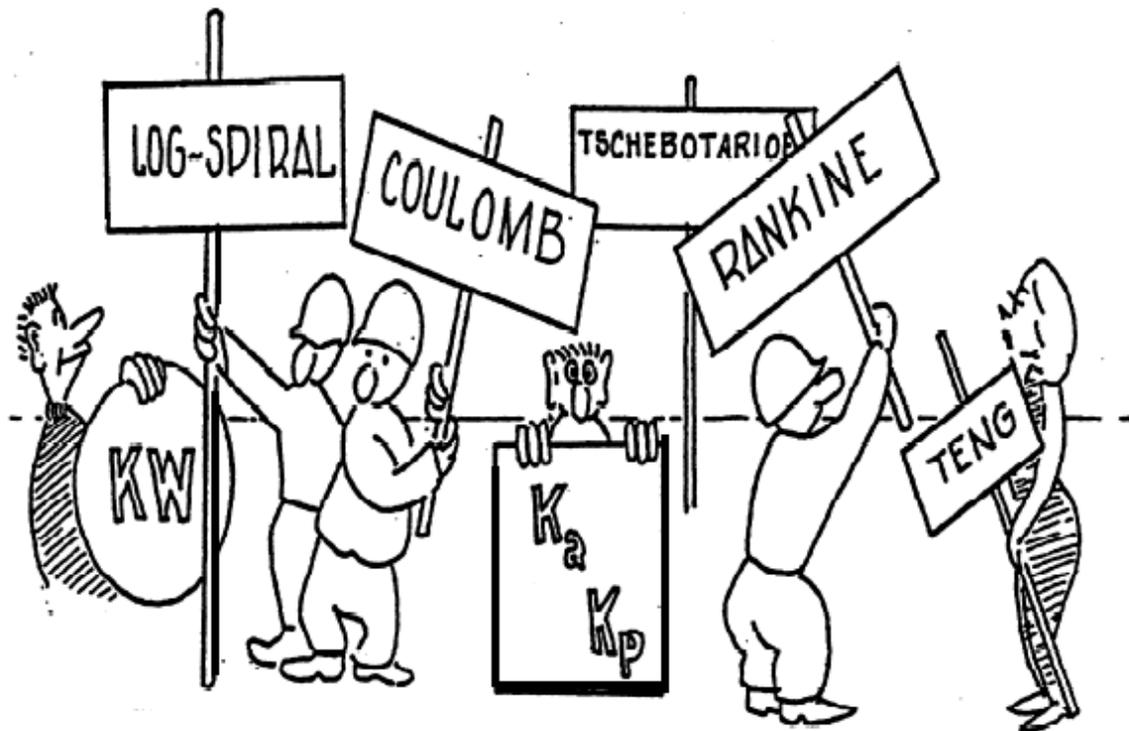


# CHAPTER 4

## EARTH PRESSURE

### THEORY AND

### APPLICATION



## **4.0 GENERAL**

All shoring systems shall be designed to withstand lateral earth pressure, water pressure and the effect of surcharge loads in accordance with the general principles and guidelines specified in this Caltrans Trenching and Shoring Manual.

### **4.1 SHORING TYPES**

Shoring systems are generally classified as unrestrained (non-gravity cantilevered), and restrained (braced or anchored). Unrestrained shoring systems rely on structural components of the wall partially embedded in the foundation material to mobilize passive resistance to lateral loads. Restrained shoring systems derive their capacity to resist lateral loads by their structural components being restrained by tension or compression elements connected to the vertical structural members of the shoring system and, additionally, by the partial embedment (if any) of their structural components into the foundation material.

#### **4.1.1 Unrestrained Shoring Systems**

Unrestrained shoring systems (non-gravity cantilevered walls) are constructed of vertical structural members consisting of partially embedded soldier piles or continuous sheet piles. This type of system depends on the passive resistance of the foundation material and the moment resisting capacity of the vertical structural members for stability; therefore its maximum height is limited by the competence of the foundation material and the moment resisting capacity of the vertical structural members. The economical height of this type of wall is generally limited to a maximum of 18 feet.

#### **4.1.2 Restrained Shoring Systems**

Restrained Shoring Systems are either anchored or braced walls. They are typically comprised of the same elements as unrestrained (non-gravity cantilevered) walls, but derive additional lateral resistance from one or more levels of braces, rakers, or anchors. These walls are typically constructed in cut situations in which construction proceeds from the top down to the base of the wall. The vertical wall elements should extend below the potential failure plane associated with the retained soil mass. For these types of walls, economical wall heights up to 80 feet are feasible.

Note - Soil Nail Walls and Mechanically Stabilized Earth (MSE) Walls are not included in this Manual. Both of these types of systems are designed by other methods that can be found on-line with FHWA or AASHTO.

## 4.2 LOADING

A major issue in providing a safe shoring system design is to determine the appropriate earth pressure loading diagram. The loads are to be calculated using the appropriate earth pressure theories. The lateral horizontal stresses ( $\sigma$ ) for both active and passive pressure are to be calculated based on the soil properties and the shoring system. Earth pressure loads on a shoring system are a function of the unit weight of the soil, location of the groundwater table, seepage forces, surcharge loads, and the shoring structure system. Shoring systems that cannot tolerate any movement should be designed for at-rest lateral earth pressure. Shoring systems which can move away from the soil mass should be designed for active earth pressure conditions, depending on the magnitude of the tolerable movement. Any movement, which is required to reach the minimum active pressure or the maximum passive pressure, is a function of the wall height and the soil type. Significant movement is necessary to mobilize the full passive pressure. The variation of lateral stress between the active and passive earth pressure values can be brought about only through lateral movement within the soil mass of the backfill as shown in Figure 4-1.

