^2 - m^2$, and $z = m^2 + n^2$. It so happens that these three positive integers x, y, and z have a special property. What is the property? Can you prove a general result?

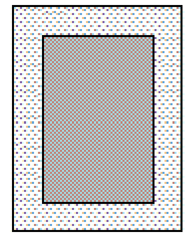
12. (II.17.13) Two automobiles each travel 60 km at steady rates. One car goes 6 kph faster than the other, thereby taking 20 minutes less time for the trip. Find the rate of the slower car.

13. (II.19.2) If I were to increase my cycling speed by 3 mph, I calculate that it would take me 40 seconds less time to cover each mile. What is my current cycling speed?

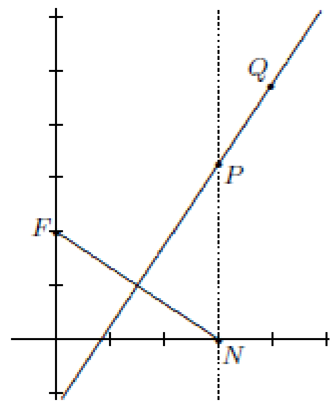
Problem-Based Mathematics I Extra Problems

1. (II.36.1) Let $A = (0, 0)$, $B = (8, 0)$, and $C = (x, x)$. Find x, given that (a) $BC = 7$; (b) $BC = 4\sqrt{2}$ (c) $BC = 10$ and CAB is a 45-degree angle.

2. (II.22.1) Robin is mowing a rectangular field that measures 24 yards by 32 yards, by pushing the mower around and around the outside of the plot. This creates a widening border that surrounds the unmowed grass in the center. During a brief rest, Robin wonders whether the job is half done yet. How wide is the uniform mowed border when Robin *is* half done?



3. (II.30.9) Let $F = (0, 2)$ and $N = (3, 0)$. Find coordinates for the point P where the perpendicular bisector of segment FN intersects the line $x = 3$.



4. (II.30.10) (Continuation) Choose a new $N \neq (3, 0)$ on the x-axis, and repeat the calculation of P: Find the intersection of the perpendicular bisector of FN with the line through N that is parallel to the y-axis. Explain why there is always an intersection point P, no matter what N is chosen.

5. (II.30.11) (Continuation) Given $Q \neq P$ on the perpendicular bisector of FN, show that the distance from Q to F exceeds the distance from Q to the x-axis. Why was it necessary to exclude the case $Q = P$?

6. (II.34.2) Given square $ABCD$, let P and Q be the points outside the square that make triangles CDP and BCQ equilateral. Prove that triangle APQ is also equilateral.

7. (II.34.7) Given square $ABCD$, let P and Q be the points outside the square that make triangles CDP and BCQ equilateral. Segments AQ and BP intersect at T. Find angle ATP.

8. (II.36.5) Let $A = (1, 1)$, $B = (3, 5)$, and $C = (7, 2)$. Explain how to cover the whole plane with non-overlapping triangles, each of which is congruent to triangle ABC.

9. (II.36.6) (Continuation) In the pattern of lines produced by your *tessellation*, you should see triangles of many different sizes. What can you say about their sizes and shapes?

10. (II.36.8) Give coordinates for a point that is 8 units from the line $y = 5$. Then find both points on the line $3x + 4y = 2$ that are 8 units from the line $y = 5$.

11. (II.36.11) Given rectangle $ABCD$, let P be the point outside $ABCD$ that makes triangle CDP equilateral, and let Q be the point outside $ABCD$ that makes triangle BCQ equilateral. Prove that triangle APQ is also equilateral.

Problem-Based Mathematics I Extra Problems

1. (II.36.14) Is it possible for a pentagon to have interior angles 120°, 120°, 120°, 90°, and 90°, in this order? What about 120°, 120°, 90°, 120°, and 90°? Are there other arrangements of the five angles that could have been considered? Do any of these pentagons *tessellate*?

2. (II.37.3) Given parallelogram *ABCD*, with diagonals *AC* and *BD* intersecting at *O*, let *POQ* be any line with *P* on *AB* and *Q* on *CD*. Prove that $AP = CQ$.

3. (II.37.5) Given that $ABCDEFG\cdots$ is a regular *n*-sided polygon, with angle *CAB* = 12 degrees, find *n*.

4. (II.37.8) In triangle *PQR*, it is given that angle *R* measures *r* degrees. The bisectors of angles *P* and *Q* are drawn, creating two acute angles where they intersect. In terms of *r*, express the number of degrees in these acute angles.

5. (II.37.9) Can two of the angle bisectors of a triangle intersect perpendicularly? Explain.

6. (II.37.10) Draw triangle *ABC* so that angles *A* and *B* are both 42 degrees. Why should *AB* be longer than *BC*? Extend *CB* to *E*, so that $CB = BE$. Mark *D* on *AB* so that $DB = BC$, then draw the line *ED*, which intersects *AC* at *F*. Find the size of angle *CFD*.

7. (II.38.4) Mark *Y* inside regular pentagon *PQRST*, so that *PQY* is equilateral. Is *RYT* straight? Explain.

8. (II.38.7) Draw a parallelogram *ABCD*, then attach equilateral triangles *CDP* and *BCQ* to the outside of the figure. Decide whether or not triangle *APQ* is equilateral. Explain.

