

Computation of Fractions

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Computation of Fractions: Considerations for Instruction

Purpose and Overview of Guide

The purpose of this guide is to provide strategies and materials for developing and implementing lessons for students who need intensive instruction in the area of fractions. Special education teachers, mathematics interventionists, and others working with students struggling in the area of fractions may find this guide helpful.

Within college- and career-ready standards, fractions are taught in Grades 3–5. This guide may be used as these concepts are introduced or with students at higher grade levels who continue to struggle with the concepts. Sample activities, worksheets, and supplemental materials also accompany this guide and are available for download at <http://www.intensiveintervention.org>.

The guide is divided into four sections:

1. The sequence of skills as defined by college- and career-ready standards
2. A list of important vocabulary and symbols
3. A brief explanation of the difficulties that students may have with fractions
4. Suggested strategies for teaching fraction computation concepts

Sequence of Skills—College- and Career-Ready Standards

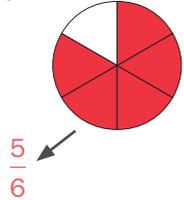
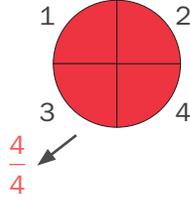
Build fractions from unit fractions—applying and extending operations of whole numbers.

- Add and subtract fractions.
- Decompose fractions in more than one way.
- Add and subtract mixed numbers with like denominators.
- Solve word problems involving addition or subtraction.
- Multiply fractions.
- Solve word problems involving multiplication.

- Use equivalent fractions.
- Add and subtract fractions with unlike denominators.
- Solve word problems involving addition or subtracting of unlike denominators.
- Divide fractions.
- Solve word problems involving division of fractions.
- Continue multiplication.

Vocabulary and Symbols

The following terms are important for students to understand when working with fractions:

<p>Fraction: A part of a whole, with all parts equivalent.</p> $\frac{1}{4}, \frac{1}{2}, \frac{2}{3}, \frac{1}{8}, \frac{4}{5}$	<p>Numerator: How many parts of the whole.</p> 	<p>Denominator: How many parts make up the whole.</p> 
<p>Common Denominator: One or more fractions have the same denominator. Necessary for adding and subtracting fractions.</p> $\frac{1}{8} + \frac{2}{8}$	<p>Equivalent Fractions: Fractions with equal value.</p> $\frac{4}{6} = \frac{2}{3} \quad \frac{2}{8} = \frac{1}{4}$	<p>Simplify/Reduce: Putting fractions in lowest terms.</p> $\frac{12}{15} = \frac{3 \times 4}{3 \times 5} = \frac{4}{5}$
<p>Least Common Multiple (LCM): The smallest common multiple of two or more denominators. Used to determine common denominator.</p> <p>Multiples of 3: 3, 6, 9, 12, 15</p> <p>Multiples of 5: 5, 10, 15, 20</p> <p>LCM is 15.</p>	<p>Greatest Common Factor (GCF): Largest common factor for the numerator and the denominator. Used to simply/reduce fractions.</p> <p>Factors of 12: 1, 2, 3, 4, 6, 12</p> <p>Factors of 15: 1, 3, 5, 15</p> <p>GCF is 3.</p>	<p>Proper Fraction: A fraction that is less than one.</p> $\frac{1}{4}, \frac{1}{2}, \frac{2}{3}, \frac{1}{8}, \frac{4}{5}$
<p>Improper Fraction: A fraction that is greater than one.</p> $\frac{6}{4}, \frac{9}{6}, \frac{14}{5}, \frac{8}{2}$	<p>Mixed Number: A number that has a whole number <i>and</i> a fraction.</p> $4 \frac{1}{4} \quad 10 \frac{2}{3}$	<p>Unit Fraction: A fraction with 1 in the numerator.</p> $\frac{1}{12}, \frac{1}{8}, \frac{1}{5}, \frac{1}{3}, \frac{1}{2}$

Common Areas of Difficulty

Prerequisite Skills Not Mastered

- Basic fact retrieval (for computation and comparison of fractions with unlike denominators)

Specific Fraction Skills

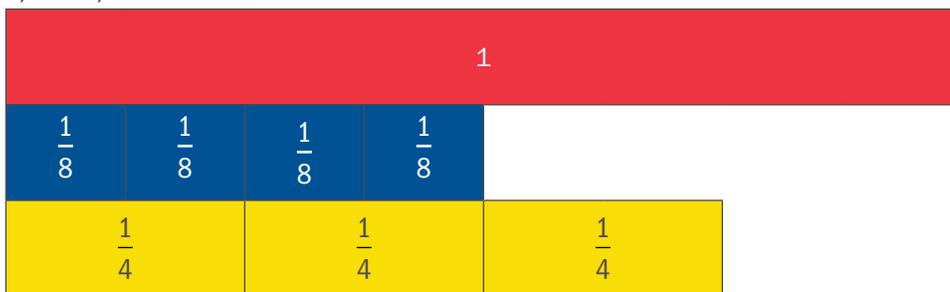
- Reading fractions
- Writing fractions
- Understanding that the larger the denominator, the smaller the value
- Poor understanding of multiples
- Understanding the four models of fractions and when to use them:
 - Area
 - Sets
 - Measurement
 - Division

Conceptual Understanding

Fraction tiles and **fraction circles** can be used to help students visualize and conceptually understand many fraction concepts. These manipulatives represent 1 whole, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{10}$, and $\frac{1}{12}$.

Adding Fractions

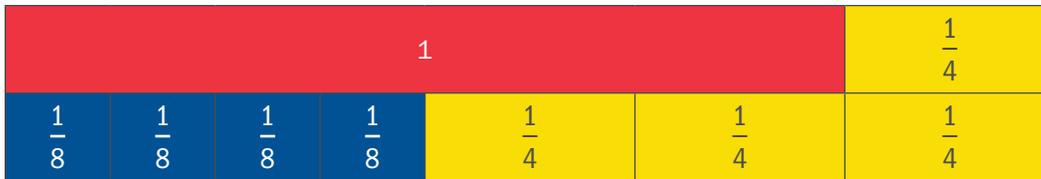
$$\frac{4}{8} + \frac{3}{4} =$$



Start with the whole.

Place $\frac{4}{8}$ under the whole.

Place $\frac{3}{4}$ under the $\frac{4}{8}$.



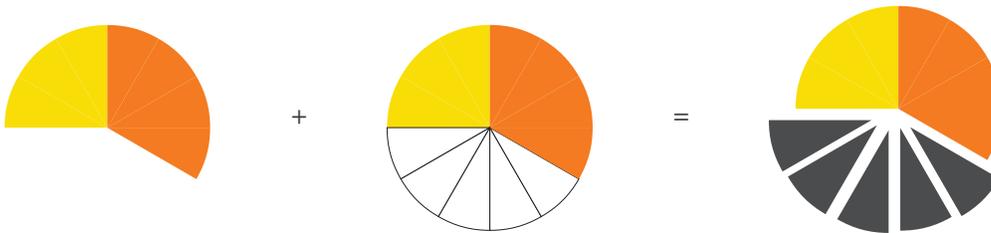
Combine $\frac{4}{8} + \frac{3}{4}$ under the whole.

Students should realize that $\frac{1}{4}$ more is needed to complete the problem.

$\frac{4}{8} + \frac{3}{4} = 1$ and $\frac{1}{4}$.

The same can be done using fraction circles.

$$\frac{1}{4} + \frac{1}{3}$$



Start with $\frac{1}{4}$ and $\frac{1}{3}$ pieces.

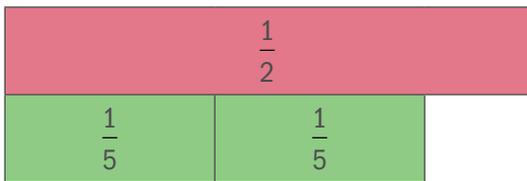
Place the pieces on the whole circle.

Determine that $\frac{1}{12}$ pieces need to be used.

Show that $\frac{5}{12}$ of the whole remains, which means $\frac{1}{4} + \frac{1}{3}$ equals $\frac{7}{12}$.

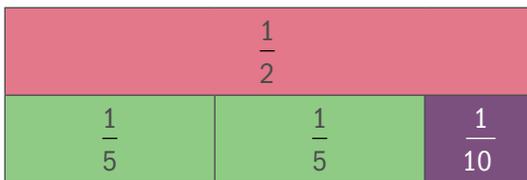
Subtracting Fractions

$$1/2 - 2/5 =$$



Start with $1/2$ and show subtracting $2/5$.

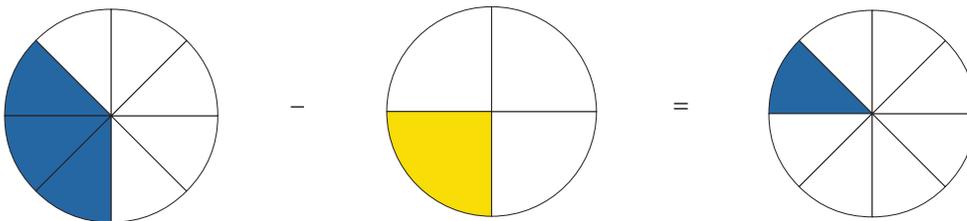
$1/2 - 2/5$ is the difference between a pink tile and two green tiles.



Place a $1/10$ next to $2/5$.

The difference is one purple tile, or $1/10$.

$$3/8 - 1/4$$



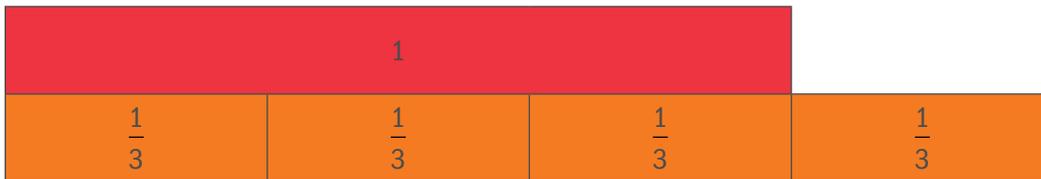
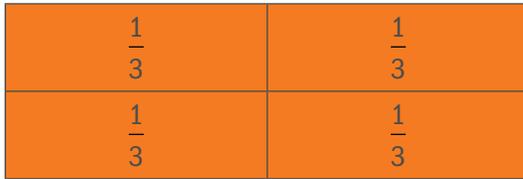
Start with $3/8$ and show subtracting $1/4$.

This shows that $1/4$ is equal to $2/8$, so the difference is $1/8$.

Multiplying Fractions

$$2 \times \frac{2}{3}$$

Show two sets of $\frac{2}{3}$ tiles.



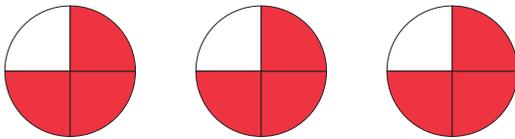
Show 1 whole.

Show two sets of $\frac{2}{3}$ tiles.