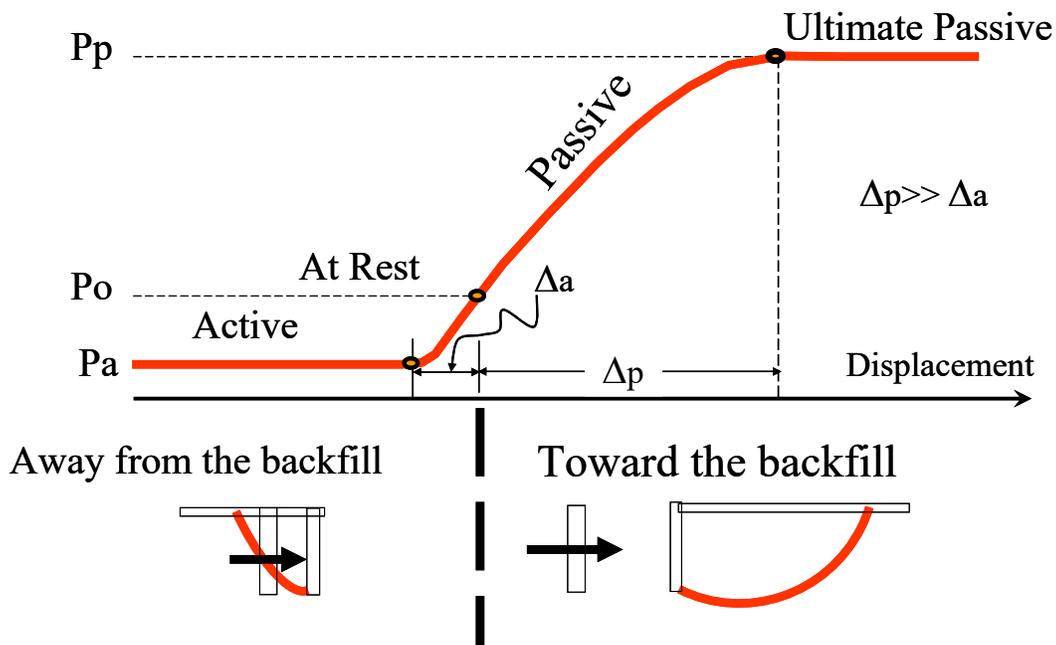


Figure 4-1. Active and passive earth pressure coefficient as a function of wall displacement

## EARTH PRESSURE THEORY AND APPLICATION

Typical values of these mobilizing movements, relative to wall height, are given in Table 4-1 (Clough 1991).

Table 4-1. Mobilized Wall Movements

Type of Backfill	Value of $\Delta/H$	
	Active	Passive
Dense Sand	0.001	0.01
Medium Dense Sand	0.002	0.02
Loose Sand	0.004	0.04
Compacted Silt	0.002	0.02
Compacted Lean Clay	0.01	0.05
Compacted Fat Clay	0.01	0.05

where:

$\Delta$  = the movement of top of wall required to reach minimum active or maximum passive pressure, by tilting or lateral translation, and

H = height of wall.

### 4.3 GRANULAR SOIL

At present, methods of analysis in common use for retaining structures are based on Rankine (1857) and Coulomb (1776) theories. Both methods are based on the limit equilibrium approach with an assumed planar failure surface. Developments since 1920, largely due to the influence of Terzaghi (1943), have led to a better understanding of the limitations and appropriate applications of classical earth pressure theories. Terzaghi assumed a logarithmic failure surface. Many experiments have been conducted to validate Coulomb's wedge theory and it has been found that the sliding surface is not a plane, but a curved surface as shown in Figure 4-2 (Terzaghi 1943).

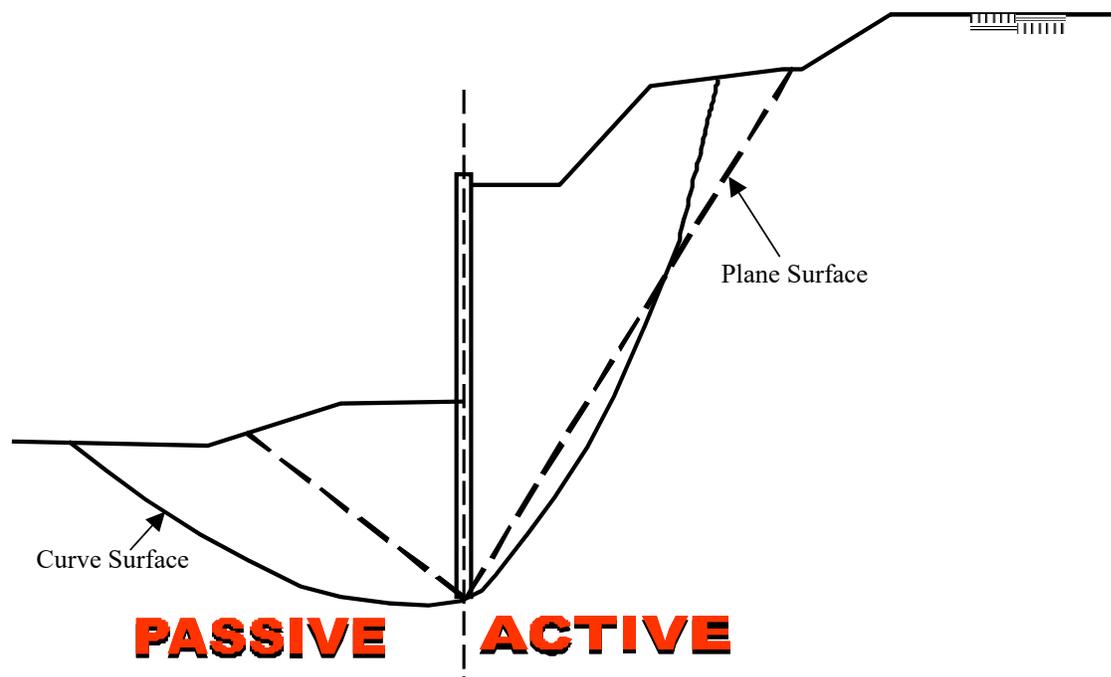


Figure 4-2. Comparison of Plane versus Curve Failure Surfaces

Furthermore, these experiments have shown that the Rankine (1857) and Coulomb (1776) earth pressure theories lead to quite accurate results for the active earth pressure. However, for the passive earth pressure, these theories are accurate only for the backfill of clean dry sand for a low wall interface friction angle.

For the purpose of the initial discussion, it is assumed that the backfills are level, homogeneous, isotropic and distribution of vertical stress ( $\sigma_v$ ) with depth is hydrostatic as shown in Figure 4-3.

## EARTH PRESSURE THEORY AND APPLICATION

The horizontal stress ( $\sigma_h$ ) is linearly proportional to depth and is a multiple of vertical stress ( $\sigma_v$ ) as shown in. Eq. 4-1.

$$\sigma_h = \sigma_v K = \gamma h K \quad \text{Eq. 4-1}$$

$$P = \frac{1}{2} \sigma_h h \quad \text{Eq. 4-2}$$

Depending on the wall movement, the coefficient  $K$  represents active ( $K_a$ ), passive ( $K_p$ ) or at-rest ( $K_o$ ) earth pressure coefficient in the above equation.

The resultant lateral earth load,  $P$ , which is equal to the area of the load diagram, shall be assumed to act at a height of  $h/3$  above the base of the wall, where  $h$  is the height of the pressure surface, measured from the surface of the ground to the base of the wall.  $P$  is the force that causes bending, sliding and overturning in the wall.