9. (II.38.8) Suppose that *ABCD* is a rhombus and that the bisector of angle *BDC* meets side *BC* at *F*. Prove that angle *DFC* is three times the size of angle *FDC*.

10. (II.40.13) Write an equation for the line that is equidistant from $5x+3y=15$ and $5x+3y=27$.

11. (II.39.10) If a quadrilateral is a rectangle, then its diagonals have the same length. What is the converse of this true statement? Is the converse true? Explain.

12. (II.40.3) Equilateral triangles *BCP* and *CDQ* are attached to the outside of regular pentagon *ABCDE*. Is quadrilateral *BPQD* a parallelogram? Justify your answer.

13. (II.40.4) A line of positive slope is drawn so that it makes a 60-degree angle where it intersects the *x*-axis. What is the slope of this line?

14. (II.40.5) Mark *P* inside square *ABCD*, so that triangle *ABP* is equilateral. Let *Q* be the intersection of *BP* with diagonal *AC*. Triangle *CPQ* looks isosceles. Is this actually true?

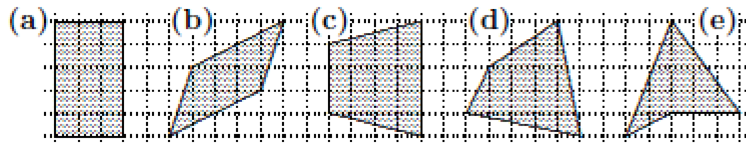
Problem-Based Mathematics I Extra Problems

1. (II.40.15) Squares *OPAL* and *KEPT* are attached to the outside of equilateral triangle *PEA*.
(a) Draw segment *TO*, then find the size of angle *TOP*.
(b) Decide whether segments *EO* and *AK* have the same length, and give your reasons.

2. (II.40.17) One stick is twice as long as another. You break the longer stick at a random point. Now you have three sticks. What is the *probability* that they form a triangle?

3. (II.42.7) Suppose that *P* is twice as far from (0, 0) as *P* is from (6, 0). Find such a point on the *x*-axis. Find another such point that is *not* on the *x*-axis.

4. (II.42.13) Which of the quadrilaterals shown at right can be used to tessellate a plane? Justify your choices.



5. (II.44.4) Is it possible for a scalene triangle to have two medians of the same length? Explain.

6. (II.46.4) Suppose that *DRONE* is a regular pentagon, and that *DRUM*, *ROCK*, *ONLY* , *NEAP*, and *EDIT* are squares attached to the outside of the pentagon. Show that decagon *ITAPLYCKUM* is equiangular. Is this decagon equilateral?

7. (II.46.6) Rearrange the letters of *doctrine* to spell a familiar mathematical word.

8. (II.46.13) Suppose that *PEANUT* is a regular hexagon, and that *PEGS*, *EACH*, *ANKL*, *NUMB*, *UTRY* , and *TPOD* are squares attached to the outside of the hexagon. Decide whether or not dodecagon *GSODRYMBKLCH* is regular, and give your reasons.

9. (II.47.8) Inside regular pentagon *JERZY* is marked point *P* so that triangle *JEP* is equilateral. Decide whether or not quadrilateral *JERP* is a parallelogram, and give your reasons.

10. (II.48.12) Draw a regular pentagon and all five of its diagonals. How many isosceles triangles can you find in your picture? How many scalene triangles can you find?

11. (II.48.13) The sides of a triangle are 6 cm, 8 cm, and 10 cm long. Find the distances from the centroid of this triangle to the three vertices.

12. (II.50.4) The diagonals of rhombus *ABCD* meet at *M*. Angle *DAB* measures 60 degrees. Let *P* be the midpoint of *AD*, and let *G* be the intersection of *PC* and *MD*. Given that $AP = 8$, find *MD*, *MC*, *MG*, *CG*, and *GP*.

13. (II.50.5) Rectangle *ABCD* has dimensions $AB = 5$ and $BC = 12$. Let *M* be the midpoint of *BC*, and let *G* be the intersection of *AM* and diagonal *BD*. Find *BG* and *AG*.

14. (II.50.6) Show that a regular dodecagon can be cut into pieces that are all regular polygons, which need not all have the same number of sides.

Problem-Based Mathematics I Extra Problems

1. (II.51.12) A regular n-sided polygon has exterior angles of m degrees each. Express m in terms of n. For how many of these regular examples is m a whole number?

2. (II.52.2) Triangle ABC has $AB = 12 = AC$ and angle A is 120 degrees. Let F and D be the midpoints of sides AC and BC, respectively, and G be the intersection of segments AD and BF. Find the lengths FD, AD, AG, BG, and BF.

3. (II.52.4) Find the side of the largest square that can be drawn inside a 12-inch equilateral triangle, one side of the square aligned with one side of the triangle.

4. (II.52.6) To the nearest tenth of a degree, how large are the congruent angles of an isosceles triangle that is exactly as tall as it is wide? (There is more than one interpretation.)

5. (II.78.9) If a line cuts a triangle into two pieces of equal area, must that line go through the centroid of the triangle? Explain your answer.