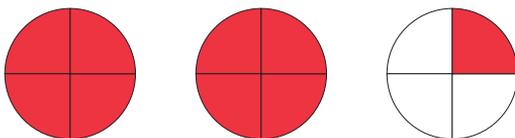
This shows $2 \times \frac{2}{3}$ equals 1 and $\frac{1}{3}$.

$$3 \times \frac{3}{4}$$

Show three sets of $\frac{3}{4}$ tiles.



Combine the sets, showing that they are equal to 2 and $\frac{1}{4}$.



Note to Teachers: Fraction tiles and circles will not work with all problems. You should ensure that the manipulatives will work for the problems you plan to demonstrate. For example, you cannot use these manipulatives to show $\frac{1}{5} + \frac{5}{6} = \frac{26}{30}$ because you do not have pieces that show 30ths.

Representing Multiplication of Fractions With Grids

$$2/3 \times 1/4$$

Using the denominators, build a grid—the first denominator represents the number of rows, and the second denominator represents the number of columns.

The numerator in each fraction tells how many rows and columns to shade in.

For $2/3$, shade in two rows.

For $1/4$, shade in one column.

The total number of boxes will be your new denominator: 12.

The total number of boxes shaded for both fractions will be your new numerator:

$$2/3 \times 1/4 = 2/12.$$

Using Fraction Bars to Determine Least Common Denominator

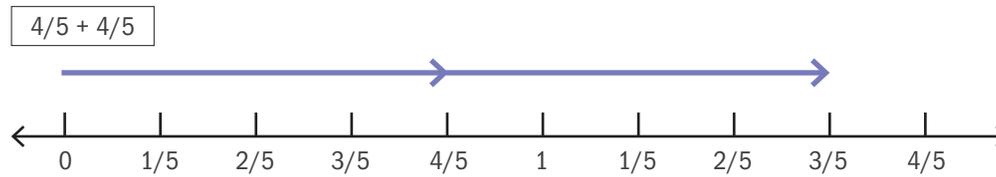
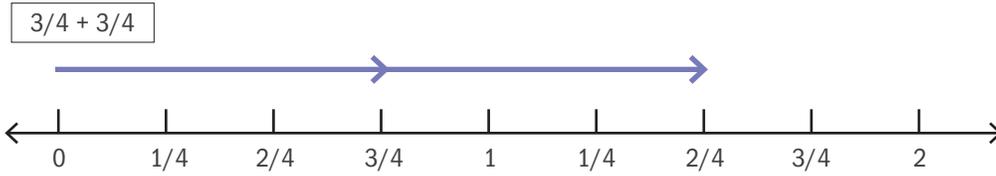
Each bar has a number, 1–9, at the beginning, and each of its multiples is listed (up to $\times 9$) on the bar.

$$2/3 + 3/4 =$$

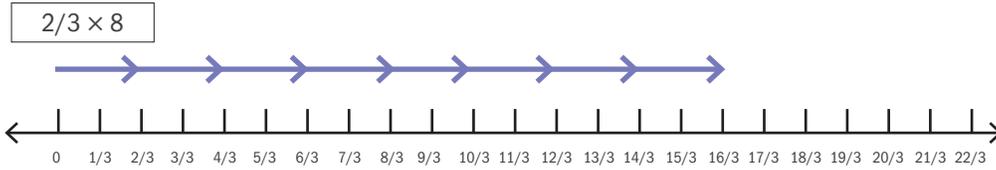
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36

1. Select the fraction bars for 3 and 4. Line them up under each other.
2. By looking at the two fraction bars, determine the LCM, which is 12.
3. Convert each fraction to a new fraction with the denominator of 12.
 - a. $2/3 = ?/12 \rightarrow 12 \div 3 = 4 \rightarrow 4 \times 2 = 8 \rightarrow 2/3 = 8/12$
 - b. $3/4 = ?/12 \rightarrow 12 \div 4 = 3 \rightarrow 3 \times 3 = 9 \rightarrow 3/4 = 9/12$
4. $2/3 + 3/4$ now becomes $8/12 + 9/12$.
5. $8/12 + 9/12 = 17/12$
6. $17/12$ can be reduced to 1 and $5/12$.

Adding Fractions With a Number Line



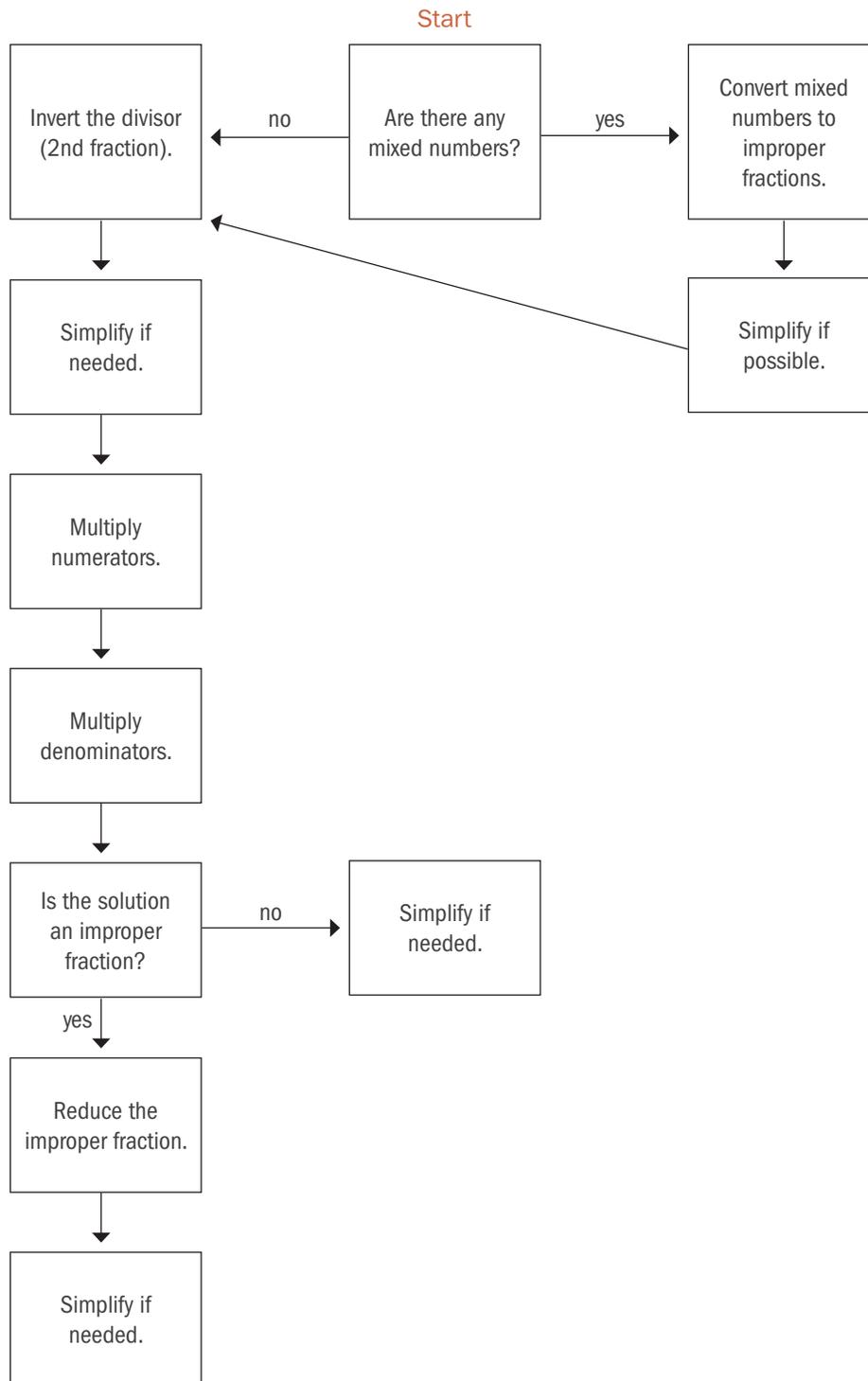
Multiplying Fractions With a Number Line



Procedural Understanding

Flowcharts can assist students in understanding the algorithm of a concept.

Flowchart for Dividing Fractions



Algorithms

Knowing **algorithms** is an important component for solving fraction problems. Students must understand the steps needed to solve each type of problem. Teachers may use these step-by-step procedures to determine where a student may be having difficulty in the algorithm. After a teacher has determined where the issue is, then teaching can begin at that point.

Adding Fractions With Like Denominators

- (a) Make sure the denominators are the same.
- (b) Add the numerators.
- (c) Put the answer over the same denominator.

(a) $\frac{3}{5} + \frac{1}{5}$

(b) $\frac{3 + 1}{5}$

(c) $\frac{4}{5}$

Subtracting Fractions With Like Denominators

- (a) Make sure the denominators are the same.
- (b) Subtract the second numerator from the first.
- (c) Put the answer over the same denominator.

(a) $\frac{7}{12} - \frac{5}{12}$

(b) $\frac{7 - 5}{12}$

(c) $\frac{2}{12}$

Adding Fractions With Unlike Denominators

- (a) Find the least common denominator (LCD) of the fractions: $4 \times 3 = 12$
- (b) Rename the first fraction so it has the LCD.
- (c) Rename the second fraction so it has the LCD.
- (d) Rewrite the problem with the new fractions.

- (e) Add the numerators.
- (f) Put the answer over the LCD.

$$\frac{1}{4} + \frac{2}{3}$$

(a) $4 \times 3 = 12 = \text{LCD}$

(b) $\frac{1 \times 3}{4 \times 3} = \frac{3}{12}$

(c) $\frac{2 \times 4}{3 \times 4} = \frac{8}{12}$

(d) $\frac{3}{12} + \frac{8}{12}$

(e) $\frac{3 + 8}{12} =$

(f) $\frac{11}{12}$

Subtracting Fractions With Unlike Denominators

- (a) Find the LCD of the fractions.
- (b) Rename the first fraction so it has the LCD.
- (c) Rename the second fraction so it has the LCD.
- (d) Rewrite the problem with the new fractions.
- (e) Subtract the second numerator from the first.
- (f) Put the answer over the LCD.

$$\frac{3}{5} - \frac{1}{2} =$$

(a) $5 \times 2 = 10 = \text{LCD}$

(b) $\frac{3 \times 2}{5 \times 2} = \frac{6}{10}$

(c) $\frac{1 \times 5}{2 \times 5} = \frac{5}{10}$

(d) $\frac{6}{10} - \frac{5}{10}$

(e) $\frac{6 - 5}{10} =$

(f) $\frac{1}{10}$

Multiplying Fractions

- (a) Multiply the numerators of the fractions.
- (b) Multiply the denominators of the fractions.
- (c) Put the product of the numerators on top of the fraction.
- (d) Put the product of the denominators on the bottom of the fraction.

$$\frac{3}{8} \times \frac{3}{4} =$$

(a) $3 \times 3 = 9$

(b) $8 \times 4 = 32$

(c, d) $\frac{9}{32}$

Dividing Fractions

- (a) Invert the second fraction (the divisor).
- (b) Change the division sign to a multiplication sign.
- (c) Multiply the numerators of the fractions.
- (d) Multiply the denominators of the fractions.
- (e) Put the product of the numerators on top of the fraction.
- (f) Put the product of the denominators on the bottom of the fraction.