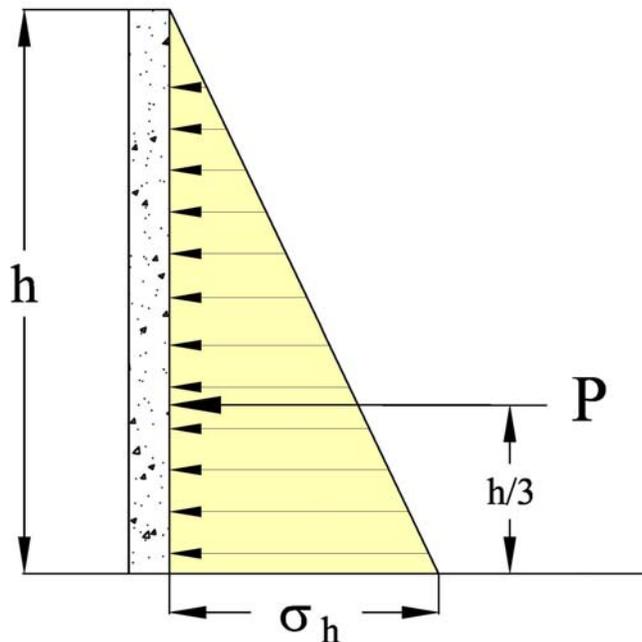


Figure 4-3. Lateral Earth Pressure Variation with Depth

Depending on the shoring system the value of the active and/or passive pressure can be determined using either the Rankine, Coulomb, Log Spiral and Trial Wedge methods.

The state of the active and passive earth pressure depends on the expansion or compression transformation of the backfill from elastic state to state of plastic equilibrium. The concept of the active and passive earth pressure theory can be explained using a continuous deadman near the

ground surface for the stability of a sheet pile wall as shown in Figure 4-4. As a result of wall deflection,  $\Delta$ , the tie rod is pulled until the active and passive wedges are formed behind and in front of the deadman. Element P, in the front of the deadman and element A, at the front of the deadman are acted on by two principal stresses, a vertical stress ( $\sigma_v$ ) and horizontal stress ( $\sigma_h$ ). In the active case, the horizontal stress ( $\sigma_a$ ) is the minor principal stress and the vertical stress ( $\sigma_v$ ) is the major principal stress. In the passive case, the horizontal stress ( $\sigma_p$ ) is the major principal stress and the vertical stress ( $\sigma_v$ ) is the minor principal stress. The resulting failure surface within the soil mass corresponding to active and passive earth pressure for the cohesionless soil is shown in Figure 4-4.



From Figure 4-4 above:

$$\sin \phi = \frac{AB}{OA} = \frac{\frac{\sigma_v - \sigma_a}{2}}{\frac{\sigma_v + \sigma_a}{2}} \quad \text{Eq. 4-3}$$

Where AB is the radius of the circle

$$\sin \phi = \frac{AB}{OA} = \frac{\sigma_v - \sigma_a}{\sigma_v + \sigma_a} \quad \text{Eq. 4-4}$$

$$\sigma_v \sin \phi + \sigma_a \sin \phi = \sigma_v - \sigma_a \quad \text{Eq. 4-5}$$

Collecting Terms:

$$\sigma_a + \sigma_a (\sin \phi) = \sigma_v - \sigma_v (\sin \phi) \quad \text{Eq. 4-6}$$

$$\sigma_a (1 + \sin \phi) = \sigma_v (1 - \sin \phi) \quad \text{Eq. 4-7}$$

$$\frac{\sigma_a}{\sigma_v} = \frac{(1 - \sin \phi)}{(1 + \sin \phi)} \quad \text{Eq. 4-8}$$

From trigonometric identities:

$$\frac{(1 - \sin \phi)}{(1 + \sin \phi)} = \tan^2 \left( 45 - \frac{\phi}{2} \right)$$

$$\frac{(1 + \sin \phi)}{(1 - \sin \phi)} = \tan^2 \left( 45 + \frac{\phi}{2} \right)$$

$$K_a = \tan^2 \left( 45 - \frac{\phi}{2} \right), \text{ where } K_a = \frac{\sigma_a}{\sigma_v} \quad \text{Eq. 4-9}$$

For the passive case:

$$K_p = \frac{\sigma_p}{\sigma_v} = \frac{1 + \sin \phi}{1 - \sin \phi} = \tan^2 \left( 45 + \frac{\phi}{2} \right) \quad \text{Eq. 4-10}$$

There are various pros and cons to the individual earth theories but briefly here is a summary:

- The Rankine formula for passive pressure can only be used correctly when the embankment slope angle,  $\beta$ , equals zero or is negative. If a large wall friction value can develop, the Rankine Theory is not correct and will give less conservative results. Rankine's theory is not intended to be used for determining earth pressures directly against a wall (friction angle does not appear in equations above). The theory is intended to be used for determining earth pressures on a vertical plane within a mass of soil.

## EARTH PRESSURE THEORY AND APPLICATION

- For the Coulomb equation, if the shoring system is vertical and the backfill slope friction angles are zero, the result will be the same as Rankine's for a level ground condition. Since wall friction requires a curved surface of sliding to satisfy equilibrium, the Coulomb formula will give only approximate results since it assumes planar failure surfaces. The accuracy for Coulomb will diminish with increased depth. For passive pressures the Coulomb formula can also give inaccurate results when there is a large back slope or wall friction angle. These conditions should be investigated and an increased factor of safety considered.
- The Log-Spiral theory was developed because of the unrealistic values of earth pressures that are obtained by theories that assume a straight line failure plane. The difference between the Log-Spiral curved failure surface and the straight line failure plane can be large and on the unsafe side for Coulomb passive pressures (especially when wall friction exceeds  $\phi/3$ ). Figure 4-2 and Figure 4-31 show a comparison of the Coulomb and Rankine failure surfaces (plane) versus the Log-Spiral failure surface (curve).
- More on Log-Spiral can be found in Section 4.7 of this Manual.

### 4.3.1 At-Rest Lateral Earth Pressure Coefficient ( $K_0$ )

For a zero lateral strain condition, horizontal and vertical stresses are related by the Poisson's ratio ( $\mu$ ) as follows:

$$K_0 = \frac{\mu}{1 - \mu} \quad \text{Eq. 4-11}$$

For normally consolidated soils and vertical walls, the coefficient of at-rest lateral earth pressure may be taken as:

$$K_0 = (1 - \sin \phi)(1 - \sin \beta) \quad \text{Eq. 4-12}$$

Where:

$\phi$  = effective friction angle of soil.

$K_0$  = coefficient of at-rest lateral earth pressure.

$\beta$  = slope angle of backfill surface behind retaining wall.

For over consolidated soils, level backfill, and a vertical wall, the coefficient of at-rest lateral earth pressure may be assumed to vary as a function of the over consolidation ratio or stress history, and may be taken as:

$$K_0 = (1 - \sin \phi)(OCR)^{\sin \phi} \quad \text{Eq. 4-13}$$

Where:

$OCR$  = over consolidation ratio

### 4.3.2 Active and/or Passive Earth Pressure

Depending on the shoring system the value of the active and/or passive pressure can be determined using either the Rankine, Coulomb or trial wedge methods.

#### 4.3.2.1 Rankine's Theory

Rankine's theory is the simplest formulation proposed for earth pressure calculations and it is based on the following assumptions:

- The wall is smooth and vertical.
- No friction or adhesion between the wall and the soil.
- The failure wedge is a plane surface and is a function of soil's friction  $\phi$  and the backfill slope  $\beta$  as shown in Eq. 4-14 and Eq. 4-17.
- Lateral earth pressure varies linearly with depth.
- The direction of the lateral earth pressure acts parallel to slope of the backfill as shown in Figure 4-5 and Figure 4-6.
- The resultant earth pressure acts at a distance equal to one-third of the wall height from the base.