6. In the spring of 2010, radio station WYEP (91.3 FM) counted down the top 913 songs from the past decade. The station began the countdown with song number 913 and played every song down to number 1. A listener heard song 848 on Tuesday at 6:00 am. The next morning at 7:00 am the listener heard song number 687 when driving to work. Estimate when the countdown began and when it ended.

7. (Continuation) The listener later learned that WYEP only counts down songs on weekdays between 6:00 am and 6:00 pm. Change your estimates in light of this new information.

8. (I.43.3) The rails on a railroad are built from thirty-foot sections. When a train wheel passes over the junction between two sections, there is an audible click. Inside a train that is traveling at 70 mph, how many clicks will a passenger hear each twenty seconds?

9. (I.48.11) A restaurant has 23 tables. Some of the tables seat 4 persons and the rest seat 2 persons. In all, 76 persons can be seated at once. How many tables of each kind are there?

10. (I.48.10) The average of two numbers is 41. If one of the numbers is 27, what is the other number? If the average of two numbers is $x + y$, and one of the numbers is x, what is the other number?

11. (I.87.5) To get from one corner of a rectangular court to the diagonally opposite corner by walking along two sides, a distance of 160 meters must be covered. By going diagonally across the court, 40 meters are saved. Find the dimensions of the court, to the nearest cm.

12. (I.91.1) A bell rope, passing through the ceiling above, just barely reaches the belfry floor. When one pulls the rope to the wall, keeping the rope taut, it reaches a point that is three inches above the floor. It is four feet from the wall to the rope when the rope is hanging freely. How high is the ceiling? It is advisable to make a clear diagram for this problem.

Problem-Based Mathematics I Extra Problems

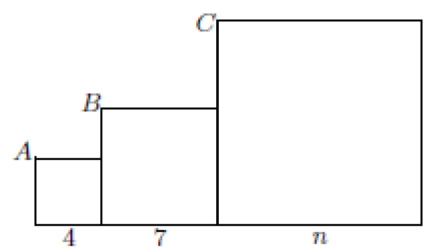
1. (I.53.1) The population for Vermont is given in the table at right.
(a) Find the average annual growth rate of this population during the time interval from 1960 to 2000.
(b) Write an equation for a line in point-slope form, using the ordered pair (1960, 389881) and the slope you found in part (a)
(c) Evaluate your equation for the years 1970 and 1980, and notice that these interpolated values do not agree with the actual table values. Find the size of each error, expressed as a percent of the actual population value.
(d) Use your point-slope equation to extrapolate a population prediction for 2010.
(e) New Hampshire has roughly the same area as Vermont, but its population reached one million a few years ago. Predict when this will happen to Vermont's population.

year	*pop*
1960	389881
1970	448327
1980	511456
1990	564964
2000	609890

2. (I.53.5) Blair runs a kiosk at the local mall that sells sweatshirts. There are two types of shirts sold. One is 100% cotton, on which the markup is \$6 per shirt. The other is a cotton and polyester blend, on which the markup is \$4 per shirt. It costs Blair \$900 per month to rent the kiosk. Let c represent the number of pure cotton sweatshirts sold in one month and b the number of blended sweatshirts sold in the same month.
(a) In terms of c and b, write an inequality that states that Blair's sales will at least meet the monthly rental expense. Sketch a graph with c on the horizontal axis and b on the vertical axis.
(b) This month, Blair could only get 20 of the pure cotton shirts from the distributor. This adds another constraint to the system. How does it affect the graph you drew in (a)?

3. (II.6.1) Three squares are placed next to each other as shown. The vertices A, B, and C are collinear. Find the dimension n.

4. (II.6.2) (Continuation) Replace the lengths 4 and 7 by m and k, respectively. Express k in terms of m and n.



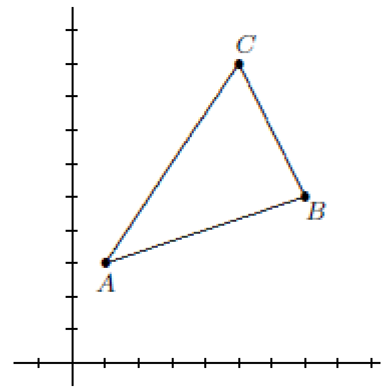
5. (II.8.6) Show that the triangle formed by the lines $y = 2x - 7$, $x + 2y = 16$, and $3x + y = 13$ is isosceles. Show also that the lengths of the sides of this triangle fit the Pythagorean equation. Can you identify the right angle just by looking at the equations?

6. (II.22.9) Let $A = (-4, 0)$, $B = (0, 6)$, and $C = (6, 0)$.
(a) Find equations for the three lines that contain the altitudes of triangle ABC.
(b) Show that the three altitudes are *concurrent*, by finding coordinates for their common point. The point of concurrence is called the *orthocenter* of triangle ABC.

7. (II.23.2) Let $A = (-4, 0)$, $B = (0, 6)$, and $C = (6, 0)$.
(a) Find equations for the three medians of triangle ABC.
(b) Show that the three medians are *concurrent*, by finding coordinates for their common point. The point of concurrence is called the *centroid* of triangle ABC.

8. (II.24.9) Let $P = (-1, 3)$. Find the point Q for which the line $2x + y = 5$ serves as the perpendicular bisector of segment PQ.

