**Trigonometry**

**Graph of a General Sine Function**

## General Form

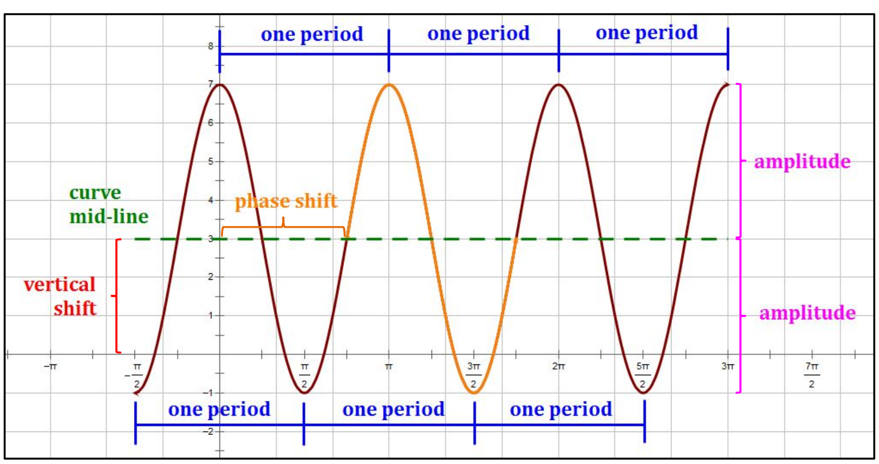
The general form of a sine function is: $\boldsymbol{y = A\ \sin(Bx - C) + D}$.

In this equation, we find several parameters of the function which will help us graph it. In particular:

- Amplitude: $\boldsymbol{Amp = |A|}$. The amplitude is the magnitude of the stretch or compression of the function from its parent function: $y = \sin x$.
- Period: $\boldsymbol{P = \frac{2\pi}{B}}$. The period of a trigonometric function is the horizontal distance over which the curve travels before it begins to repeat itself (i.e., begins a new cycle). For a sine or cosine function, this is the length of one complete wave; it can be measured from peak to peak or from trough to trough. Note that 2π is the period of $y = \sin x$.
- Phase Shift: $\boldsymbol{PS = \frac{C}{B}}$. The phase shift is the distance of the horizontal translation of the function. Note that the value of $\boldsymbol{C}$ in the general form has a minus sign in front of it, just like $\boldsymbol{h}$ does in the vertex form of a quadratic equation: $\boldsymbol{y = (x - h)^2 + k}$. So,
    - A minus sign in front of the $\boldsymbol{C}$ implies a translation to the right, and
    - A plus sign in front of the $\boldsymbol{C}$ implies a implies a translation to the left.
- Vertical Shift: $\boldsymbol{VS = D}$. This is the distance of the vertical translation of the function. This is equivalent to $\boldsymbol{k}$ in the vertex form of a quadratic equation: $\boldsymbol{y = (x - h)^2 + k}$.

**Example:** $\boldsymbol{y = 4\sin\left(2x - \frac{3}{2}\pi\right) + 3}$

The midline has the equation $\mathrm{y = D}$. In this example, the midline is: $\mathrm{y = 3}$. One wave, shifted to the right, is shown in orange below.




**For this example:**

$\boldsymbol{A = 4;\ \ B = 2;\ \ C = \frac{3}{2}\pi;\ \ D = 3}$

Amplitude: $\boldsymbol{Amp = |A| = |4| = 4}$

Period: $\boldsymbol{P = \frac{2\pi}{B} = \frac{2\pi}{2} = \pi}$

Phase Shift: $\boldsymbol{PS = \frac{C}{B} = \frac{\frac{3}{2}\pi}{2} = \frac{3}{4}\pi}$

Vertical Shift: $\boldsymbol{VS = D = 3}$

## Trigonometry

## Graphing a Sine Function with No Vertical Shift: $y = A\sin(Bx - C)$

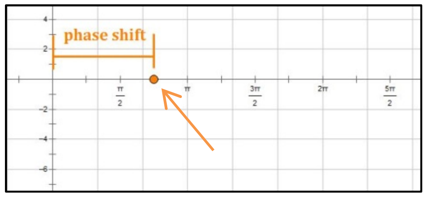
A wave (cycle) of **the sine function** has three zero points (points on the x-axis) – at the beginning of the period, at the end of the period, and halfway in-between.