(a) $\frac{2}{5} \div \frac{3}{4}$

(b) $\frac{2}{5} \times \frac{4}{3}$

(c, d) $\frac{2 \times 4}{5 \times 3} =$

(e, f) $\frac{8}{15}$

Adding Mixed Numbers

- Convert the first mixed number to an improper fraction.
- Multiply the denominator by the whole number.
- Put the product over the denominator.
- Add the new fraction's numerator to the original fraction's numerator.
- Use the same denominator for the new numerator.
- Convert the second mixed number to an improper fraction.
- Multiply the denominator by the whole number.
- Put the product over the denominator.
- Add the new fraction's numerator to the original fraction's numerator.
- Use the same denominator for the new numerator.
- Rewrite the problem with improper fractions.
- Find the LCD of the fractions.
- Rename the first fraction so it has the LCD.
- Rename the second fraction so it has the LCD.
- Rewrite the problem with the LCD.
- Add the numerators.
- Put the answer over the LCD.
- Simplify. Reduce improper fraction to a mixed number.

$$2\frac{3}{4} + 6\frac{3}{8}$$

$$(a) 2\frac{3}{4}$$

$$(b) 4 \times 2 = 8$$

$$(c) \frac{8}{8}$$

$$(d) \frac{4}{8} + 3 =$$

$$(e) \frac{11}{4}$$

$$(f) 6\frac{3}{8}$$

$$(g) 8 \times 6$$

$$(h) = \frac{48}{8}$$

$$(i) \frac{48 + 3}{8} =$$

$$(j) \frac{51}{8}$$

$$(k) \frac{11}{4} + \frac{51}{8}$$

$$(l) 4 \times 2 = 8, 8 \times 1 = 8 = \text{LCD}$$

$$(m) \frac{11 \times 2}{4 \times 2} = \frac{22}{8}$$

$$(n) \frac{51 \times 1}{8 \times 1} = \frac{51}{8}$$

$$(o) \frac{22}{8} + \frac{51}{8}$$

$$(p) \frac{22 + 51}{8} =$$

$$(q) \frac{73}{8} =$$

$$(r) 9\frac{1}{8}$$

Multiplying Mixed Numbers

- Convert the first mixed number to an improper fraction.
- Multiply the denominator by the whole number.
- Put the product over the denominator.
- Add the new fraction's numerator to the original fraction's numerator.
- Use the same denominator for the new numerator.
- Convert the second mixed number to an improper fraction.
- Multiply the denominator by the whole number.
- Put the product over the denominator.
- Add the new fraction's numerator to the original fraction's numerator.
- Use the same denominator for the new numerator.
- Rewrite the problem with the improper fractions.
- Multiply the numerators of the improper fractions.
- Multiply the denominators of the improper fractions.
- Put the product of the numerators on top of the fraction.
- Put the product of the denominators on the bottom of the fraction.
- Simplify. Reduce improper fraction to a mixed number.

Example:

$$3\frac{2}{9} \times 4\frac{3}{4} =$$

(a) $3\frac{2}{9}$

(b) $9 \times 3 = 27$

(c) $\frac{27}{9}$

(d) $\frac{27 + 2}{9} =$

(e) $\frac{29}{9}$

(f) $4\frac{3}{4}$

(g) $4 \times 4 = 16$

(h) $\frac{16}{4}$

(i) $\frac{16 + 3}{4} =$

(j) $\frac{19}{4}$

(k) $\frac{29}{9} \times \frac{19}{4}$

(l) $29 \times 19 = 551$

(m) $9 \times 4 = 36$

(n, o) $\frac{551}{36} =$

(p) $15\frac{11}{36}$

Dividing Mixed Numbers

- (a) Convert the first mixed number to an improper fraction.
- (b) Multiply the denominator by the whole number.
- (c) Put the product over the denominator.
- (d) Add the new fraction's numerator to the original fraction's numerator.
- (e) Use the same denominator for the new numerator.
- (f) Convert the second mixed number to an improper fraction.
- (g) Multiply the denominator by the whole number.
- (h) Put the product over the denominator.
- (i) Add the new fraction's numerator to the original fraction's numerator.
- (j) Use the same denominator for the new numerator.
- (k) Rewrite the problem with the improper fractions.
- (l) Invert the second fraction (the divisor).
- (m) Change the division sign to a multiplication sign.
- (n) Multiply the numerators of the fractions.
- (o) Multiply the denominators of the fractions.
- (p) Put the product of the numerators on the top of the fraction.
- (q) Put the product of the denominators on the bottom of the fraction.
- (r) Simplify. Reduce improper fraction to a mixed number.

Example:

$$6\frac{1}{2} \div 3\frac{2}{3} =$$

(a) $6\frac{1}{2}$

(b) $2 \times 6 = 12$

(c) $\frac{12}{2}$

(d) $\frac{12 + 1}{2} =$

(e) $\frac{13}{2}$

(f) $3\frac{2}{3}$

(g) $3 \times 3 = 9$

(h) $\frac{9}{3}$

(i) $\frac{9 + 2}{3} =$

(j) $\frac{11}{3}$

(k) $\frac{13}{2} \div \frac{11}{3}$

(l, m) $\frac{13}{2} \times \frac{3}{11}$

(n) $13 \times 3 = 39$

(o) $2 \times 11 = 22$

(p, q) $\frac{39}{22} =$

(r) $1\frac{17}{22}$

2. Fraction Addition and Subtraction Concepts

Sample Activities

- a. Activity One: Using Fraction Tiles and Fraction Circles**
- b. Activity Two: Using Fraction Tiles and Fraction Circles**
- c. Activity Three: Using Fraction Tiles and Fraction Circles**

Worksheets

- a. Fraction Addition**
- b. Fraction Subtraction**

Sample Fraction Addition and Subtraction Concepts Activities 1–3

College- and Career-Ready Standards:

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

4.NF.3. Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.

- Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

Activity One: Using Fraction Tiles and Fraction Circles

Purpose:

- To show addition concepts (joining) with fraction tiles (or circles).
- Give the student a visual representation of adding fractions along with an equation that matches the visual.

Principles of Intensive Intervention Illustrated:

- Provide concrete learning opportunities (including use of manipulatives).
- Provide explicit error correction and have the student repeat the correct process.
- Use precise, simple language to teach key concepts or procedures.
- Use explicit instruction and modeling with repetition to teach a concept or demonstrate steps in a process.
- Provide repeated opportunities to practice each step correctly.

Materials (available for download from NCII):

- Fraction tiles or fraction circles (see Supplemental Materials Section)
- Fraction addition flash cards (see Supplemental Materials Section)
- Worksheet: Fraction Addition (for extra practice)

Modeling:



1. Place one tile in front of the student and review numerator and denominator vocabulary.
2. Place four of the $\frac{1}{8}$ tiles in front of the student (not pushed together).
3. Explain that one tile is $\frac{1}{8}$.
4. **Sample language:** “When you put two tiles together, you have $\frac{2}{8}$. By joining the two $\frac{1}{8}$ pieces, you are *adding* them to get $\frac{2}{8}$.” (Show equation saying $\frac{1}{8} + \frac{1}{8} = \frac{2}{8}$.)
5. **Sample language:** “When you add a third $\frac{1}{8}$ tile, you get $\frac{3}{8}$. By joining the three $\frac{1}{8}$ pieces, you are adding them to get $\frac{3}{8}$.” (Show equation saying $\frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$.)
6. Repeat for steps to show $\frac{4}{8}$.
7. Explain: To get any fraction with a number that is greater than 1 in the numerator, you join or add together fractions with 1 in the numerator.

Guided Practice:



1. Place all six of the $\frac{1}{6}$ pieces in front of the student. Also place a piece of paper in front of the student so that he or she can record equations.
2. Ask the student to show $\frac{1}{6}$.
3. Have the student record $\frac{1}{6} = \frac{1}{6}$.
4. Ask the student to show $\frac{2}{6}$.
5. Have the student record $\frac{1}{6} + \frac{1}{6} = \frac{2}{6}$.
6. Repeat until all combinations with 6 in the denominator have been recorded.

(Note: This activity can be used with any/all denominators. Sixths was used for explicative purposes only.) Repeat this activity with at least two more examples (e.g., fourths, thirds) or use the worksheet provided for additional practice.

Corrective Feedback:

Sample incorrect student response: $1/6 + 1/6 = 2/12$.

Teacher feedback: “Remember, when we add fractions with the same denominator, we add each part (point to the numerator). With $1/6 + 1/6$, we add the 1 part plus 1 part to get 2 parts. The answer is $2/6$. $2/12$ would be $1/12 + 1/12$. (Demonstrate with tiles.) Remember, to get fractions with numbers greater than 1 in the numerator, we add fractions with 1 in the numerator that have *the same denominator*. In the new fraction, the denominator stays the same.”

Have the student demonstrate the correct process before moving on.



Activity Two: Using Fraction Tiles and Fraction Circles

Purpose:

- Show subtraction concepts (separating) with fraction tiles (or circles).
- Give the student a visual representation of subtracting fractions.

Principles of Intensive Intervention:

- Provide concrete learning opportunities (including use of manipulatives).
- Provide explicit error correction and have the student repeat the correct process.
- Use precise, simple language to teach key concepts or procedures.
- Use explicit instruction and modeling with repetition to teach a concept or demonstrate steps in a process.
- Provide repeated opportunities to practice each step correctly.

Materials (available for download from NCII):

- Fraction tiles or fraction circles (see Supplemental Materials Section)
- Fraction subtraction flash cards (see Supplemental Materials Section)
- Worksheet: Fraction Subtraction (for extra practice)

Modeling:



1. Place four of the $\frac{1}{8}$ tiles (pushed together) in front of the student.
2. Remind the student that $\frac{4}{8}$ equals $\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$.
3. **Sample language:** “When you have four of the $\frac{1}{8}$ tiles together, you have $\frac{4}{8}$. By separating one of the $\frac{1}{8}$ pieces, you are subtracting $\frac{4}{8} - \frac{1}{8}$ to get $\frac{3}{8}$.”
4. Show equation by separating or removing the $\frac{1}{8}$ tile, saying $\frac{4}{8} - \frac{1}{8} = \frac{3}{8}$.

5. **Sample language:** “When you subtract another $\frac{1}{8}$ tile, you get $\frac{2}{8}$. By separating another $\frac{1}{8}$ piece, you are subtracting to get $\frac{2}{8}$. (Show equation by separating/removing the $\frac{1}{8}$ tile, saying $\frac{3}{8} - \frac{1}{8} = \frac{2}{8}$.)
6. Repeat steps to show $\frac{2}{8} - \frac{1}{8} = \frac{1}{8}$.
7. Explain: When you start with a fraction with a number greater than 1 in the numerator, you can separate the parts. Each part is a fraction with 1 in the numerator.