Problem-Based Mathematics I Extra Problems



1. (II.26.2) Let $A = (1, 3)$, $B = (7, 5)$, and $C = (5, 9)$. Answer the item below that is determined by *the first letter of your last name*. Find coordinates for the requested point.
(a-e) Show that the three medians of triangle ABC are concurrent at a point G.
(f-m) Show that the three altitudes of triangle ABC are concurrent at a point H.
(n-z) Show that the perpendicular bisectors of the sides of triangle ABC are concurrent at a point K. What special property does K have?

2. (II.26.3) (Continuation for class discussion) It looks like G, H, and K are collinear. Are they?

3. (II.27.5) It is given that $a + b = 6$ and $ab = 7$.
(a) Find the value of $a^2 + b^2$. Can you do this without finding values for a and b ?
(b) Make up a geometry word problem that corresponds to the question in part (a).

4. (II.28.8) Find the lengths of *all* the altitudes of the triangle whose vertices are (0, 0), (3, 0), and (1, 4).

5. (II.28.9) Form a triangle using three lattice points of your choosing. Verify that the medians of your triangle are concurrent.

6. (II.33.9) By making a straight cut through one vertex of an isosceles triangle, Dylan dissected the triangle into two smaller isosceles triangles. Find the angle sizes of the original triangle. There is more than one possibility. How can you be sure that you have found them all?

7. (II.40.9) Suppose that $ABCD$ is a square, with $AB = 6$. Let N be the midpoint of CD and F be the intersection of AN and BD. What is the length of AF?

8. (II.34.9) Make an accurate drawing of an acute-angled, non-equilateral triangle ABC and its circumcenter K. Use your protractor to measure (a) angles A and BKC; (b) angles B and CKA; (c) angles C and AKB. Do you notice anything?

9. (II.39.14) The sides of a triangle have lengths 9, 12, and 15. (This is a special triangle!)
(a) Find the lengths of the medians of the triangle.
(b) The medians intersect at the centroid of the triangle. How far is the centroid from each of the vertices of the triangle?

10. (II.39.15) (Continuation) Apply the same questions to the equilateral triangle of side 6.

11. (II.41.1) The lengths of the sides of triangle ABC are $AB = 15 = AC$ and $BC = 18$. Find the distance from A to **(a)** the centroid of ABC; **(b)** the circumcenter of ABC.

Problem-Based Mathematics I Extra Problems

1. (II.42.1) Triangle PQR is isosceles, with $PQ = 13 = PR$ and $QR = 10$. Find the distance from P to the centroid of PQR. Find the distance from Q to the centroid of PQR.

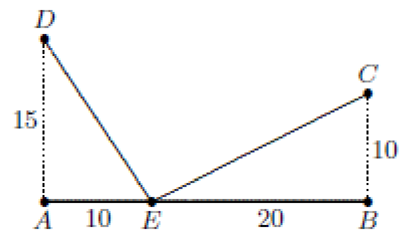
2. (II.42.2) (Continuation) Find the distance from P to the circumcenter of triangle PQR.

3. (II.42.3) For what triangles is it true that the circumcenter and the centroid are the same point?

4. (II.44.6) Triangle ABC has $AB = 10 = AC$ and $BC = 12$. Find the distance from A to
(a) the centroid of ABC; (b) the circumcenter of ABC.

5. (II.46.3) Suppose that a quadrilateral is measured and found to have a pair of equal nonadjacent sides and a pair of equal nonadjacent angles. Is this enough evidence to conclude that the quadrilateral is a parallelogram? Explain.

6. (II.1.8) In the diagram, AEB is straight and angles A and B are right. Calculate the total distance $DE + EC$.



7. (II.1.9) (Continuation) If $AE = 15$ and $EB = 15$ instead, would $DE + EC$ be the same?

8. (II.1.10) (Continuation) You have seen that the value chosen for AE determines the value of $DE + EC$. One also says that *$DE + EC$ is a function of AE*. Letting x stand for AE (and $30 - x$ for EB), write a formula for this function. Then enter this formula into your calculator, graph it, and find the value of x that produces the *shortest* path from D to C through E. Draw an accurate picture of *this* path and make a conjecture about *angles AED* and BEC. Use your protractor to test your conjecture.

9. (II.28.15) A stop sign—a regular *octagon*—can be formed from a 12-inch square sheet of metal by making four straight cuts that snip off the corners. How long, to the nearest 0.01 inch, are the sides of the resulting polygon?

10. (II.32.7) Triangle ABC has a 34-degree angle at A. The bisectors of angles B and C meet at point I. What is the size of angle BIC? Answer this question **(a)** assuming that ABC is right; **(b)** assuming that ABC is isosceles; **(c)** choosing sizes for angles B and C. Hmm. . .

11. (II.34.5) Although you have used the converse of the Pythagorean Theorem, it has not yet been proved in this book. State the converse and prove it if you can.