**Example:**

$y = 4\sin\left(2x - \frac{3}{2}\pi\right).$

**Step 1:** Phase Shift: $PS = \frac{C}{B}$.

The first wave begins at the point **$PS$ units to the right of the Origin**.

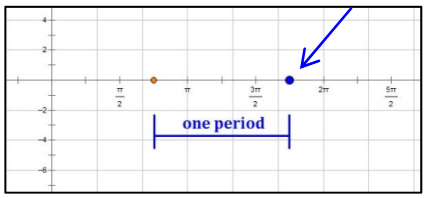



$PS = \frac{C}{B} = \frac{\frac{3}{2}\pi}{2} = \frac{3}{4}\pi.$

The point is: $\left(\frac{3}{4}\pi, 0\right)$

**Step 2:** Period: $P = \frac{2\pi}{B}$.

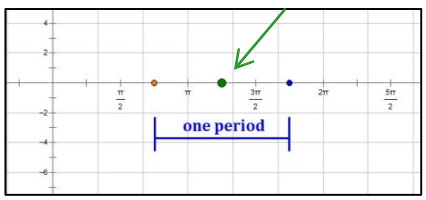
The first wave ends at the point **$P$ units to the right of where the wave begins**.




$P = \frac{2\pi}{B} = \frac{2\pi}{2} = \pi.$ The first wave ends at the point:

$\left(\frac{3}{4}\pi + \pi, 0\right) = \left(\frac{7}{4}\pi, 0\right)$

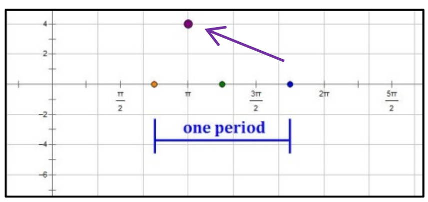
**Step 3:** The third zero point is located halfway between the first two.




The point is:

$\left(\frac{\frac{3}{4}\pi + \frac{7}{4}\pi}{2}, 0\right) = \left(\frac{5}{4}\pi, 0\right)$

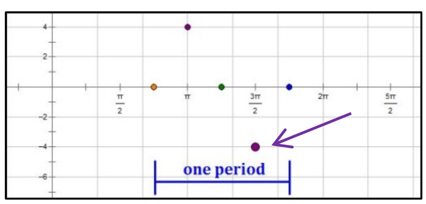
**Step 4:** The $y$-value of the point halfway between the left and center zero points is "$A$".




The point is:

$\left(\frac{\frac{3}{4}\pi + \frac{5}{4}\pi}{2}, 4\right) = (\pi, 4)$

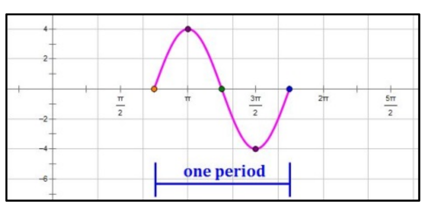
**Step 5:** The $y$-value of the point halfway between the center and right zero points is "$-A$".




The point is:

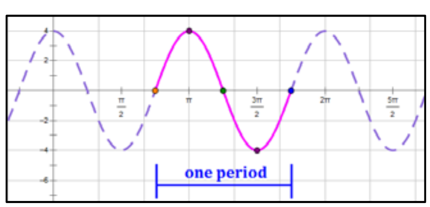
$\left(\frac{\frac{5}{4}\pi + \frac{7}{4}\pi}{2}, -4\right) = \left(\frac{3}{2}\pi, -4\right)$

**Step 6:** Draw a smooth curve through the five key points.




This will produce the graph of one wave of the function.

**Step 7:** Duplicate the wave to the left and right as desired.




Note: If $D \neq 0$, all points on the curve are shifted vertically by $D$ units.