Guided Practice:



1. Place all six of the $\frac{1}{6}$ pieces in front of the student. Also place a piece of paper in front of the student so that he or she can record equations.
2. Ask the student to show $\frac{6}{6}$ with the tiles.
3. Have the student separate $\frac{1}{6}$ from $\frac{6}{6}$ with the tiles.
4. Have the student record the subtraction equation $\frac{6}{6} - \frac{1}{6} = \frac{5}{6}$.
5. Have the student separate $\frac{1}{6}$ from $\frac{5}{6}$ with the tiles.
6. Have the student record the subtraction equation $\frac{5}{6} - \frac{1}{6} = \frac{4}{6}$.
7. Repeat until all separations with 6 in the denominator have been recorded.

Repeat this activity with at least two more examples (e.g., fourths, thirds) or use the worksheet provided for additional practice.

Corrective Feedback:

Sample student response: The student is unable to write an equation that matches the separating action shown with the tiles.

Teacher feedback: “Remember, when we separate one of the parts, we subtract. You need to write a subtraction equation. What fraction did you start with?”

The student responds.

Teacher feedback: “Good. So that is the first fraction in the equation. How many did you take away?”

The student responds.

Teacher feedback: “Good. So we write minus $1/6$. Now to figure out the answer, we count how many tiles are left. How many are there on the table?”

The student responds.

Teacher feedback: “Good. Remember, when we subtract a fraction with 1 in the numerator, you are separating 1 part. Your answer has *the same denominator*, and the numerator should be only 1 less than what you started with.”

Read the correct number sentence. Have the student demonstrate the correct process before moving on.

Activity Three: Using Fraction Tiles and Fraction Circles

College- and Career-Ready Standards Addressed:

4.NF.3. Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.

- Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples:* $3/8 = 1/8 + 1/8 + 1/8$; $3/8 = 1/8 + 2/8$; $2\ 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8$.

Purpose:

- Show how a fraction can be decomposed.
- Give the student a visual representation of breaking down a fraction into parts.

Principles of Intensive Intervention:

- Provide concrete learning opportunities (including use of manipulatives).
- Provide explicit error correction and have the student repeat the correct process.
- Use precise, simple language to teach key concepts or procedures.
- Use explicit instruction and modeling with repetition to teach a concept or demonstrate steps in a process.
- Provide repeated opportunities to practice each step correctly.

Materials (available for download from NCII):

- Fraction tiles or fraction circles (see Supplemental Materials section)
- Fraction flash cards (see Supplemental Materials section)
- Blank paper to record each decomposition

(Note: Fraction tiles may be preferable for this activity at first because unit fractions are written on the tiles.)

Modeling:

1. Display fraction to be decomposed (use $\frac{3}{5}$ as example).
2. Explain that this fraction is $\frac{3}{5}$ and show it with the tiles pushed together.
3. Tell the student that you will make different fraction combinations that equal $\frac{3}{5}$.
4. Separate the $\frac{3}{5}$ tiles. Write the equation $\frac{3}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$.
5. Now push two of the tiles together. Write the equation $\frac{3}{5} = \frac{2}{5} + \frac{1}{5}$.

Guided Practice:

1. Place a fraction in front of the student (use $\frac{5}{8}$ as example).
2. Ask the student to make the different fraction combinations that equal $\frac{5}{8}$.
3. The student should show the combinations with the tiles and then write the equation to record each combination.
4. Combinations for $\frac{5}{8}$ include the following:

$$\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{5}{8}$$

$$\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{2}{8} = \frac{5}{8}$$

$$\frac{1}{8} + \frac{1}{8} + \frac{3}{8} = \frac{5}{8}$$

$$\frac{1}{8} + \frac{4}{8} = \frac{5}{8}$$

$$\frac{2}{8} + \frac{3}{8} = \frac{5}{8}$$

$$\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$$

$$\frac{4}{8} + \frac{1}{8} = \frac{5}{8}$$

Corrective Feedback:

Sample incorrect student response 1: The student is unable to separate tiles appropriately to form a new combination for $\frac{5}{8}$.

Teacher feedback: Demonstrate a new combination and explain how it is different from the previous combination(s). For example: "In that combination, we have all of the $\frac{1}{8}$ tiles separated to show $\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{5}{8}$. In a new combination, I push two of the $\frac{1}{8}$ tiles together. This shows $\frac{2}{8}$. Now we have $\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{2}{8} = \frac{5}{8}$. Let's write the equation."

Have the student write the equation correctly before moving on.

Sample incorrect student response 2: The student is unable to write the equation that matches the tile combination $1/8 + 1/8 + 1/8 + 2/8 = 5/8$.

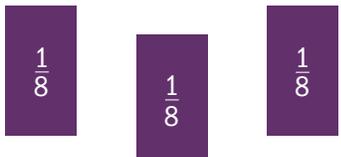
Teacher feedback: “Look at the tiles. You have $1/8$ alone, $1/8$ alone, $1/8$ alone, and then two $1/8$ tiles pushed together. The $1/8$ tiles pushed together are $2/8$. So let’s count together. We have $1/8 + 1/8 + 1/8 + 2/8 = 5/8$. That is the equation you write on the paper. Try again.”

Read the correct number sentence. Have the student demonstrate the correct process before moving on.

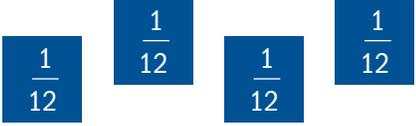
Worksheet

Fraction Addition

Objective: Show a fraction using fraction tiles. Write the addition equation represented by that fraction.

Show the Fraction	Write the Equation
<p>Example: $\frac{3}{8}$</p> 	$\frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$
$\frac{2}{3}$	
$\frac{4}{5}$	
$\frac{3}{4}$	

Objective: Show a fraction using fraction tiles. Write the addition equation represented by that fraction.

Show the Fraction	Write the Equation
<p style="text-align: center;">Example: $\frac{4}{12}$</p> 	$\frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} = \frac{4}{12}$
$\frac{6}{8}$	
$\frac{3}{10}$	
$\frac{5}{6}$	

Worksheet

Fraction Subtraction

Directions: Show a fraction using fraction tiles (on the worksheet or separately). Write and solve the subtraction equation represented by that fraction.

Show the Fraction	Write the Equation
<p style="text-align: center;">Example: $\frac{4}{4}$</p> <div style="display: flex; justify-content: center; gap: 10px;"> <div style="border: 1px solid black; padding: 5px; text-align: center;">$\frac{1}{4}$</div> <div style="border: 1px solid black; padding: 5px; text-align: center;">$\frac{1}{4}$</div> <div style="border: 1px solid black; padding: 5px; text-align: center;">$\frac{1}{4}$</div> <div style="border: 1px solid black; padding: 5px; text-align: center;">$\frac{1}{4}$</div> </div>	<p>Subtract $\frac{1}{4}$: $\frac{4}{4} - \frac{1}{4} = \frac{3}{4}$</p> <div style="display: flex; justify-content: center; gap: 10px;"> <div style="border: 1px solid black; padding: 5px; text-align: center;">$\frac{1}{4}$</div> <div style="border: 1px solid black; padding: 5px; text-align: center;">$\frac{1}{4}$</div> <div style="border: 1px solid black; padding: 5px; text-align: center;">$\frac{1}{4}$</div> <div style="border: 1px solid black; padding: 5px; text-align: center;">$\frac{1}{4}$</div> </div>
	<p>Subtract $\frac{1}{4}$ again: $\frac{3}{4} - \frac{1}{4} = \frac{2}{4}$</p> <div style="display: flex; justify-content: center; gap: 10px;"> <div style="border: 1px solid black; padding: 5px; text-align: center;">$\frac{1}{4}$</div> <div style="border: 1px solid black; padding: 5px; text-align: center;">$\frac{1}{4}$</div> <div style="border: 1px solid black; padding: 5px; text-align: center;">$\frac{1}{4}$</div> </div>
<p style="text-align: center;">$\frac{3}{3}$</p>	<p>Subtract $\frac{1}{3}$.</p>
	<p>Subtract $\frac{1}{3}$ again.</p>

Show the Fraction	Write the Equation
$\frac{5}{5}$	Subtract $\frac{1}{5}$.
	Subtract $\frac{1}{5}$ again.
	Subtract $\frac{1}{5}$ again.



Directions: Show a fraction using fraction tiles (on the worksheet or separately). Write and solve the subtraction equation represented by that fraction.

Show the Fraction	Write the Equation
$\frac{6}{6}$	Subtract $\frac{1}{6}$. Example: $\frac{6}{6} - \frac{1}{6} = \frac{5}{6}$
	Subtract $\frac{1}{6}$ again.
	Subtract $\frac{1}{6}$ again.
	Subtract $\frac{1}{6}$ again.



Show the Fraction	Write the Equation
$\frac{8}{8}$	Subtract $\frac{1}{8}$.
	Subtract $\frac{1}{8}$ again.

3. Addition and Subtraction With Unlike Denominators

Sample Activities

- a. Activity One: Writing Equivalent Fractions (Finding a Common Denominator)**
- b. Activity Two: Writing Equivalent Fractions to Solve Addition Problems (Finding a Common Denominator)**
- c. Activity Three: Writing Equivalent Fractions to Solve Subtraction Problems (Finding a Common Denominator)**

Worksheets

- a. Adding and Subtracting Fractions With Unlike Denominators**

Sample Adding and Subtracting With Unlike Denominators Activities 1–3

College- and Career-Ready Standards Addressed: Use equivalent fractions as a strategy to add and subtract fractions (with unlike denominators).

5.NF.1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example,* $2/3 + 5/4 = 8/12 + 15/12 = 23/12$. (In general, $a/b + c/d = (ad + bc)/bd$.)

Activity One: Writing Equivalent Fractions (Finding a Common Denominator)

Purpose: Practice finding common denominators (prerequisite skill for solving addition and subtraction problems with unlike denominators).

Principles of Intensive Intervention:

- Use precise, simple language to teach key concepts or procedures.
- Use explicit instruction and modeling with repetition to teach a concept or demonstrate steps in a process.
- Provide concrete learning opportunities (including use of manipulatives).
- Provide explicit error correction and have the student repeat the correct process.

Materials (available for download from NCII):

- Paper and pencil
- Multiplication Chart (optional; see Supplemental Materials section)

Modeling 1 (only one fraction is changed):

1. Present two fractions with different denominators (e.g., $\frac{2}{6}$ and $\frac{3}{12}$).
2. Point to the denominators (6 and 12) and say, “These are not the same.”
3. Explain that we need to change one or both of the fractions so that the denominators are the same. Note that when we rewrite a fraction, it *must always* be equivalent.
4. Explain that you should look at the smaller denominator first to see if it is a factor of the larger denominator.
5. Explain that 6 is a factor of 12 because 6 times 2 equals 12. (If you are using a Multiplication Chart, show 6 times 2 equals 12 on the chart.)
6. Explain that to write an equivalent fraction, you multiply the numerator and the denominator by the same number.
7. Explain that we multiply $\frac{2}{6}$ times $\frac{2}{2}$ to rewrite $\frac{2}{6}$ as an equivalent fraction with 12 in the denominator.
8. Demonstrate setting up the multiplication.
9. Solve the multiplication to get $\frac{4}{12}$ as the answer.
10. Explain that now $\frac{3}{12}$ and $\frac{4}{12}$ have the same denominator.