Problem-Based Mathematics I Reference

absolute value: The absolute value of x is denoted $|x|$ and is the distance between x and zero on a number line. The absolute value of a quantity is never negative. [22]

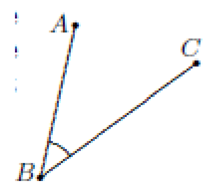
additive inverse: See *opposite*.

adjacent: Two vertices of a polygon that are connected by an edge. Two edges of a polygon that intersect at a vertex. Two angles of a polygon that have a common side. [56]

altitude: In a triangle, an altitude is a segment that joins one of the three vertices to a point on the opposite side, the intersection being perpendicular. In some triangles, it may be necessary to extend the side to meet the altitude. The *length* of this segment is also called an altitude, as is the distance that separates the parallel sides of a trapezoid. [57]

Angle-Angle-Side (corresponding): When the parts of one triangle can be matched with the parts of another triangle, so that two pairs of corresponding angles have the same sizes, and so that one pair of corresponding sides has the same length, then the triangles are congruent. This rule of evidence is abbreviated to AAS. [55]

angle bisector: Given an angle, this ray divides the angle into two equal parts. [59]



angles can often be identified by a single letter, but sometimes three letters are necessary. The angle shown can be called B, ABC, or CBA. [53]

Angle-Side-Angle: When the parts of one triangle can be matched with the parts of another triangle, so that two pairs of corresponding angles have the same sizes, and so that the (corresponding) shared sides have the same length, then the triangles are congruent. This rule of evidence is abbreviated to ASA. [55]

average speed: The average speed during a time interval is $\dfrac{\text{total distance}}{\text{total time}}$. [13]

average a list of numbers: Add them and divide by how many numbers in the list. [7-Extra]

balance diagram: A diagram displaying a scale that is in equilibrium. [4]

binomial: The sum of two unlike monomials, *e.g.* $x+2$ or $3x^3y-7z^5$.[26, 37]

bisect: Divide into two pieces that are, in some sense, equal. [53, 58, 59, 62]

cc: Abbreviation for cubic centimeter. See *conversions*.

centroid: The medians of a triangle are concurrent at this point, which is the balance point (also known as the *center of gravity*) of the triangle. [Extra--24]

circumcenter: The perpendicular bisectors of the sides of a triangle are concurrent at this point, which is equidistant from the vertices of the triangle. [65]

circumcircle: When possible, the circle that goes through all the vertices of a polygon.

coefficient: See *monomial*.

collinear: Three (or more) points that all lie on a single line are collinear. [24]

combine over a common denominator: To create a single fraction that is equal to a given sum of fractions. [11]

common denominator: Given a set of fractions, a common denominator is divisible by every one of the given denominators. [11]

common monomial factor: A *monomial* that divides every term of a *polynomial*. [37]

complementary: Two angles that fit together to form a right angle are called complementary. Each angle is the *complement* of the other. [53]

completing the square: Adding a quantity to a trinomial so that the new trinomial can be factored as a perfect square. [43]

concurrent: Three (or more) lines that go through a common point are *concurrent*. [Extra--24]

congruent: When the points of one figure can be matched with the points of another figure, so that corresponding parts have the same size, then the figures are called *congruent*, which means that they are considered to be equivalent. [52, 54]

consecutive integers: Two integers are consecutive if their difference is 1. [4]

continuous: A variable whose values fill an *interval*. Continuous variables represent quantities that are divisible, such as time and distance. See also *discrete*. [13]

converse: The converse of a statement of the form "if [something] then [something else]" is the statement "if [something else] then [something]." [62]

conversions: 1 mile = 5280 feet; 1 foot = 12 inches; 1 inch = 2.54 centimeters; one liter is 1000 milliliters; a milliliter is the same as a cubic centimeter.

coordinate: A number that locates a point on a number line or describes the position of a point in the plane with respect to two number lines (axes). [5, 51]

corresponding: Describes parts of figures (such as angles or segments) that have been matched by means of a transformation. [58, 60]

CPCTC: *Corresponding Parts of Congruent Triangles are themselves Congruent*. [58]

decagon: A polygon that has ten sides. [64]

degree: For a monomial, this counts how many variable factors would appear if the monomial were written without using exponents. The degree of a polynomial is the largest degree found among its monomial terms. [46]

diagonal: A segment that connects two nonadjacent vertices of a polygon. [55]

direct variation: Two quantities *vary directly* if one quantity is a constant multiple of the other. Equivalently, the ratio of the two quantities is constant. The graph of two quantities that vary directly is a straight line passing through the origin. [13]

discrete: A variable that is restricted to integer values. [9-Extra]

distance formula: The distance from (x_1, y_1) to (x_2, y_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. These formulas are consequences of the *Pythagorean Theorem*. [48]

distributive property: Short form of "multiplication distributes over addition," a special property of arithmetic. In algebraic code: $a(b+c)$ and $ab+ac$ are equivalent, as are $(b+c)a$ and $ba+ca$, for any three numbers a, b, and c. [1] Multiplication also distributes over subtraction, of course.

endpoint convention: If an interval includes an endpoint (as in $6 \le x$ or $y \le -4$), this point is denoted graphically by filling in a circle. If an interval excludes an endpoint (as in $6 < x$ or $y < -4$), this point is denoted by drawing an empty circle. [8]

equation: A statement that two expressions are equivalent. For example, $3x+5 = 2x-4$, $\frac{3}{4} = \frac{15}{20}$, and $(x+3)^2 = x^2 + 6x + 9$ are all equations. [5] The last one is an *identity*.

equiangular: A polygon all of whose angles are the same size. [50]

equidistant: A shortened form of *equally distant*. [51]

equilateral: A polygon all of whose sides have the same length. [50]

Euclidean geometry (also known as plane geometry) is characterized by its parallel postulate, which states that, *given a line, exactly one line can be drawn parallel to it through a point not on the given line*. A more familiar version of this assumption states that *the sum of the angles of a triangle is a straight angle*. [53,61] The Greek mathematician Euclid, who flourished about 2300 years ago, wrote many books, and established a firm logical foundation for geometry.