Values for the coefficient of active lateral earth pressure using the Rankine Theory may be taken as shown in Eq. 4-14:

$$K_a = \cos\beta \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}} \quad \text{Eq. 4-14}$$

And the magnitude of active earth pressure can be determined as shown in Figure 4-5 and Eq. 4-15:

$$P_a = \frac{1}{2}(\gamma)(h^2)(K_a) \quad \text{Eq. 4-15}$$

The failure plane angle  $\alpha$  can be determined as shown in Eq. 4-16:

$$\alpha = \left(45 + \frac{\phi}{2}\right) - \frac{1}{2} \left( \text{Arc sin} \left( \frac{\sin\beta}{\sin\phi} \right) - \beta \right) \quad \text{Eq. 4-16}$$

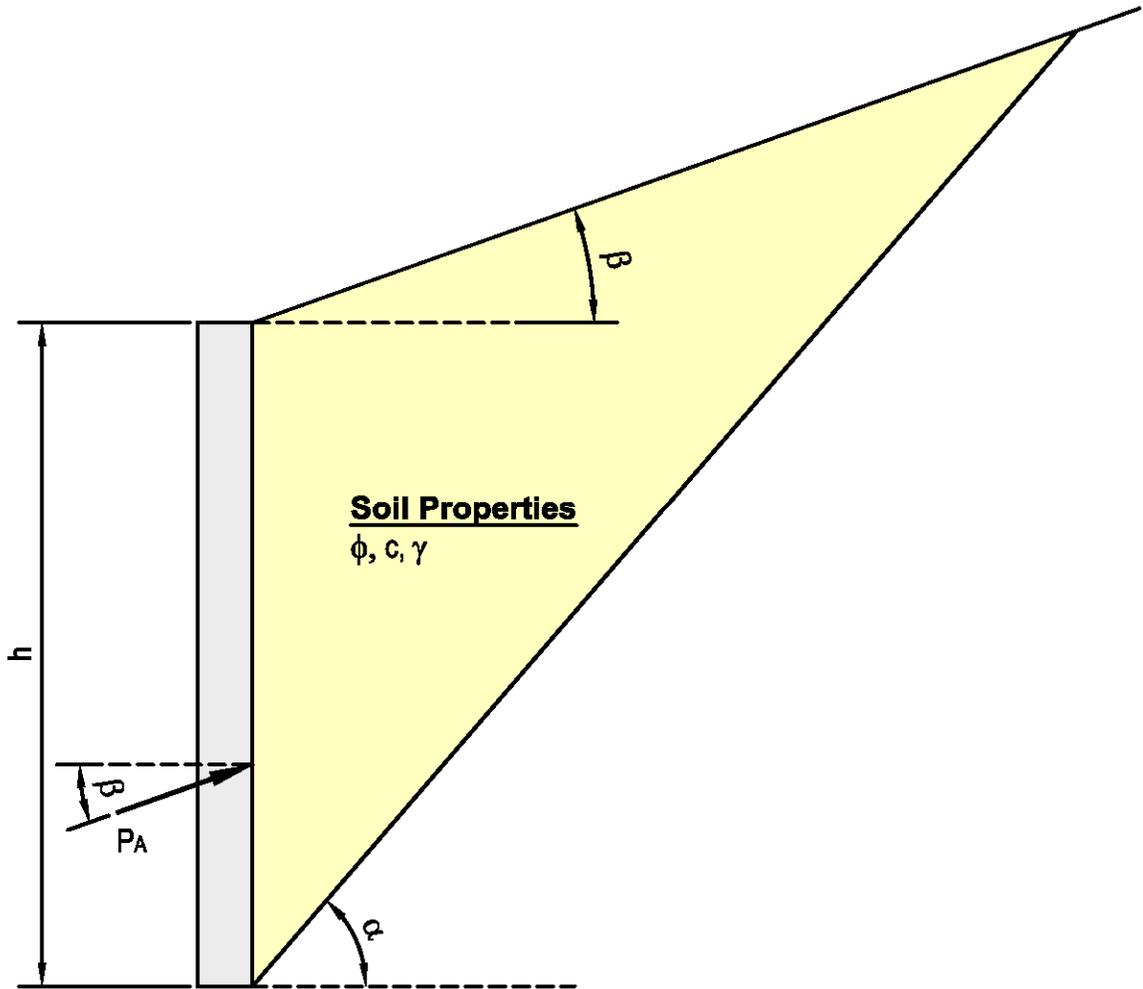


Figure 4-5. Rankine's active wedge

Rankine made similar assumptions to his active earth pressure theory to calculate the passive earth pressure. Values for the coefficient of passive lateral earth pressure may be taken as:

$$K_p = \cos\beta \frac{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}} \quad \text{Eq. 4-17}$$

And the magnitude of passive earth pressure can be determined as shown in Figure 4-6 and Eq. 4-18:

$$P_p = \frac{1}{2}(\gamma)(h^2)(K_p) \quad \text{Eq. 4-18}$$

The failure plane angle  $\alpha$  can be determined as shown in Eq. 4-19:

$$\alpha = \left( 45 - \frac{\phi}{2} \right) + \frac{1}{2} \left( \text{Arc sin} \left( \frac{\sin \beta}{\sin \phi} \right) + \beta \right) \quad \text{Eq. 4-19}$$

