GRAPHING LINEAR EQUATIONS IN TWO VARIABLES

The graphs of linear equations in two variables are straight lines. Linear equations may be written in several forms:

Slope-Intercept Form: y = mx+ b

In an equation of the form y = mx + b, such as y = −2x − 3, the slope is **m** and the y-intercept is the point **(0, b)**. To graph equations of this form, construct a table of values (**Method 1**) or use the slope and y-intercept (**Method 3**) (see Examples 1 and 6).

General Form: ax + by = c

To graph equations of this form, such as 3x − 2y = −6, find the **x-** and **y-intercepts** (**Method 2**), or solve the equation for y to write it in the form y = mx + b and construct a table of values (see Example 2).

Horizontal Lines: y = b

The graph of **y = b** is a horizontal line passing through the point **(0, b)** on the y-axis. To graph an equation of this form, such as y = 4, plot the point (0, b) on the y-axis and draw a horizontal line through it (see Example 4). If the equation is not in the form y = b, solve the equation for y.

Vertical Lines: x = a

The graph of **x = a** is a vertical line passing through the point **(a, 0)** on the **x-axis**. To graph a vertical line, such as 4x + 12 = 0, solve the equation for x to write it in the form **x = a**, plot the point **(a, 0)** on the x-axis, and draw a vertical line through it (see Example 5).

METHOD 1: CONSTRUCT A TABLE OF VALUES

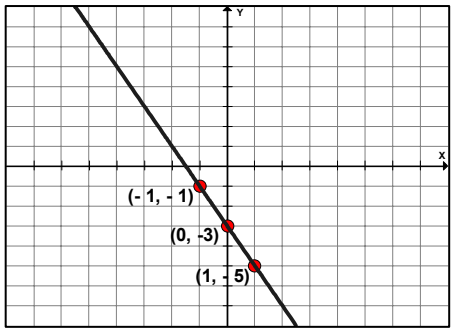
To graph equations of the form **y = mx** and **y = mx + b**,

1) **Choose** three values for **x**. Substitute these values in the equation and solve to find the corresponding y-coordinates.
2) Plot the ordered pairs found in step 1.
3) Draw a straight line through the plotted points. If the points do not line up, a mistake has been made.

Example 1: Graph y = −2x − 3

To graph the equation, choose three values for x and list them in a table. (Hint: choose values that are easy to calculate, like −1, 0, and 1.) Substitute each value in the equation and simplify to find the corresponding y-coordinate. Plot the ordered pairs and draw a straight line through the points.

x	y = −2x − 3	(x, y)
−1	y = −2(−1) – 3 = 2 – 3 = −1	(−1, −1)
0	y = −2(0) – 3 = –3	(0, –3)
1	y = −2(1) – 3 = −2 – 3 = −5	(1, −5)



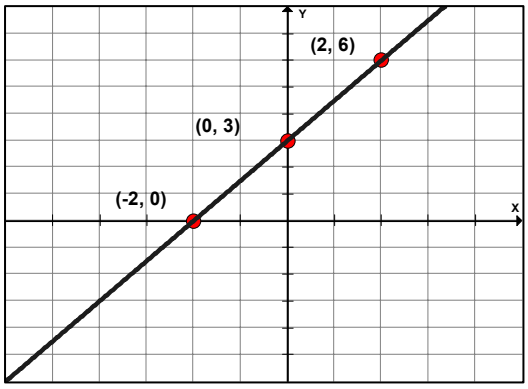
Example 2: Graph $3x - 2y = -6$

The equation $3x - 2y = -6$ is written in the general form. To graph this equation with a table of values, first solve the equation for y to write it in the form **y = mx + b**, as shown:

$$\begin{aligned} 3x - 2y &= -6 \\ 3x - 2y - 3x &= -3x - 6 \\ -2y &= -3x - 6 \\ \frac{-2y}{-2} &= \frac{-3x}{-2} - \frac{6}{-2} \\ y &= \frac{3}{2}x + 3 \end{aligned}$$

Next, choose three values for x and calculate the corresponding y-coordinates. (Hint: to cancel fractions, choose multiples of the denominator.) Plot the points in the table and draw a line through them.

x	$\mathbf{y = \frac{3}{2}x + 3}$	**(x, y)**
−2	$y = \frac{3}{2}(-2) + 3 = -3 + 3 = 0$	(−2, 0)
0	$y = \frac{3}{2}(0) + 3 = 0$	(0, 3)
2	$y = \frac{3}{2}(2) + 3 = 3 + 3 = 6$	(1, 6)



METHOD 2: FIND THE X- AND Y-INTERCEPTS

In Example 2, the line crosses the x-axis at (−2, 0) and y-axis at (0, 3). The **point** where the line crosses the **x-axis** is called the **x-intercept**. At this point, the y-coordinate is **0**. The **point** were the line crosses the **y-axis** is called the **y-intercept**. At this point, the x-coordinate is **0**.

When an equation is written in the general form, such as $-2x + 4y = 8$, it is easier to graph the equation by finding the intercepts.

1) To find the **x-intercept**, let **y = 0** then substitute 0 for y in the equation and solve for x.

2) To find the **y-intercept**, let **x = 0** then substitute 0 for x in the equation and solve for y.

3) Plot the intercepts, label each point, and draw a straight line through these points.

Example 3: Graph −2x + 4y = 8

1) To graph the equation, find the x- and y-intercepts.

To find the **x-intercept**, let **y = 0** and solve the equation for x.

$y = 0, \quad -2x + 4y = 8$

$-2x + 4(0) = 8$

$-2x = 8$

$x = -4$

The x-intercept is (−4, 0).