Euler line: The centroid, the circumcenter, and the orthocenter of any triangle are collinear. [60] The Swiss scientist Leonhard Euler (1707-1783) wrote copiously on both mathematics and physics, and knew the *Aeneid* by heart.

evaluate: Find the numerical value of an expression by *substituting* numerical values for the *variables*. For example, to evaluate $2t + 3r$ when $t = 7$ and $r = -4$, substitute the values 7 and −4 for t and r, respectively. [2]

exponent: An integer that indicates the number of equal factors in a product. For example, the exponent is 3 in the expression w^3, which means $w \cdot w \cdot w$. [33]

exponents, rules of: These apply when there is a *common base*: $a^m \cdot a^n = a^{m+n}$ and $\frac{a^m}{a^n} = a^{m-n}$; when there is a *common exponent*: $a^m \cdot b^m = (ab)^m$ and $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$; or when an exponential expression is raised to a power: $\left(a^m\right)^n = a^{mn}$. Notice the special case of the common-base rules: $a^0 = 1$.[36, 38, 39, 41]

exterior angle: An angle that is formed by a side of a polygon and the extension of an adjacent side. It is supplementary to the adjacent interior angle. [61]

Exterior-Angle Theorem: An exterior angle of a triangle is the sum of the two nonadjacent interior angles. [61]

extrapolate: To enlarge a table of values by going outside the given range of data. [6-Extra]

factor: *Noun*: a number or expression that divides another number or expression without remainder. For example, 4 is a factor of 12, $2x$ is a factor of $4x^2 + 6xy$. *Verb*: to rewrite a number or an expression as a product of its factors. For example, 12 can be factored as $2 \cdot 2 \cdot 3$, and $4x^2 + 6xy$ can be factored as $2x(2x + 3y)$. [32]

factored form: Written as a product of factors. For example, $(x - 3)(2x + 5) = 0$ is written in factored form. If an equation is in factored form it is particularly easy to find the solutions, which are $x = 3$ and $x = -2.5$ in this example. [35]

function: A function is a rule that describes how the value of one thing is determined uniquely by the value of another thing. [50]

greatest common (integer) factor: Given a set of integers, this is the largest integer that divides all of the given integers. Also called the *greatest common divisor*.

greatest common (monomial) factor: Given a set of *monomials*, this is the largest monomial that divides all of the given monomials. [37]

Greek letters appear often in mathematics. Some of the common ones are α (alpha), β (beta), Δ or δ (delta), θ (theta), Λ and λ (lambda), μ (mu), π (pi), and Ω or ω (omega).

guess-and-check: A method for creating equations to solve word problems. In this approach, the *equation* emerges as the way to check a *variable* guess. Initial practice is with constant guesses, so that the checking can be done with ordinary arithmetic. [8, 9]

hexagon: a polygon that has six sides. [50]

hypotenuse: In a right triangle, the side opposite the right angle. This is the longest side of a right triangle. [46]

Hypotenuse-Leg: When the hypotenuses of two right triangles have the same length, and a leg of one triangle has the same length as a leg of the other, then the triangles are congruent. This rule of evidence is abbreviated to HL. [57]

identity: An *equation*, containing at least one *variable*, that is true for all possible values of the variables that appear in it. For example, $x(x+y)=x^2+xy$ is true no matter what values are assigned to x and y. [30]

inequality: A statement that relates the positions of two quantities on a number line. For example, $5 < x$ or $t \le 7$. [9]

integer: A whole number—positive, negative, or zero. [1, 52]

interpolate: To enlarge a table of values by staying within the given range of data. [6-Extra]

interval: A connected piece of a number line. It might extend infinitely far in the positive direction (as in $-1 < x$), extend infinitely far in the negative direction (as in $t \le 7$), or be confined between two endpoints (as in $2 < m \le 7$).

irrational number: A number that cannot be expressed exactly as the ratio of two integers. Two familiar examples are π and $\sqrt{2}$. See *rational number*. [41]

isosceles triangle: A triangle that has two sides of the same length. [52] The word is derived from the Greek *iso* + *skelos* (equal + leg)

Isosceles-Triangle Theorem: If a triangle has two sides of equal length, then the angles opposite those sides are also the same size. [59]

kite: A quadrilateral that has two disjoint pairs of congruent adjacent sides. [51, 58]

lattice point: A point both of whose coordinates are integers. The terminology derives from the rulings on a piece of graph paper, which form a lattice. [25, 52]

leg: The perpendicular sides of a right triangle are called its legs. [49, 57, 63]

like terms: These are *monomials* that have the same variables, each with the same exponents, but possibly different numerical coefficients. Like terms can be combined into a single monomial; unlike terms cannot. [6]

linear: A polynomial, equation, or function of the first degree. For example, $y = 2x - 3$ defines a linear function, and $2x + a = 3(x - c)$ is a linear equation. [18, 19]

linear combinations: A method for solving systems of linear equations. [35]

lowest terms: A fraction is in lowest terms if the greatest common factor of the numerator and denominator is 1. For example, $\frac{14}{21}$ is not in lowest terms because 14 and 21 have 7 as a common factor. When numerator and denominator are each divided by 7 the resulting fraction $\frac{2}{3}$ is equal to $\frac{14}{21}$ and is in lowest terms.