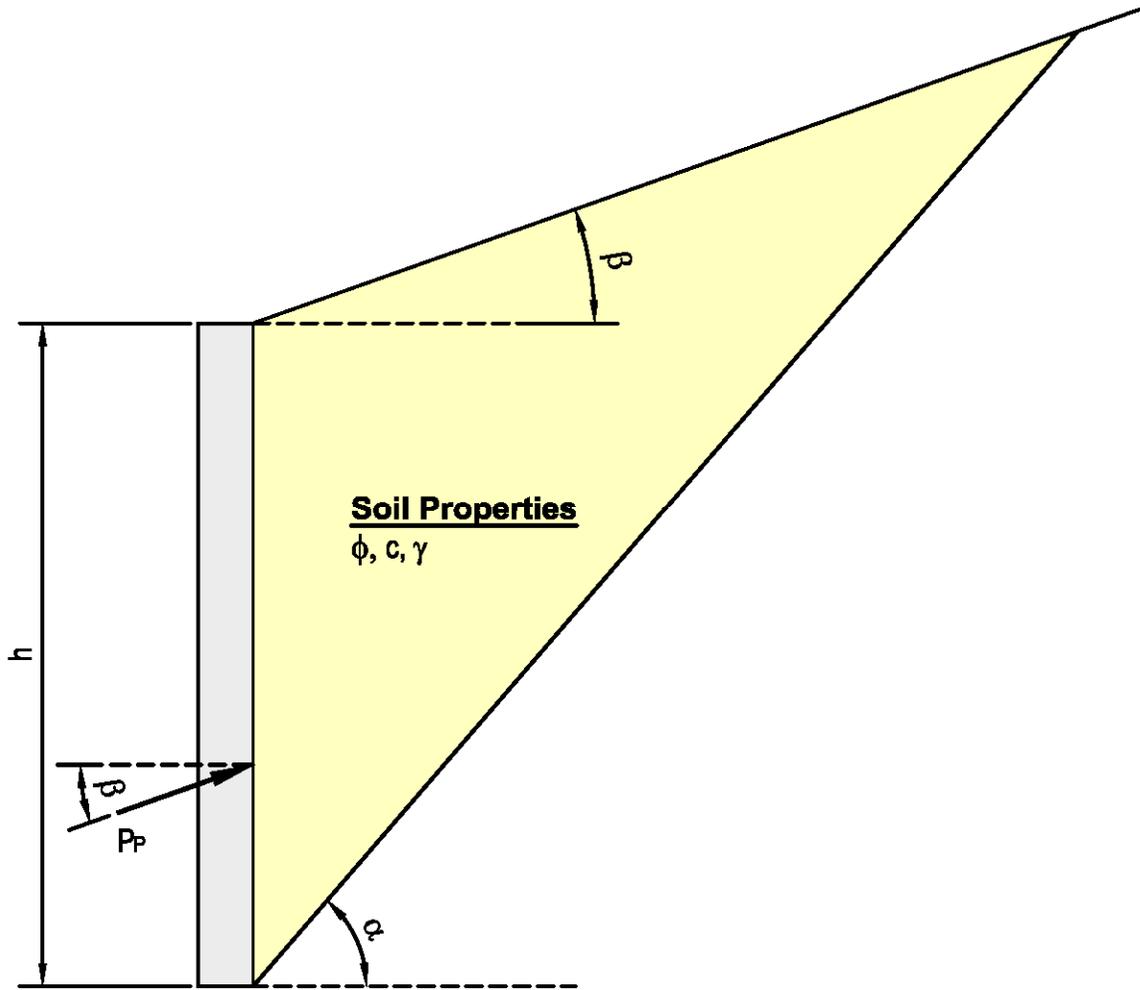


Figure 4-6. Rankine's passive wedge

Where:

$h$  = height of pressure surface on the wall.

$P_a$  = active lateral earth pressure resultant per unit width of wall.

$P_p$  = passive lateral earth pressure resultant per unit width of wall.

$\beta$  = angle from backfill surface to the horizontal.

$\alpha$  = failure plane angle with respect to horizontal.

$\phi$  = effective friction angle of soil.

$K_a$  = coefficient of active lateral earth pressure.

$K_p$  = coefficient of passive lateral earth pressure.

$\gamma$  = unit weight of soil.

Although Rankine's equation for the passive earth pressure is provided above, one should not use the Rankine method to calculate the passive earth pressure when the backfill angle is greater than zero ( $\beta > 0$ ). As a matter of fact the  $K_p$  value for both positive ( $\beta > 0$ ) and negative ( $\beta < 0$ ) backfill slope is identical. This is clearly not correct. Therefore, avoid using the Rankine equation to calculate the passive earth pressure coefficient for sloping ground.

#### 4.3.2.2 Coulomb's Theory

Coulomb's (1776) earth pressure theory is based on the following assumptions:

- The wall is rough.
- There is friction or adhesion between the wall and the soil.
- The failure wedge is a plane surface and is a function of the soil friction  $\phi$ , wall friction  $\delta$ , the backfill slope  $\beta$  and the slope of the wall  $\omega$ .
- Lateral earth pressure varies linearly with depth.
- The direction of the lateral earth pressure acts at an angle  $\delta$  with a line that is normal to the wall.
- The resultant earth pressure acts at a distance equal to one-third of the wall height from the base.

## EARTH PRESSURE THEORY AND APPLICATION

Values for the coefficient of active lateral earth pressure may be taken as shown in Eq. 4-20:

$$K_a = \frac{\cos^2(\phi - \omega)}{\cos^2 \omega \cos(\delta + \omega) \left[ 1 + \sqrt{\frac{\sin(\delta + \phi) \sin(\phi - \beta)}{\cos(\delta + \omega) \cos(\omega - \beta)}} \right]^2} \quad \text{Eq. 4-20}$$

And the magnitude of active earth pressure can be determined as shown in Figure 4-7 and Eq. 4-21:

$$P_a = \frac{1}{2}(\gamma)(h^2)(K_a) \quad \text{Eq. 4-21}$$

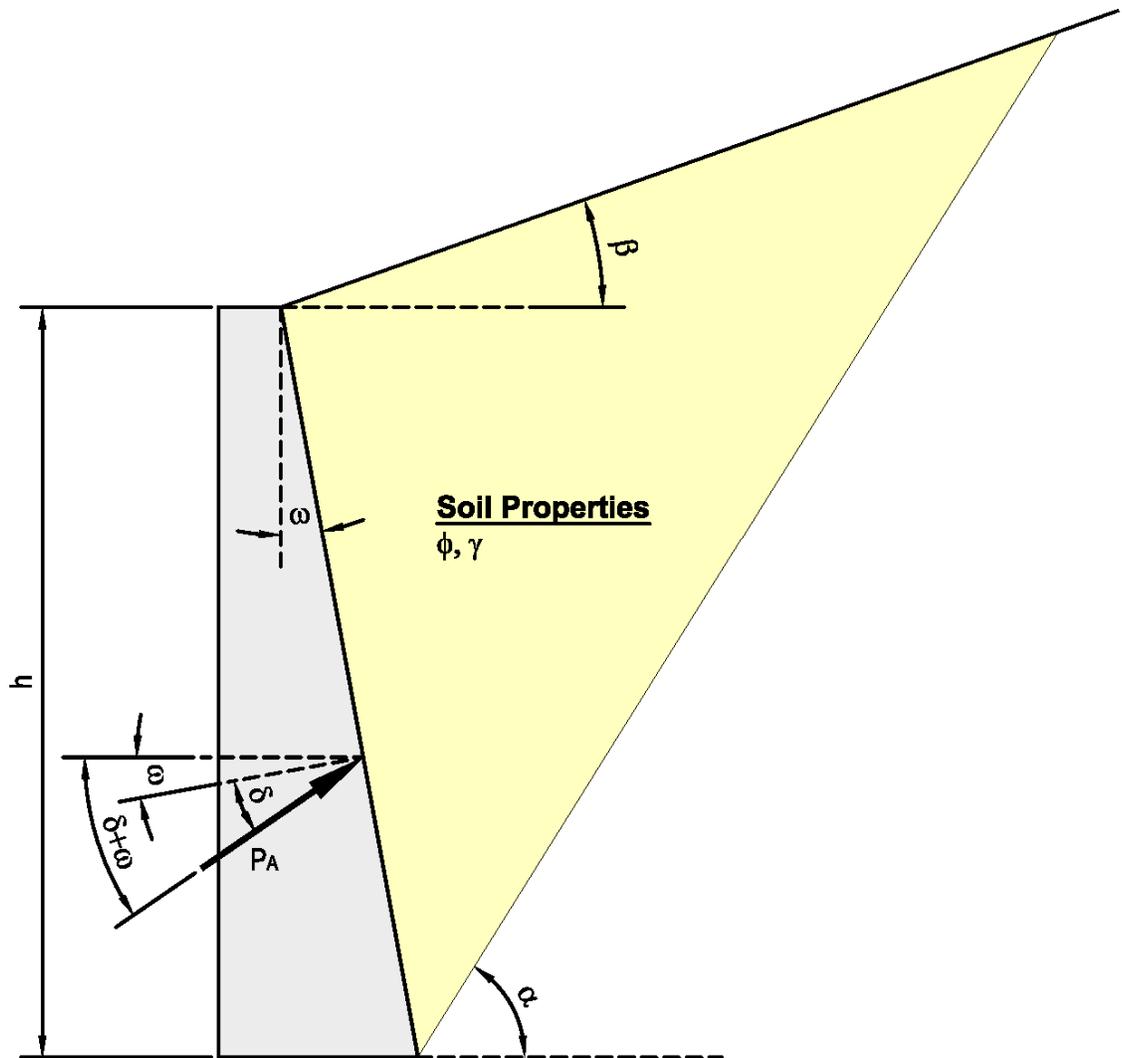


Figure 4-7. Coulomb's active wedge

Coulomb's passive earth pressure is derived similar to his active earth pressure except the inclination of the force is as shown in Figure 4-7. Values for the coefficient of passive lateral earth pressure may be taken as shown in Eq. 4-22:

$$K_p = \frac{\cos^2(\phi + \omega)}{\cos^2 \omega \cos(\delta - \omega) \left[ 1 - \sqrt{\frac{\sin(\delta + \phi) \sin(\phi + \beta)}{\cos(\delta - \omega) \cos(\beta - \omega)}} \right]^2} \quad \text{Eq. 4-22}$$

And the magnitude of passive earth pressure can be determined as shown in Figure 4-8 and Eq. 4-23:

$$P_p = \frac{1}{2} (\gamma) (h^2) (K_p) \quad \text{Eq. 4-23}$$

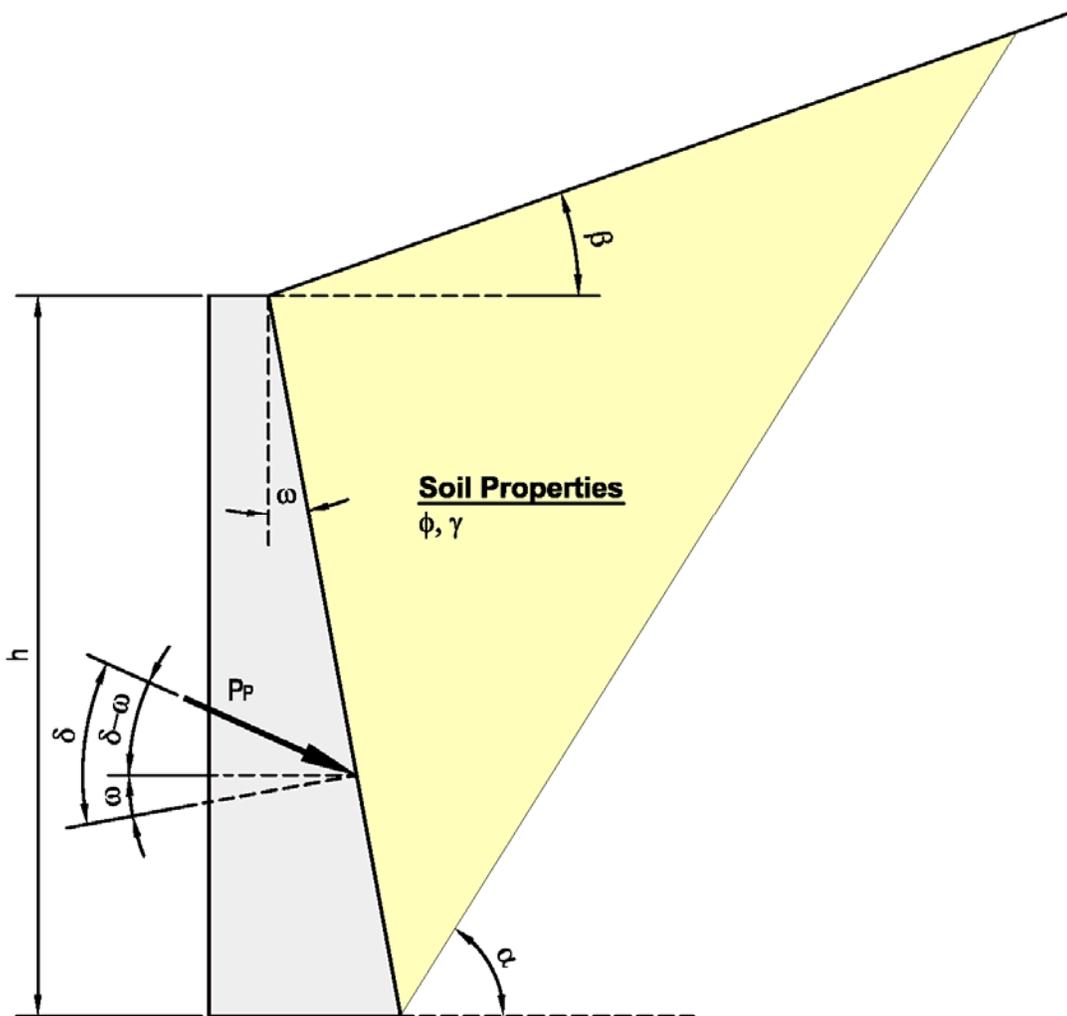


Figure 4-8. Coulomb's passive wedge

## EARTH PRESSURE THEORY AND APPLICATION

Where:

$h$  = height of pressure surface on the wall.

$P_a$  = active lateral earth pressure resultant per unit width of wall.

$P_p$  = passive lateral earth pressure resultant per unit width of wall.

$\delta$  = friction angle between backfill material and face of wall. (See Table 4-2)

$\beta$  = angle from backfill surface to the horizontal.

$\alpha$  = failure plane angle with respect to the horizontal.

$\omega$  = angle from the face of wall to the vertical.

$\phi$  = effective friction angle of soil.

$K_a$  = coefficient of active lateral earth pressure.

$K_p$  = coefficient of passive lateral earth pressure.

$\gamma$  = unit weight of soil.