Modeling 2 (both fractions are changed):

1. Present two fractions with different denominators (e.g., $\frac{2}{3}$ and $\frac{1}{4}$).
2. Point to the denominators (3 and 4) and say, “These are not the same.”
3. Explain that we need to change one or both of the fractions so that the denominators are the same. When we rewrite a fraction, it *must* be equivalent.
4. Explain that you should look at the smaller denominator first to see if it is a factor of the larger denominator.
5. Explain that 3 is *not* a factor of 4. You cannot divide 4 by 3 and get a whole number. (If you are using a Multiplication Chart, show the student that 3 is not a factor of 4.)
6. Explain that you need to write equivalent fractions for *both* fractions and have to decide on the least common denominator (LCD).

7. For each fraction, you will multiply the numerator and the denominator by the denominator of the other fraction.

$$\frac{2}{3} \frac{(4)}{(4)} \text{ and } \frac{1}{4} \frac{(3)}{(3)}$$

8. Explain that to write an equivalent fraction, you multiply the numerator and the denominator by the *same number*.
9. Demonstrate setting up the multiplication.
10. Explain that we multiply $\frac{2}{3}$ times $\frac{4}{4}$ to rewrite $\frac{2}{3}$ as an equivalent fraction. The new fraction is $\frac{8}{12}$.
11. Explain that we multiply $\frac{1}{4}$ times $\frac{3}{3}$ to rewrite $\frac{1}{4}$ as an equivalent fraction. The new fraction is $\frac{3}{12}$.
12. Both fractions are rewritten with 12 in the denominator.
13. Explain that now $\frac{8}{12}$ and $\frac{3}{12}$ have the same denominator, so addition or subtraction can be performed.

Guided Practice:

1. Present two fractions with different denominators (e.g., $\frac{1}{4} + \frac{3}{8}$ or $\frac{1}{2} + \frac{3}{5}$).
2. Ask the student to look at the denominators. Ask, "Are they the same?"
3. The student decides that the denominators are not the same.
4. Direct the student to see if the smaller denominator is a multiple of the larger denominator.
5. If yes ($\frac{1}{4} + \frac{3}{8}$), the student multiplies the numerator and the denominator of the fraction with the smaller denominator by a factor to make the denominators the same.
6. If no ($\frac{1}{2} + \frac{3}{5}$), the student multiplies the numerator and the denominator of each fraction with the denominator of the other fraction. (See sample under Modeling 2.)
7. The student finds the new equivalent fractions so that both have the same denominator.

Corrective Feedback:

Sample incorrect student response 1: The student cannot determine if the smaller denominator is a factor of the larger denominator.

Teacher feedback:

Option 1: The teacher should multiply the smaller denominator (review the facts) by 1, 2, 3, and so on until the other denominator is an answer (showing it is a multiple) or until the other denominator gets surpassed (showing it is not a multiple).

Option 2: The teacher should use the Multiplication Chart to show the multiples in numerical order to determine if it is a multiple of the larger denominator.

Sample incorrect student response 2: The student cannot recall multiplication facts when writing the new equivalent fraction.

Teacher feedback: Use the Multiplication Chart as an aid as needed.

Always have the student demonstrate the correct procedure before moving on.

Activity Two: Writing Equivalent Fractions to Solve Addition Problems (Finding a Common Denominator)

Purpose:

- Solve a computation problem with unlike denominators.
- Recognize when fractions need to be rewritten so that the computation problem can be solved.

Principles of Intensive Intervention Illustrated:

- Use precise, simple language to teach key concepts or procedures.
- Use explicit instruction and modeling with repetition to teach a concept or demonstrate steps in a process.
- Provide explicit error correction and have the student repeat the correct process.

Materials (available for download from NCII):

- Paper and pencil
- Multiplication Chart (optional; see Supplemental Materials section)
- Worksheet: Adding and Subtracting Fractions With Unlike Denominators (for extra practice)

Modeling:

1. Present an addition problem with different denominators (e.g., $1/3 + 2/5$).
2. Point to the denominators (3 and 5) and say, “These are not the same. I need to write equivalent fractions to make these denominators the same.”
3. Follow the steps from Activity One to convert the fractions to $5/15$ and $6/15$, respectively. This should be a review.
4. Explain that now the fractions have the same denominator, so we can add.
5. Add the numerators: $5 + 6 = 11$. The denominator stays the same: 15.
6. The answer is $11/15$.

Guided Practice:

1. Present an addition problem with different denominators (e.g., $1/4 + 5/8$).
2. Ask the student to find the denominators and check if they are the same.
3. Have the student determine if the fractions need to be rewritten.
4. The student determines that only one fraction needs to be rewritten.
5. Follow the steps from Activity One.
6. Remind the student that now $2/8$ and $5/8$ have the same denominator, so we can add.
7. Add the numerators: $2 + 5 = 7$. The denominator stays the same: 8.
8. The answer is $7/8$.

Corrective Feedback:

Sample incorrect student response 1: The student cannot determine if the smaller denominator is a factor of the larger denominator.

Teacher Feedback:

Option 1: The teacher should multiply the smaller denominator (review the facts) by 1, 2, 3, and so on until the other denominator is an answer (showing it is a multiple) or until the other denominator gets surpassed (showing it is not a multiple).

Option 2: The teacher should use the Multiplication Chart to show the multiples in numerical order to determine if it is a multiple of the larger denominator.

Sample incorrect student response 2: The student added the denominators after changing them to be the same (misconception about what adding fractions means).

Teacher Feedback:

Option 1: Explain to or remind the student that we do not add the denominators. Use the tiles or circles to demonstrate adding.

Option 2: Go back to the adding activities where the denominators are the same to review adding fractions.

Always have the student demonstrate the correct procedure before moving on.

Activity Three: Writing Equivalent Fractions to Solve Subtraction Problems (Finding a Common Denominator)

Purpose:

- Find common denominators (equivalent fractions).
- Rewrite subtraction equations with unlike denominators so that they can be solved.

Principles of Intensive Intervention Illustrated:

- Use precise, simple language to teach key concepts or procedures.
- Use explicit instruction and modeling with repetition to teach a concept or demonstrate steps in a process.
- Provide explicit error correction and have the student repeat the correct process.

Materials (available for download from NCII):

- Paper and pencil
- Multiplication Chart (optional; see Supplemental Materials section)
- Worksheet: Adding and Subtracting Fractions With Unlike Denominators (for extra practice)

Modeling:

1. Present a subtraction problem with different denominators (e.g., $2/3 - 2/5$).
2. Point to the denominators (3 and 5) and say, “These are not the same. I need to write equivalent fractions to make these denominators the same.”
3. Follow the steps from Activity One (should be a review).
4. Explain that now $10/15$ and $6/15$ have the same denominator, so we can subtract.
5. Subtract the numerators: $10 - 6 = 4$. The denominator stays the same: 15.
6. The answer is $4/15$.

Guided Practice:

1. Present a subtraction problem with different denominators (e.g., $3/4 - 1/3$).
2. Ask the student to find the denominators. Ask if they are the same.
3. Have the student determine if the fractions need to be rewritten.
4. The student determines that both fractions need to be rewritten.
5. Follow the steps from Activity One (should be a review).
6. Remind the student that now $9/12$ and $4/12$ have the same denominator, so we can subtract.
7. Subtract the numerators: $9 - 4 = 5$. The denominator stays the same: 12.
8. The answer is $5/12$.

Corrective Feedback:

Sample incorrect student response 1: The student cannot determine if the smaller denominator is a factor of the larger denominator.

Teacher Feedback:

Option 1: The teacher should multiply the smaller denominator (review the facts) by 1, 2, 3, and so on until the other denominator is an answer (showing it is a multiple) or until the other denominator gets surpassed (showing it is not a multiple).

Option 2: The teacher should use the Multiplication Chart to show the multiples in numerical order to determine if it is a multiple of the larger denominator or not.

Sample incorrect student response 2: The student subtracted the denominators after changing them to be the same and ended with 0 in the answer (misconception about what subtracting fractions means).

Teacher Feedback:

Option 1: Explain to or remind the student that we do not subtract the denominators. Use the tiles or circles to demonstrate subtracting.

Option 2: Go back to the subtracting activities where the denominators are the same to review subtracting fractions.

Always have the student demonstrate the correct procedure before moving on.

Worksheet

Adding and Subtracting Fractions With Unlike Denominators

Objective: Given two fractions with unlike denominators, rewrite fractions with a common denominator and add or subtract to solve.

Steps:

1. Look at the two fractions. Can you add or subtract them or do you need to find a common denominator?
2. Multiply to find a common denominator, if necessary.
3. Add or subtract the numerator to solve the problem.

Only One Fraction Change

Problem	Rewrite With Common Denominator	Solve	Answer
$\frac{1}{2} + \frac{1}{4} =$			
$\frac{5}{12} - \frac{1}{3} =$			

Problem	Rewrite With Common Denominator	Solve	Answer
$\frac{3}{8} - \frac{1}{4} =$			
$\frac{2}{5} + \frac{3}{10} =$			

Both Fractions Change

Problem	Rewrite With Common Denominator	Solve	Answer
$\frac{1}{3} + \frac{1}{2} =$			
$\frac{3}{4} - \frac{1}{6} =$			



Problem	Rewrite With Common Denominator	Solve	Answer
$\frac{2}{7} + \frac{2}{3} =$			
$\frac{3}{5} - \frac{1}{4} =$			
$\frac{3}{8} + \frac{2}{5} =$			
$\frac{2}{3} - \frac{3}{10} =$			

4. Using Addition and Subtraction to Convert Mixed Numbers and Improper Fractions

Sample Activities

- a. Using Addition and Subtraction to Convert Mixed Numbers and Improper Fractions**

Worksheets

- a. Using Addition and Subtraction to Convert Mixed Numbers and Improper Fractions**
- b. Using Addition and Subtraction to Convert Mixed Numbers and Improper Fractions (Scaffolded)**

Sample Activity: Using Addition and Subtraction to Convert Mixed Numbers and Improper Fractions

College- and Career-Ready Standards Addressed: This activity does not directly correlate to one specific domain. It is however, relevant for understanding fractions greater than 1 and the ways to represent them.

Mathematics | Grade 4

4NF: Overall statement on Fractions: Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., $15/9 = 5/3$), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

- Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.
- Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

Mathematics | Grade 5

5NF: Use equivalent fractions as a strategy to add and subtract fractions.

Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, $2/3 + 5/4 = 8/12 + 15/12 = 23/12$. (In general, $a/b + c/d = (ad + bc)/bd$.)*

Purpose: Introduce a mathematical procedure for converting improper fractions and mixed numbers.

Note: Converting improper fractions and mixed numbers using addition and subtraction may be easier for initial instruction than using multiplication and division concepts, particularly for students with intensive needs in mathematics. Students also should be taught to convert in that way after they have mastered the addition and subtraction procedure.