To find the **y-intercept**, let **x = 0** and solve the equation for y.

$x = 0, \quad -2x + 4y = 8$

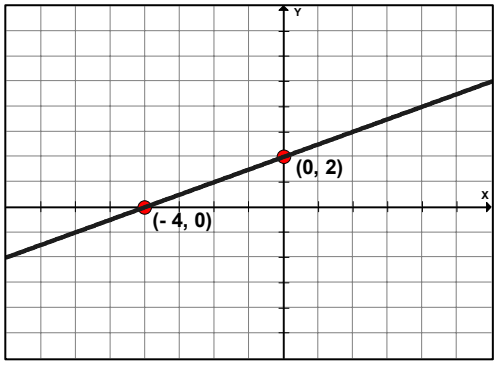
$-2(0) + 4y = 8$

$4y = 8$

$y = 2$

The y-intercept is (0, 2).

2) Next plot each intercept, label the points, and draw a line through them.

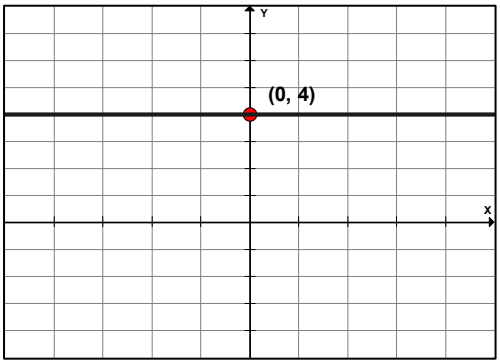


Graphing Horizontal and Vertical Lines

The graph of **y = b** is a **horizontal line** passing through the point **(0, b)**, the **y-intercept**. The graph of **x = a** is a **vertical line** passing through the point **(a, 0)**, the **x-intercept**.

Example 4: Graph y = 4

To graph the equation, plot the intercept on the y-axis, label the point, and draw a horizontal line through the point.

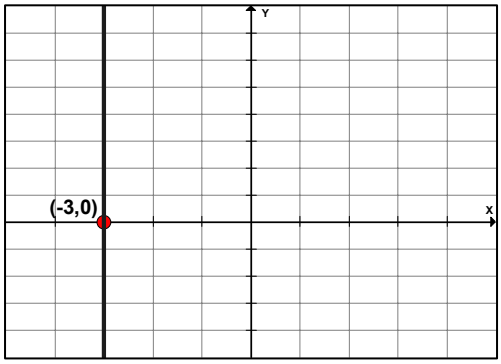


Example 5: Graph 4x + 12 = 0

First, solve the equation for x to write it in the form **x = a**.

$$\begin{aligned} 4x + 12 &= 0 \\ -12 \quad &-12 \\ 4x &= -12 \\ x &= -3 \end{aligned}$$

The **x-intercept** is (−3, 0). Plot this point on the x-axis, label the point, and draw a vertical line through the point.



METHOD 3: USE THE SLOPE AND Y-INTERCEPT

To graph an equation using the slope and y-intercept,

1) Write the equation in the form **y = mx + b** to find the slope **m** and the y-intercept **(0, b)**.

2) Next, plot the y-intercept.

3) From the y-intercept, move up or down and left or right, depending on whether the slope is positive or negative. Draw a point, and from there, move up or down and left or right again to find a third point.

4) Draw a straight line through all three points.

Example 6: Graph 2x + 5y = 10.

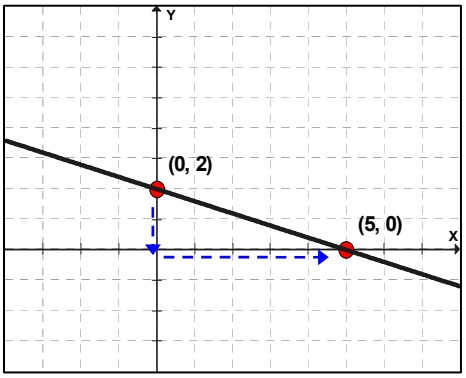
To graph the equation using the slope and y-intercept, write the equation in the form **y = mx + b** to find the slope **m** and the y-intercept **(0, b)**.

$$\begin{aligned} 2x + 5y &= 10 \\ 2x + 5y - 2x &= -2x + 10 \\ 5y &= -2x + 10 \\ \frac{5y}{5} &= \frac{-2x}{5} + \frac{10}{5} \\ y &= -\frac{2}{5}x + 2 \end{aligned}$$

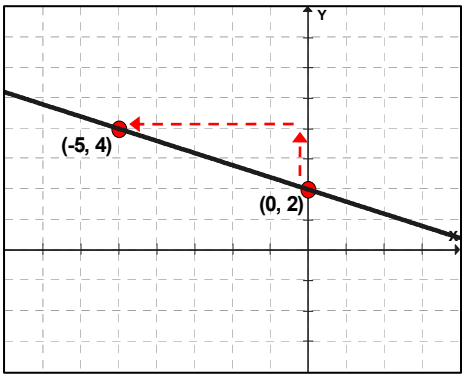
The slope **m =** $\frac{\text{the change in y}}{\text{the change in x}} = \frac{\text{rise}}{\text{run}} = -\frac{2}{5}$ and the **y-intercept** is (0, 2).

Now, plot the y-intercept. From there, *move up or down two units* (the rise) then *move right or left five units to the right* (the run) to find additional points.

When the slope is *negative*, make the change in y *negative* to locate points to the *right* of the y-intercept; make the change in x *negative* to locate points to the *left* of the y-intercept.



$m = \frac{-2}{5}$



$m = \frac{2}{-5}$