Table 4-2. Wall friction

**ULTIMATE FRICTION FACTOR FOR DISSIMILAR MATERIALS**

INTERFACE MATERIALS	FRICTION ANGLE, $\delta$ (°)
Mass concrete on the following foundation materials:	
• Clean sound rock ·····	35
• Clean gravel, gravel-sand mixtures, coarse sand ·····	29 to 31
• Clean fine to medium sand, silty medium to coarse sand, silty or clayey gravel ·····	24 to 29
• Clean fine sand, silty or clayey fine to medium sand ·····	19 to 24
• Fine sandy silt, nonplastic silt ·····	17 to 19
• Very stiff and hard residual or preconsolidated clay ·····	22 to 26
• Medium stiff and stiff clay and silty clay ·····	17 to 19
Masonry on foundation materials has same friction factors.	
Steel sheet piles against the following soils:	
• Clean gravel, gravel-sand mixtures, well-graded rock fill with spalls ·····	22
• Clean sand, silty sand-gravel mixture, single-size hard rock fill ·····	17
• Silty sand, gravel or sand mixed with silt or clay ·····	14
• Fine sandy silt, nonplastic silt ·····	11
Formed or precast concrete or concrete sheet piling against the following soils:	
• Clean gravel, gravel-sand mixture, well-graded rock fill with spalls ·····	22 to 26
• Clean sand, silty sand-gravel mixture, single-size hard rock fill ·····	17 to 22
• Silty sand, gravel or sand mixed with silt or clay ·····	17
• Fine sandy silt, nonplastic silt ·····	14
Various structural materials:	
• Masonry on masonry, igneous and metamorphic rocks:	
○ dressed soft rock on dressed soft rock ·····	35
○ dressed hard rock on dressed soft rock ·····	33
○ dressed hard rock on dressed hard rock ·····	29
• Masonry on wood in direction of cross grain ·····	26
• Steel on steel at sheet pile interlocks ·····	17
This table is a reprint of Table 3.11.5.3-1, AASHTO LRFD BDS, 4 <sup>th</sup> ed, 2007	

Further discussion of Wall Friction is included in Section 4.6.

#### 4.4 COHESIVE SOIL

Neither Coulomb's nor Rankine's theories explicitly incorporated the effect of cohesion in the lateral earth pressure computations. Bell (1952) modified Rankine's solution to include the effect of the backfill with cohesion. The derivation of Bell's equations for the active and passive earth pressure follows the same steps as were used in Section 4.3 as shown below.

For the cohesive soil Figure 4-9 can be used to derive the relationship for the active and passive earth pressures.

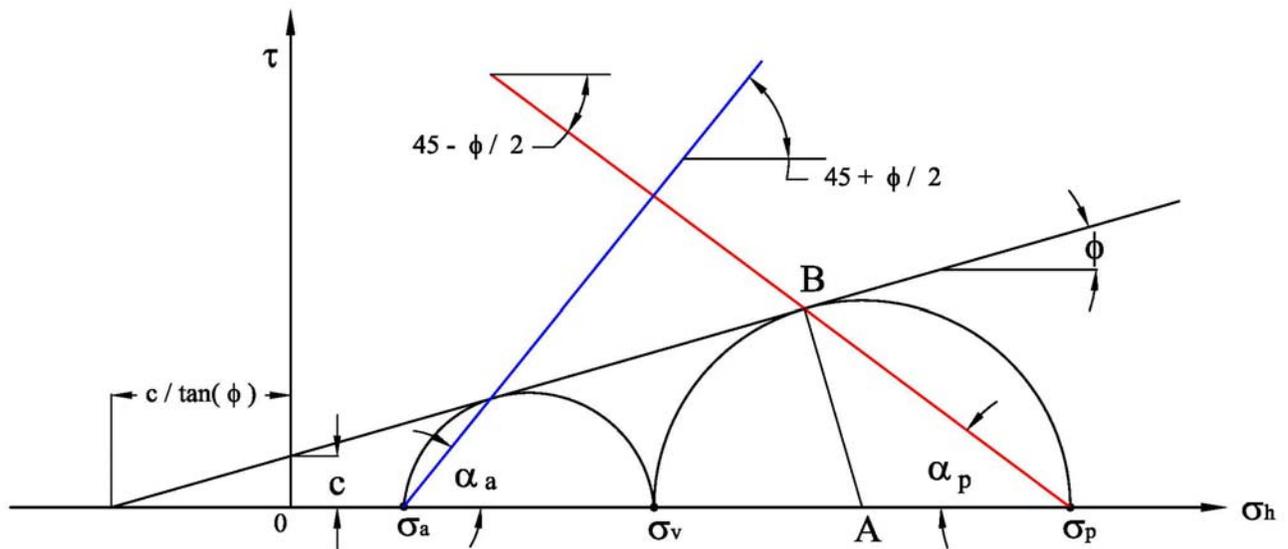


Figure 4-9. Mohr Circle Representation of Earth Pressure for Cohesive Backfill

For the Active case:

$$\sin \phi = \frac{\frac{\sigma_v - \sigma_a}{2}}{\frac{\sigma_v + \sigma_a}{2} + \frac{c}{\tan \phi}} \quad \text{Eq. 4-24}$$

Then,

$$\sigma_v \sin \phi + \sigma_a \sin \phi + 2c(\cos \phi) = \sigma_p - \sigma_a \quad \text{Eq. 4-25}$$

Collecting Terms:

$$\sigma_v(1 - \sin \phi) = \sigma_a(1 + \sin \phi) + 2c(\cos \phi) \quad \text{Eq. 4-26}$$

Solving for  $\sigma_a$

$$\sigma_a = \frac{\sigma_v(1 - \sin \phi)}{(1 + \sin \phi)} - \frac{2c(\cos \phi)}{(1 + \sin \phi)} \quad \text{Eq. 4-27}$$

Using the trigonometric identities from above:

$$\sigma_a = \sigma_v \tan^2\left(45 - \frac{\phi}{2}\right) - 2c \tan\left(45 - \frac{\phi}{2}\right) \quad \text{Eq. 4-28}$$

$$\sigma_a = \sigma_v K_a - 2c\sqrt{K_a}, \text{ where } \sigma_v = \gamma z \quad \text{Eq. 4-29}$$

For the passive case:

Solving for  $\sigma_p$

$$\sigma_p = \frac{\sigma_v(1 + \sin \phi)}{(1 - \sin \phi)} + \frac{2c(\cos \phi)}{(1 - \sin \phi)} \quad \text{Eq. 4-30}$$

$$\sigma_p = \sigma_v \tan^2\left(45 + \frac{\phi}{2}\right) + 2c \tan\left(45 + \frac{\phi}{2}\right) \quad \text{Eq. 4-31}$$

$$\sigma_p = \sigma_v K_p - 2c\sqrt{K_p}, \text{ where } \sigma_v = \gamma z \quad \text{Eq. 4-32}$$

Extreme caution is advised when using cohesive soil to evaluate soil stresses. The evaluation of the stress induced by cohesive soils is highly uncertain due to their sensitivity to shrinkage-swell, wet-dry and degree of saturation. Tension cracks (gaps) can form, which may considerably alter the assumptions for the estimation of stress. The development of the tension cracks from the surface to depth,  $h_{cr}$ , is shown in Figure 4-10.