Materials (available for download from NCII):

- Cards with improper fractions or mixed numbers (see Supplemental Materials section)
- Prompt cards for converting fractions (see Supplemental Materials section)
- Worksheet: Using Addition and Subtraction to Convert Mixed Numbers and Improper Fractions (scaffolded or nonscaffolded version, for extra practice)

Prerequisite Vocabulary: equivalent, numerator, denominator, improper fraction, mixed number

Prerequisite Skills and Knowledge:

- Fractions equal to 1 include those when the numerator and the denominator are the same (e.g., 1/1, 2/2, 3/3, and 4/4).
- An understanding of simple addition and subtraction.
- Understanding addition and subtraction of fractions (denominators must be the same before the operation can be performed).

Prompt Card Steps:

Converting Improper Fractions to Mixed Numbers Using Subtraction

1. Subtract a fraction equal to 1 (e.g., 2/2 or 4/4) from the improper fraction. Make sure the denominators are the same.
2. The mixed number equals 1 **combined with** the new proper fraction:

$$\frac{8}{5}$$

$$\frac{8}{5} - \frac{5}{5} = \frac{3}{5}$$

$$\frac{8}{5} = 1\frac{3}{5}$$

Changing Mixed Numbers to Improper Fractions

1. Change the 1 whole part of the mixed number to a fraction with the same denominator.
2. Add the fraction equal to 1 to the proper fraction.
3. The improper fraction equals the answer to the addition problem:

$$1\frac{3}{5}$$
$$\frac{5}{5} + \frac{3}{5} = \frac{8}{5}$$
$$\mathbf{1\frac{3}{5} = \frac{8}{5}}$$

Modeling (when the improper fraction and the mixed number are less than 2):

Improper Fraction to Mixed Number Using Subtraction

1. The teacher shows $5/4$ and explains that it is an improper fraction.
2. The teacher explains that the fraction is improper because it is greater than 1.
3. The fraction is greater than 1 because the numerator (5) is greater than the denominator (4).
4. The teacher explains that improper fractions have a mixed number that is equivalent.
5. The teacher explains that we can find the equivalent mixed number without using manipulatives (as in the conceptual activity).
6. The teacher displays the prompt card (Improper Fraction to Mixed Number Using Subtraction) and explains that the two steps on the card will be followed.
7. The teacher reads the first step.
8. The teacher determines that a fraction equal to 1 with 4 in the denominator is $4/4$.
9. The teacher writes $5/4 - 4/4$.
10. The teacher explains that the answer is $1/4$.
11. The teacher reads the second step.
12. The teacher explains that the new mixed number is 1 and $1/4$.
13. This means that $5/4$ is the same as 1 and $1/4$.

Mixed Number to Improper Fraction Using Addition

1. The teacher shows $1 \text{ and } \frac{3}{4}$ and explains that it is a mixed number.
2. The teacher explains that this is a mixed number because it has a whole number and a proper fraction.
3. The teacher explains that mixed numbers have an improper fraction that is equivalent.
4. The teacher explains that we can find the equivalent mixed number without using manipulatives.
5. The teacher displays the prompt card and explains that the steps on the card will be followed.
6. The teacher reads the first step.
7. The teacher determines that a fraction equal to 1 with 4 in the denominator is $\frac{4}{4}$.
8. The teacher writes $\frac{4}{4}$.
9. The teacher reads the second step.
10. The teacher sets up the addition problem: $\frac{4}{4} + \frac{3}{4}$.
11. The teacher solves the addition problem.
12. The teacher explains that the improper fraction equivalent to $1 \text{ and } \frac{3}{4}$ is $\frac{7}{4}$.
13. This means that $1 \text{ and } \frac{3}{4}$ is the same as $\frac{7}{4}$.

Guided Practice (when the improper fraction and the mixed number are less than 2):

Improper Fraction to Mixed Number Using Subtraction

1. The student shows $\frac{5}{3}$ and explains that it is an improper fraction.
2. The student explains that this fraction is improper because it is greater than 1.
3. The fraction is greater than 1 because the numerator (5) is greater than the denominator (3).
4. The student explains that improper fractions have a mixed number that is equivalent.

5. The student follows the prompt card.
6. The student reads the first step.
7. The student determines that a fraction equal to 1 with 3 in the denominator is $\frac{3}{3}$.
8. The student writes $\frac{5}{3} - \frac{3}{3}$.
9. The student explains that the answer is $\frac{2}{3}$.
10. The student reads the second step.
11. The student explains that the new mixed number is 1 and $\frac{2}{3}$.
12. This means that $\frac{5}{3}$ is the same as 1 and $\frac{2}{3}$.

Mixed Number to Improper Fraction Using Addition

1. The student shows 1 and $\frac{1}{6}$ and explains that this is a mixed number.
2. The student explains that this is a mixed number because it has a whole number and a proper fraction.
3. The student explains that mixed numbers have an improper fraction that is equivalent.
4. The student follows the prompt card.
5. The student reads the first step.
6. The student determines that a fraction equal to 1 with 6 in the denominator is $\frac{6}{6}$.
7. The student writes $\frac{6}{6}$.
8. The student reads the second step.
9. The student sets up the addition problem: $\frac{6}{6} + \frac{1}{6}$.
10. The student solves the addition problem.
11. The student explains that the improper fraction equivalent to 1 and $\frac{1}{6}$ is $\frac{7}{6}$.
12. This means 1 and $\frac{1}{6}$ is the same as $\frac{7}{6}$.

Note: Prompt cards should be faded as students become more comfortable with converting fractions. Teachers should use discretion on when to do this based on each student's individual needs.

Corrective Feedback:

Sample student error 1: The student has difficulty articulating, following, or reading the prompt card steps.

Teacher Feedback: Provide help with the explanation; this concept is difficult when it is first taught. The students will likely require repeated practice before being able to complete the activity without teacher assistance.

Sample student error 2: The student has difficulty recognizing mixed numbers versus improper fractions.

Teacher Feedback: Remind the student of the rules: (1) Mixed numbers *always* have a whole number with a fraction. (2) Improper fractions have a numerator that is greater than the denominator.

Sample student error 3: The student has difficulty finding the correct size of fraction pieces.

Teacher Feedback: Remind the student to look at the denominator in the problem and choose the fraction pieces with the same number as the value of the denominator.

Note: If you are working on improper fractions and mixed numbers greater than 2, see the following prompt card modifications. Improper fractions and mixed numbers greater than 2 should not be taught until students are accurate and fluent converting those between 1 and 2.

Modified Prompt Card Steps (for numbers greater than 2):

Improper Fraction to Mixed Number Using Subtraction

1. Subtract a fraction equal to 1 from the improper fraction with the same denominator.
2. Check to see if the remaining fraction is a proper fraction.
3. If so, the mixed number equals 1 **combined with** the new proper fraction.
4. If not, repeat Steps 1 and 2 until the answer to the subtraction problem is a proper fraction.
5. Count how many times a fraction equal to 1 was subtracted, which equals the whole number in the mixed number.
6. The new mixed number equals the whole number **and** the proper fraction.

Mixed Number to Improper Fraction Using Addition

1. Change the whole number part of the mixed number to fractions equal to 1 with the same denominator.
2. For example, if the mixed number is 3 and $\frac{1}{4}$, you should write $\frac{4}{4} + \frac{4}{4} + \frac{4}{4}$, which is three “1’s.”
3. Add the fractions equal to 1 so that $\frac{4}{4} + \frac{4}{4} + \frac{4}{4} = \frac{12}{4}$.
4. Add the proper fraction to the improper fraction so that $\frac{12}{4} + \frac{1}{4} = \frac{13}{4}$.
5. The improper fraction equals the answer to the *final* addition problem.

Worksheet

Using Addition and Subtraction to Convert Mixed Numbers and Improper Fractions

Directions: Decide if each number in the left column is a mixed number or an improper fraction. Then convert between mixed numbers and improper fractions. Use your prompt card to help if you need it.

Fraction	Show Your Work	Answer
$\frac{11}{7}$		
$\frac{8}{5}$		
$1\frac{2}{3}$		
$\frac{7}{4}$		
$1\frac{4}{9}$		

Fraction	Show Your Work	Answer
$1\frac{2}{5}$		
$\frac{13}{8}$		
$\frac{8}{6}$		
$1\frac{1}{9}$		
$\frac{4}{3}$		
$1\frac{7}{9}$		
$1\frac{1}{4}$		

Worksheet

Using Addition and Subtraction to Convert Mixed Numbers and Improper Fractions (Scaffolded)

Directions: Use the steps on your prompt card to convert mixed numbers to improper fractions and improper fractions to mixed numbers.

Mixed Number to Improper Fraction

Mixed Number	Show Your Work	Improper Fraction
$1\frac{2}{3}$		
$1\frac{1}{8}$		

Mixed Number	Show Your Work	Improper Fraction
$1\frac{3}{5}$		
$1\frac{4}{7}$		
$1\frac{5}{6}$		

Improper Fraction to Mixed Number

Improper Fraction	Show Your Work	Mixed Number
$\frac{9}{5}$		



Improper Fraction	Show Your Work	Mixed Number
$\frac{6}{4}$		
$\frac{3}{2}$		
$\frac{15}{8}$		
$\frac{5}{3}$		

Both Types

Fraction	Show Your Work	Answer
$\frac{11}{7}$		
$\frac{8}{5}$		
$1\frac{2}{3}$		
$\frac{7}{4}$		