median of a triangle: A segment that joins a vertex of a triangle to the midpoint of the opposite side. [57]

midpoint: The point on a segment that is equidistant from the endpoints of the segment. If the endpoints are (a, b) and (c, d), the midpoint is $\left(\frac{a+c}{2}, \frac{b+d}{2}\right)$. [53]

model: An equation (or equations) that describe a context quantitatively. [5]

monomial: A product of variables (no additions, subtractions, or divisions) times an optional constant, called the *coefficient*. For example, ax^2 and $3x^5$ are monomials. [37] See also *binomial*, *polynomial*, and *trinomial*.

multiplicative inverse: See *reciprocal*.

negative reciprocal: One number is the negative reciprocal of another if the product of the two numbers is -1. [53]

number line: A line on which two points have been designated to represent 0 and 1. This sets up a one-to-one correspondence between numbers and points on the line. [5]

octagon: a polygon that has eight sides. [60]

opposite: When the sum of two quantities is zero, they are called opposites (or *additive inverses*); each is the opposite of the other. On a number line, zero is exactly midway between any number and its opposite. [3]

or: Unless you are instructed to do otherwise, interpret this word *inclusively* in mathematical situations. Thus a phrase ". . . (something is true) or (something else is true) . . . " allows for the possibility that *both* (something is true) and (something else is true).

orthocenter: The altitudes of a triangle are concurrent at this point. [Extra--24]

parallel: Coplanar lines that do not intersect. When drawn in a coordinate plane, they are found to have the same slope, or else no slope at all. The shorthand $\parallel$ is often used. [31, 32]

parallelogram: A quadrilateral that has two pairs of parallel sides. [56, 61]

perfect-square trinomial: A trinomial which can be factored as the square of a binomial. [40]

perimeter: The total length of the sides of a figure. The perimeter of a rectangle is twice the length plus twice the width. In algebraic code, $p = 2l + 2w = 2(l + w)$. [8]

period of a pendulum: The time needed for a pendulum to swing back and forth once. [22, 46]

perpendicular: Coplanar lines that intersect to form a right angle. If m_1 and m_2 are the slopes of two lines in the *xy*-plane, neither line parallel to a coordinate axis, and if $m_1m_2 = -1$, then the lines are perpendicular. [53, 57]

perpendicular bisector: Given a line segment, this is the line that is perpendicular to the segment and that goes through its *midpoint*. The points on this line are all *equidistant* from the endpoints of the segment. [53]

point of intersection: A point where one line or curve meets another. The coordinates of a point of intersection must satisfy the equations of the intersecting curves. [28]

point-slope form: The line with slope m that passes through the point (h, k) can be described in point-slope form by either $y - k = m(x - h)$ or $y = m(x - h) + k$. [22, 52]

polynomial: A sum of *monomials*. See also *binomial* and *trinomial*. [37]

postulate: A statement that is accepted as true, without proof. [53]

probability: A number between 0 and 1, often expressed as a percent, that expresses the likelihood that a given event will occur. For example, the probability that two coins will *both* fall showing heads is 25%. [Extra-22]

proportion: An *equation* stating that two *ratios* are equal. For example, $\frac{4}{6} = \frac{6}{9}$ is a proportion.

Problem-Based Mathematics I Reference

Pythagorean Theorem: The square on the hypotenuse of a right triangle equals the sum of the squares on the legs. If a and b are the lengths of the legs of a right triangle, and if c is the length of the hypotenuse, then these lengths fit the Pythagorean equation $a^2+b^2 = c^2$. [47] Little is known about the Greek figure Pythagoras, who flourished about 2500 years ago, except that he probably did not discover the theorem that bears his name. [46]

quadrant: one of the four regions formed by the coordinate axes. Quadrant I is where both coordinates are positive, and the other quadrants are numbered (using Roman numerals) in a counterclockwise fashion. [51]

quadratic equation: A polynomial equation of degree 2. [43]

quadratic formula: The solution to the quadratic equation $ax^2 + bx + c = 0$, which can be written as $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

quadrilateral: a four-sided polygon. [50, 51, 55]

radical expression: An expression containing roots, like $\sqrt{2}$ or $\sqrt{x-3}$. [42]

rate (of change): Rate often denotes speed, *i.e.* units of distance per unit of time. For example, 60 miles per hour, 50 feet per second, 67 furlongs per fortnight. A general rate of change is similar: number of units of A *per* one unit of B. For example, 5 liters per student, 24 angels per pinhead, 1.3 thousand persons per year, 70 passengers per lifeboat. [1, 2, 21, 52]

ratio: The ratio of *a to b* is the expression $\dfrac{a}{b}$ also written a:b or a/b or $a \div b$. [9]

rational number: A number that can be written as the ratio of two integers. For example, $5, \dfrac{7}{13}, \text{ and } 0.631$ are rational numbers. See also *irrational number*. [41]

reciprocal: When the product of two quantities is 1, they are called reciprocals (or *multiplicative inverses*); each is the reciprocal of the other. For example, 0.2 is the reciprocal of 5, and $\dfrac{a}{b}$ is the reciprocal of $\dfrac{b}{a}$. Any nonzero number has a reciprocal. [4]

rhombus: An equilateral quadrilateral. [50, 55]

right angle: An angle that is its own supplement. [44]

segment: That part of a line that lies between two designated points. [47]