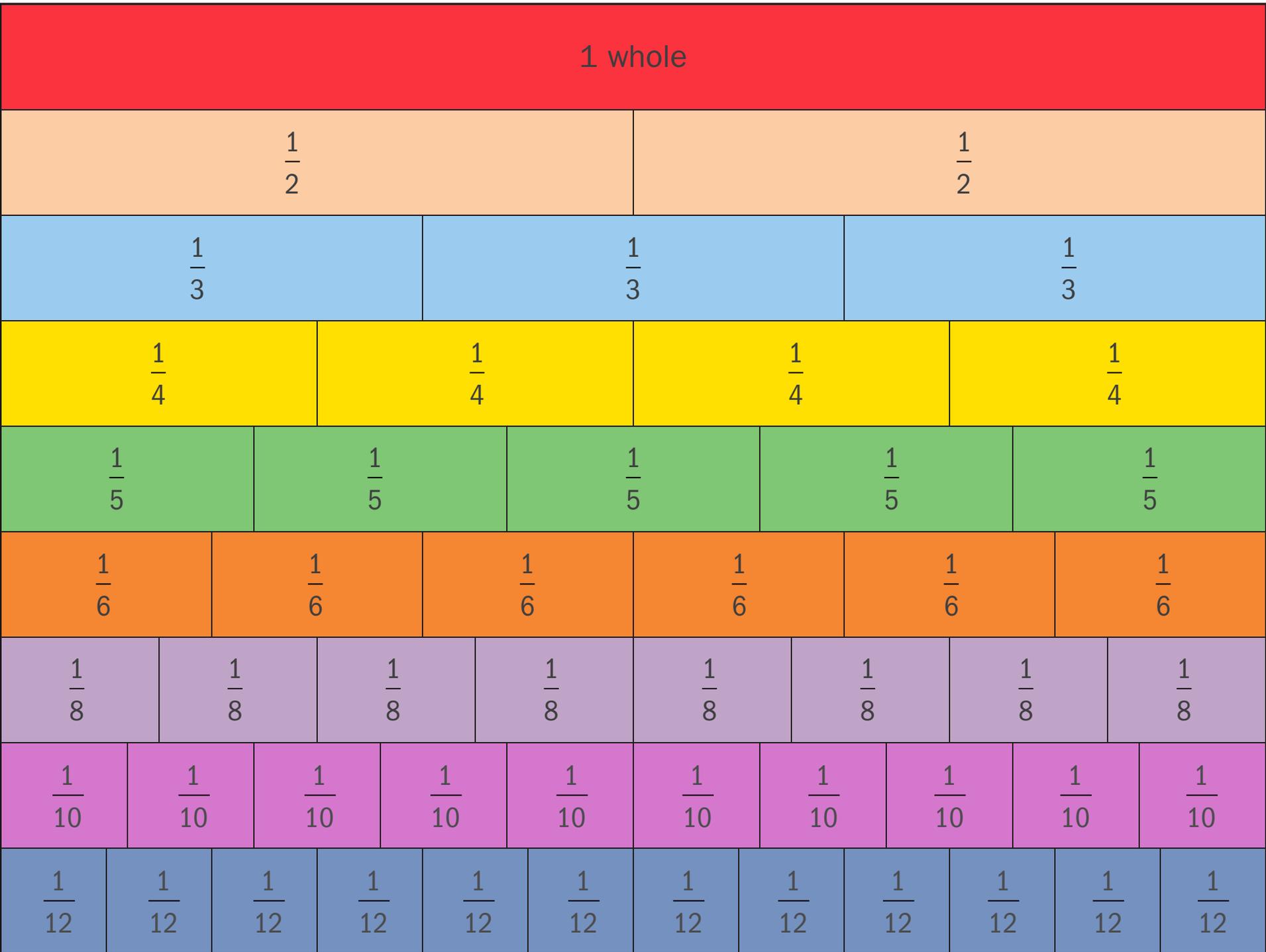
Fraction	Show Your Work	Answer
$1\frac{4}{9}$		
$1\frac{2}{5}$		
$\frac{13}{8}$		
$\frac{8}{6}$		

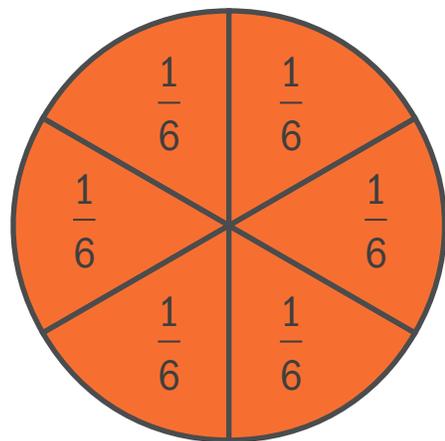
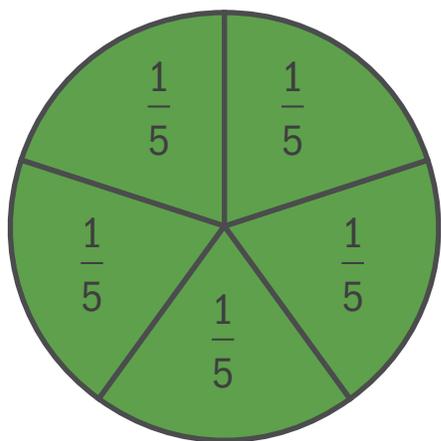
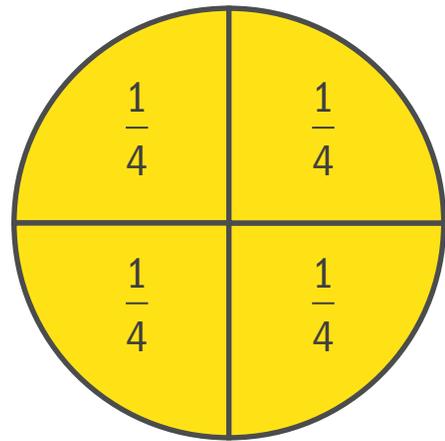
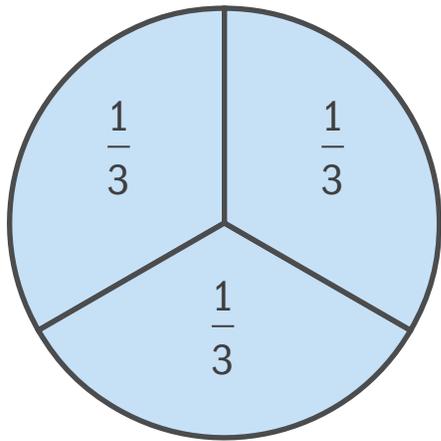
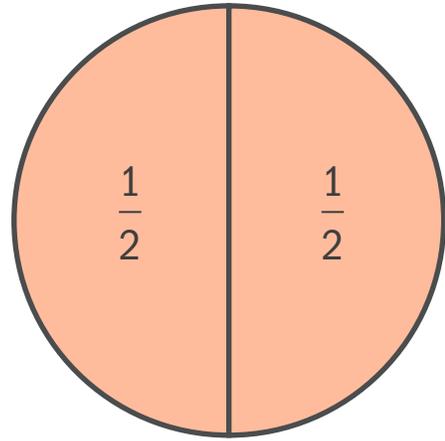
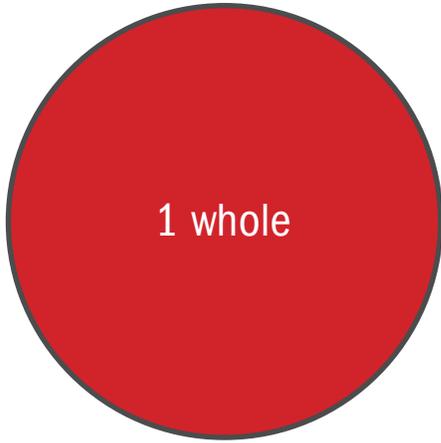
Fraction	Show Your Work	Answer
$1\frac{1}{9}$		
$\frac{4}{3}$		
$1\frac{7}{9}$		
$1\frac{1}{4}$		
$\frac{6}{5}$		

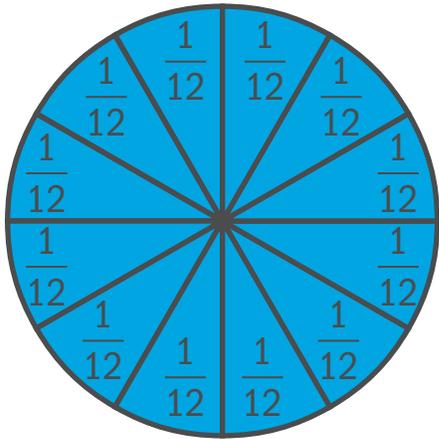
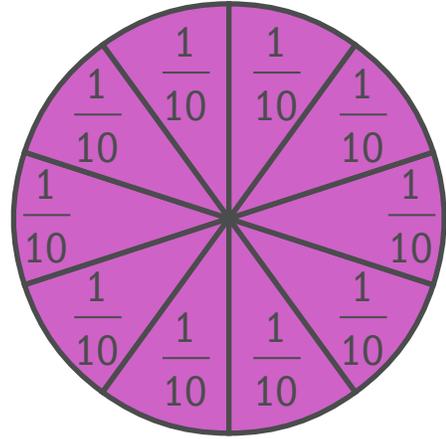
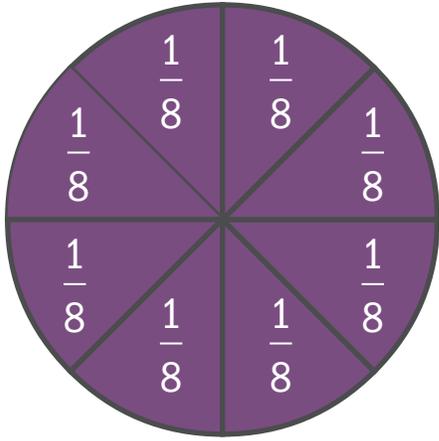


5. Supplemental Materials

- a. Fractions Tiles**
- b. Fraction Circles**
- c. Fraction Addition Flash Cards**
- d. Fraction Subtraction Flash Cards**
- e. Fraction Cards**
- f. Multiplication Chart**
- g. Improper Fraction Cards**
- h. Mixed Number Cards**
- i. Prompt Cards for Converting Improper Fractions
and Mixed Numbers**







$$\frac{1}{2} + \frac{1}{2}$$

$$\frac{1}{3} + \frac{1}{3}$$

$$\frac{2}{3} + \frac{1}{3}$$

$$\frac{1}{4} + \frac{2}{4}$$

$$\frac{1}{4} + \frac{3}{4}$$

$$\frac{1}{5} + \frac{1}{5}$$

$$\frac{1}{5} + \frac{4}{5}$$

$$\frac{2}{5} + \frac{2}{5}$$

$$\frac{2}{3}$$

$$\frac{2}{2} = 1$$

$$\frac{3}{4}$$

$$\frac{3}{3} = 1$$

$$\frac{2}{5}$$

$$\frac{4}{4} = 1$$

$$\frac{4}{5}$$

$$\frac{5}{5} = 1$$

$$\frac{1}{6} + \frac{2}{6}$$

$$\frac{2}{6} + \frac{2}{6}$$

$$\frac{3}{6} + \frac{2}{6}$$

$$\frac{5}{6} + \frac{0}{6}$$

$$\frac{2}{8} + \frac{5}{8}$$

$$\frac{1}{8} + \frac{4}{8}$$

$$\frac{3}{8} + \frac{5}{8}$$

$$\frac{0}{8} + \frac{7}{8}$$

$$\frac{4}{6}$$

$$\frac{3}{6}$$

$$\frac{5}{6}$$

$$\frac{5}{6}$$

$$\frac{5}{8}$$

$$\frac{7}{8}$$

$$\frac{7}{8}$$

$$\frac{8}{8} = 1$$

$$\frac{2}{10} + \frac{6}{10}$$

$$\frac{3}{10} + \frac{1}{10}$$

$$\frac{4}{10} + \frac{5}{10}$$

$$\frac{7}{10} + \frac{3}{10}$$

$$\frac{3}{12} + \frac{5}{12}$$

$$\frac{7}{12} + \frac{5}{12}$$

$$\frac{1}{12} + \frac{6}{12}$$

$$\frac{10}{12} + \frac{0}{12}$$

$$\frac{4}{10}$$

$$\frac{8}{10}$$

$$\frac{10}{10} = 1$$

$$\frac{9}{10}$$

$$\frac{12}{12} = 1$$

$$\frac{8}{12}$$

$$\frac{10}{12}$$

$$\frac{7}{12}$$

$$\begin{array}{r} 2 \\ - \\ 2 \end{array} - \begin{array}{r} 1 \\ - \\ 2 \end{array}$$

$$\begin{array}{r} 3 \\ - \\ 3 \end{array} - \begin{array}{r} 1 \\ - \\ 3 \end{array}$$