Scandinavian flags are all based on the *Dannebrog*. [Extra-16]

scatter plot: The graph of a discrete set of data points. [19]

Scientific Notation: A form for expressing a number as a product of a number between 1 and 10 and a power of 10. [3]

Sentry Theorem: The sum of the exterior angles (one at each vertex) of any polygon is 360 degrees. [64]

Side-Angle-Side: When the parts of one triangle can be matched with the parts of another triangle, so that two pairs of corresponding sides have the same lengths, and so that the (corresponding) angles they form are also the same size, then the triangles are congruent. This rule of evidence is abbreviated to just SAS. [55]

Side-Side-Angle: Insufficient grounds for congruence. [55] See *Hypotenuse-Leg*, however.

Side-Side-Side: When the parts of one triangle can be matched with the parts of another triangle, so that all three pairs of corresponding sides have the same lengths, then the triangles are congruent. This rule of evidence is abbreviated to just SSS. [55]

similar: Two figures are similar if their points can be matched in such a way that all ratios of corresponding lengths are proportional to a fixed *ratio of similarity*. Corresponding angles of similar figures must be equal in size. [52]

simplest radical form: An expression $a\sqrt{b}$ is in simplest radical form if b is a positive integer that has no factors that are perfect squares. For example, $18\sqrt{5}$ is in simplest radical form, but $5\sqrt{18}$ is not. [42]

simultaneous solution: A solution to a system of equations must satisfy *every* equation in the system. [28]

slope: The slope of a line is a measure of its steepness. It is computed by the ratio $\dfrac{\text{rise}}{\text{run}}$ or $\dfrac{\text{change in } y}{\text{change in } x}$. A line with positive slope rises as the value of x increases. If the slope is negative, the line drops as the value of x increases. [15]

slope-intercept form: The line whose slope is m and whose y-intercept is b can be described in slope-intercept form by $y = mx + b$. [19]

solve: To find the numerical values of the variables that make a given equation or inequality a true statement. Those values are called *solutions*. [5]

square: *Noun*: an equilateral and equiangular quadrilateral. *Verb*: To multiply a number by itself; *i.e.* b^2 is the square of *b*.

square root: A square root of a nonnegative number *k* is a number whose square is *k*. If *k* is positive, there are two such roots. The positive root is denoted $\sqrt{k}$, and sometimes called "the square root of *k*." The negative root is denoted $-\sqrt{k}$.

standard form: A linear equation in the form $ax + by = c$. [23]

stop sign: [60]

substitution: Replacing one algebraic expression by another of equal value. [31]

supplementary: Two angles that fit together to form a straight line are called *supplementary*. Each angle is the *supplement* of the other. [53]

system of equations: A set of two or more equations. The solution to a system of linear equations is the coordinates of the point where the lines meet. The solution is the values of the variables that satisfy all the equations of the system at the same time. [28]

tessellate: To fit non-overlapping tiles together to cover a planar region. [Extra-21]

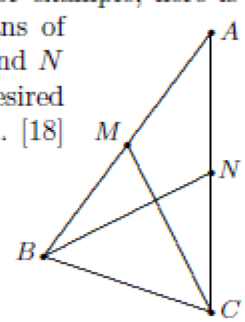
transversal: A line that intersects two other lines in a diagram. [60]

triangle inequality: For any *P*, *Q*, and *R*, $PQ \le PR + RQ$. It says that any side of a triangle is less than the sum of the other two sides. [20]

trinomial: The sum of three unlike monomials, e.g. $x^2 - x + 2$ or $3x^3y - 7x^5 + 8qrs$. [37]

two-column proof: A way of outlining a geometric deduction. Steps are in the left column, and supporting reasons are in the right column. For example, here is how one might show that an isosceles triangle ABC has two medians of the same length. It is given that $AB = AC$ and that M and N are the midpoints of sides AB and AC, respectively. The desired conclusion is that medians CM and BN have the same length. [18]

$AB = AC$	given
$AM = AN$	M and N are midpoints
$\angle MAC = \angle NAB$	shared angle
$\Delta MAC \cong \Delta NAB$	SAS
$CM = BN$	CPCTC



variable: A letter (such as *x*, *y*, or *n*) used to represent a number. A few letters (such as *m* and *n*) tend to be associated with integers, but this is not a rule. [2]

vertical angles: Puzzling terminology that is often used to describe a pair of nonadjacent angles formed by two intersecting lines. [56]

***x*-intercept**: The x-coordinate of a point where a line or curve meets the x-axis. [18]

***y*-intercept**: The y-coordinate of a point where a line or curve meets the y-axis. [18]