$$\begin{array}{r} 3 \\ - \\ 3 \end{array} - \begin{array}{r} 2 \\ - \\ 3 \end{array}$$

$$\begin{array}{r} 4 \\ - \\ 4 \end{array} - \begin{array}{r} 3 \\ - \\ 4 \end{array}$$

$$\begin{array}{r} 3 \\ - \\ 4 \end{array} - \begin{array}{r} 2 \\ - \\ 4 \end{array}$$

$$\begin{array}{r} 4 \\ - \\ 5 \end{array} - \begin{array}{r} 1 \\ - \\ 5 \end{array}$$

$$\begin{array}{r} 5 \\ - \\ 5 \end{array} - \begin{array}{r} 3 \\ - \\ 5 \end{array}$$

$$\begin{array}{r} 6 \\ - \\ 6 \end{array} - \begin{array}{r} 2 \\ - \\ 6 \end{array}$$

$$\begin{array}{r} 2 \\ - \\ 3 \end{array}$$

$$\begin{array}{r} 1 \\ - \\ 2 \end{array}$$

$$\begin{array}{r} 1 \\ - \\ 4 \end{array}$$

$$\begin{array}{r} 1 \\ - \\ 3 \end{array}$$

$$\begin{array}{r} 3 \\ - \\ 5 \end{array}$$

$$\begin{array}{r} 1 \\ - \\ 4 \end{array}$$

$$\begin{array}{r} 4 \\ - \\ 6 \end{array}$$

$$\begin{array}{r} 2 \\ - \\ 5 \end{array}$$

$$\begin{array}{r} 5 \\ \hline 6 \end{array} - \begin{array}{r} 1 \\ \hline 6 \end{array}$$

$$\begin{array}{r} 3 \\ \hline 6 \end{array} - \begin{array}{r} 1 \\ \hline 6 \end{array}$$

$$\begin{array}{r} 7 \\ \hline 8 \end{array} - \begin{array}{r} 2 \\ \hline 8 \end{array}$$

$$\begin{array}{r} 8 \\ \hline 8 \end{array} - \begin{array}{r} 6 \\ \hline 8 \end{array}$$

$$\begin{array}{r} 5 \\ \hline 8 \end{array} - \begin{array}{r} 5 \\ \hline 8 \end{array}$$

$$\begin{array}{r} 6 \\ \hline 8 \end{array} - \begin{array}{r} 1 \\ \hline 8 \end{array}$$

$$\begin{array}{r} 10 \\ \hline 10 \end{array} - \begin{array}{r} 3 \\ \hline 10 \end{array}$$

$$\begin{array}{r} 10 \\ \hline 10 \end{array} - \begin{array}{r} 8 \\ \hline 10 \end{array}$$

$$\frac{2}{6}$$

$$\frac{4}{6}$$

$$\frac{2}{8}$$

$$\frac{5}{8}$$

$$\frac{5}{8}$$

$$\frac{0}{8} = 0$$

$$\frac{2}{10}$$

$$\frac{7}{10}$$

$$\begin{array}{r} 7 \\ \hline 10 \end{array} - \begin{array}{r} 2 \\ \hline 10 \end{array}$$

$$\begin{array}{r} 8 \\ \hline 10 \end{array} - \begin{array}{r} 6 \\ \hline 10 \end{array}$$

$$\begin{array}{r} 9 \\ \hline 10 \end{array} - \begin{array}{r} 1 \\ \hline 10 \end{array}$$

$$\begin{array}{r} 12 \\ \hline 12 \end{array} - \begin{array}{r} 1 \\ \hline 12 \end{array}$$

$$\begin{array}{r} 12 \\ \hline 12 \end{array} - \begin{array}{r} 3 \\ \hline 12 \end{array}$$

$$\begin{array}{r} 8 \\ \hline 12 \end{array} - \begin{array}{r} 6 \\ \hline 12 \end{array}$$

$$\begin{array}{r} 5 \\ \hline 12 \end{array} - \begin{array}{r} 2 \\ \hline 12 \end{array}$$

$$\begin{array}{r} 11 \\ \hline 12 \end{array} - \begin{array}{r} 6 \\ \hline 12 \end{array}$$

$$\frac{2}{10}$$

$$\frac{5}{10}$$

$$\frac{11}{12}$$

$$\frac{8}{10}$$

$$\frac{2}{12}$$

$$\frac{9}{12}$$

$$\frac{5}{12}$$

$$\frac{3}{12}$$

$$\frac{1}{2}$$

$$\frac{2}{2}$$

$$\frac{1}{3}$$

$$\frac{2}{3}$$

$$\frac{3}{3}$$

$$\frac{1}{4}$$

$$\frac{2}{4}$$

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$$\frac{6}{6}$$

$$\frac{1}{8}$$

$$\frac{2}{8}$$

$$\frac{3}{8}$$

$$\frac{4}{8}$$

$$\frac{5}{8}$$

$$\frac{6}{8}$$

$$\frac{7}{8}$$

$$\frac{8}{8}$$

$$\frac{1}{10}$$

$$\frac{2}{10}$$

$$\frac{3}{10}$$

$$\frac{4}{10}$$

$$\frac{5}{10}$$

$$\frac{6}{10}$$

$$\frac{7}{10}$$

$$\frac{8}{10}$$

$$\frac{9}{10}$$

$$\frac{10}{10}$$

$$\frac{1}{12}$$

$$\frac{2}{12}$$

$$\frac{3}{12}$$

$$\frac{4}{12}$$

$$\frac{5}{12}$$

$$\frac{6}{12}$$

$$\frac{7}{12}$$

$$\frac{8}{12}$$

$$\frac{9}{12}$$

$$\frac{10}{12}$$

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

$$\frac{5}{4}$$

$$\frac{3}{1}$$

$$\frac{4}{1}$$

$$\frac{6}{1}$$

$$\frac{9}{1}$$

$$\frac{13}{1}$$

$$\frac{20}{1}$$

$$\frac{2}{1}$$

$$\frac{3}{2}$$

$$\frac{4}{2}$$

$$\frac{5}{2}$$

$$\frac{7}{2}$$

$$\frac{4}{3}$$

$$\frac{6}{3}$$

$$\frac{7}{3}$$

$$\frac{15}{3}$$

$$\frac{5}{4}$$

$$\frac{7}{4}$$

$$\frac{11}{4}$$

$$\frac{16}{4}$$

$$\frac{6}{5}$$

$$\frac{7}{5}$$

$$\frac{10}{5}$$

$$\frac{12}{5}$$

$$\frac{7}{6}$$

$$\frac{10}{6}$$

$$\frac{11}{6}$$

$$\frac{18}{6}$$

$$\frac{8}{7}$$

$$\frac{10}{7}$$

$$\frac{14}{7}$$

$$\frac{17}{7}$$

$$\frac{9}{8}$$

$$\frac{10}{8}$$

$$\frac{16}{8}$$

$$\frac{19}{8}$$

$$\frac{11}{10}$$

$$\frac{13}{10}$$

$$\frac{16}{10}$$

$$\frac{20}{10}$$

13

—

12

15

—

12

20

—

12

24

—

12

$$1\frac{1}{2}$$

$$2\frac{1}{2}$$

$$3\frac{1}{2}$$

$$4\frac{1}{2}$$

$$1\frac{2}{3}$$

$$1\frac{1}{3}$$

$$2\frac{2}{3}$$

$$3\frac{1}{3}$$

$$1 \frac{1}{3}$$

$$1 \frac{2}{3}$$

$$2 \frac{2}{3}$$

$$4 \frac{1}{3}$$

$$1 \frac{3}{4}$$

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$$4 \frac{1}{5}$$

$$1 \frac{1}{6}$$

$$1 \frac{4}{6}$$

$$1 \frac{5}{6}$$

$$2 \frac{3}{6}$$

$$1 \frac{1}{7}$$

$$1 \frac{4}{7}$$

$$1 \frac{5}{7}$$

$$3 \frac{2}{7}$$

$$1 \frac{1}{8}$$

$$1 \frac{3}{8}$$

$$1 \frac{7}{8}$$

$$2 \frac{3}{8}$$

$$5 \frac{2}{3}$$

$$4 \frac{3}{4}$$

$$3 \frac{2}{5}$$

$$6 \frac{2}{6}$$

$$2 \frac{6}{7}$$

$$3 \frac{6}{8}$$

$$2 \frac{1}{9}$$

$$3 \frac{3}{10}$$

Converting an Improper Fraction to a Mixed Number Using Subtraction (when the improper fraction is less than 2)

Step	Example
Start with an improper fraction .	$\frac{8}{5}$
Subtract a fraction equal to 1 from the improper fraction . Make sure the denominators are the same.	$\frac{8}{5} - \frac{5}{5} = \frac{3}{5}$
The mixed number is equal to 1 and the new proper fraction .	$\frac{8}{5} = 1\frac{3}{5}$

Converting a Mixed Number to an Improper Fraction Using Addition (when the mixed number is less than 2)

Step	Example
Start with a mixed number .	$1\frac{3}{5}$
Change the 1 in the mixed number to a fraction with the same denominator .	$1 = \frac{5}{5}$
Add the fraction equal to 1 to the proper fraction to get the improper fraction .	$\frac{5}{5} + \frac{3}{5} = \frac{8}{5}$

Converting an Improper Fraction to a Mixed Number Using Subtraction (when the improper fraction is greater than 2)

Step	Example
Start with an improper fraction .	$\frac{16}{5}$
Subtract a fraction equal to 1 from the improper fraction . Make sure the denominators are the same.	$\frac{16}{5} - \frac{5}{5} = \frac{11}{5}$
Check to see if the new fraction is a proper fraction or an improper fraction .	$\frac{11}{5}$ is an improper fraction.
If the new number is still an improper fraction , repeat the previous steps. Continue until the new fraction is a proper fraction .	$\frac{11}{5} - \frac{5}{5} = \frac{6}{5}$ $\frac{6}{5}$ is an improper fraction. $\frac{6}{5} - \frac{5}{5} = \frac{1}{5}$ $\frac{1}{5}$ is a proper fraction.
Count how many times a fraction equal to one was subtracted from an improper fraction . That will be the whole number in the new mixed number .	$\frac{5}{5}$ was subtracted from an improper fraction three times, so the whole number is 3.
The new mixed number equals the whole number combined with the new proper fraction .	$\frac{16}{5} = 3\frac{1}{5}$

Converting a Mixed Number to an Improper Fraction Using Addition (when the mixed number is greater than 2)

Step	Example
Start with a mixed number .	$3\frac{1}{4}$
Change the whole number part of the mixed number to fractions equal to 1 with the same denominator as the proper fraction part of the mixed number .	$3 = 1 + 1 + 1 = \frac{4}{4} + \frac{4}{4} + \frac{4}{4}$
Add the fractions equal to one together.	$\frac{4}{4} + \frac{4}{4} + \frac{4}{4} = \frac{12}{4}$
Add to that sum the proper fraction part of the mixed number to get your new improper fraction .	$\frac{12}{4} + \frac{1}{4} = \frac{13}{4}$