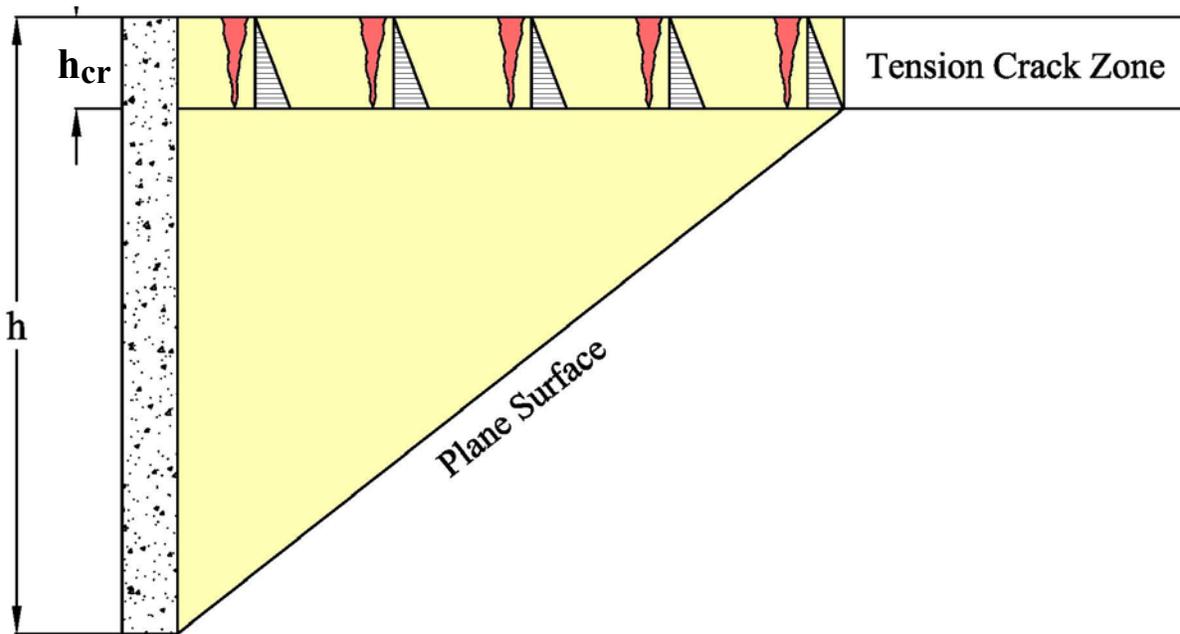


Figure 4-10. Tension crack with hydrostatic water pressure

## EARTH PRESSURE THEORY AND APPLICATION

As shown in Figure 4-11, the active earth pressure ( $\sigma_a$ ) normal to the back of the wall at depth,  $h$ , is equal to:

$$\sigma_a = \gamma h K_a - 2C\sqrt{K_a} \quad \text{Eq. 4-33}$$

$$P_a = \frac{1}{2} \gamma h^2 K_a - 2C\sqrt{K_a}(h) \quad \text{Eq. 4-34}$$

According to Eq. 4-33 the lateral stress ( $\sigma_a$ ) at some point along the wall is equal to zero, therefore,

$$\gamma h K_a - 2C\sqrt{K_a} = 0 \quad \text{Eq. 4-35}$$

$$h = h_{cr} = \frac{2C\sqrt{K_a}}{\gamma K_a} \quad \text{Eq. 4-36}$$

As shown in Figure 4-11, the passive earth pressure ( $\sigma_p$ ) normal to the back of the wall at depth,  $h$ , is equal to:

$$\sigma_p = \gamma h K_p + 2C\sqrt{K_p} \quad \text{Eq. 4-37}$$

$$P_p = \frac{1}{2} \gamma h^2 K_p + 2C\sqrt{K_p}(h) \quad \text{Eq. 4-38}$$

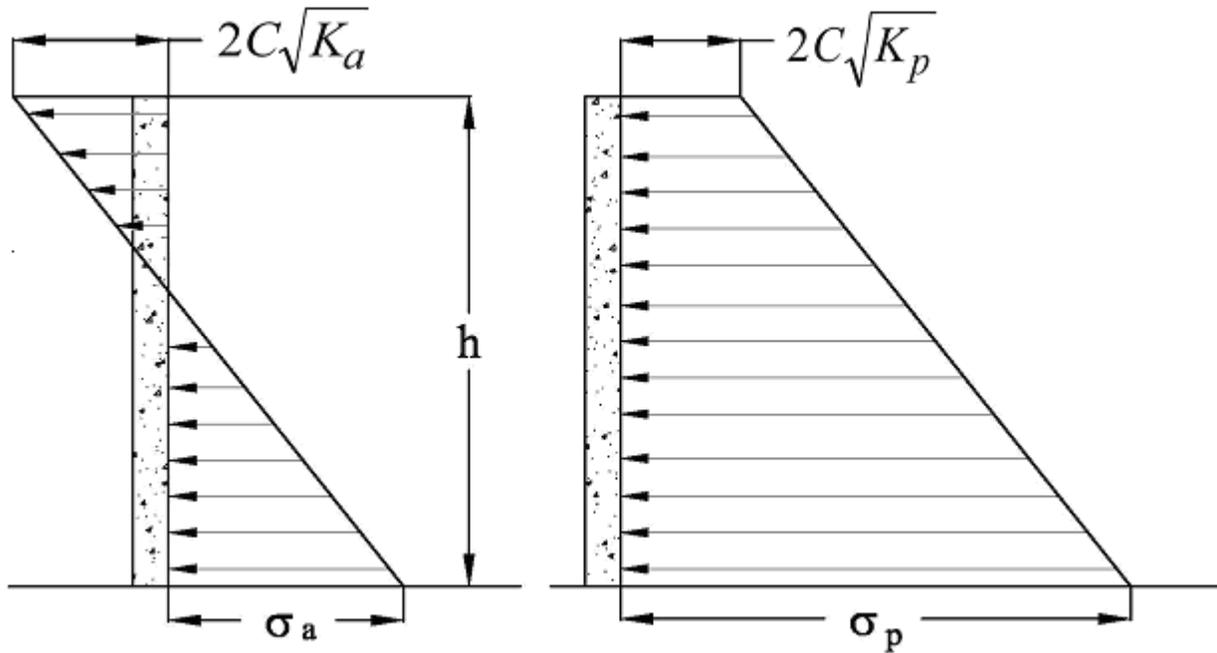


Figure 4-11. Cohesive Soil Active Passive Earth Pressure Distribution

The effect of the surcharges and ground water are not included in the above figure. In the presence of water, the hydrostatic pressure in the tension crack needs to be considered.

For shoring systems which support cohesive backfill, the height of the tension zone,  $h_{cr}$ , should be ignored and the simplified lateral earth pressure distribution acting along the entire wall height,  $h$ , including presence of water pressure within the tension zone as shown in Figure 4-12 shall be used.

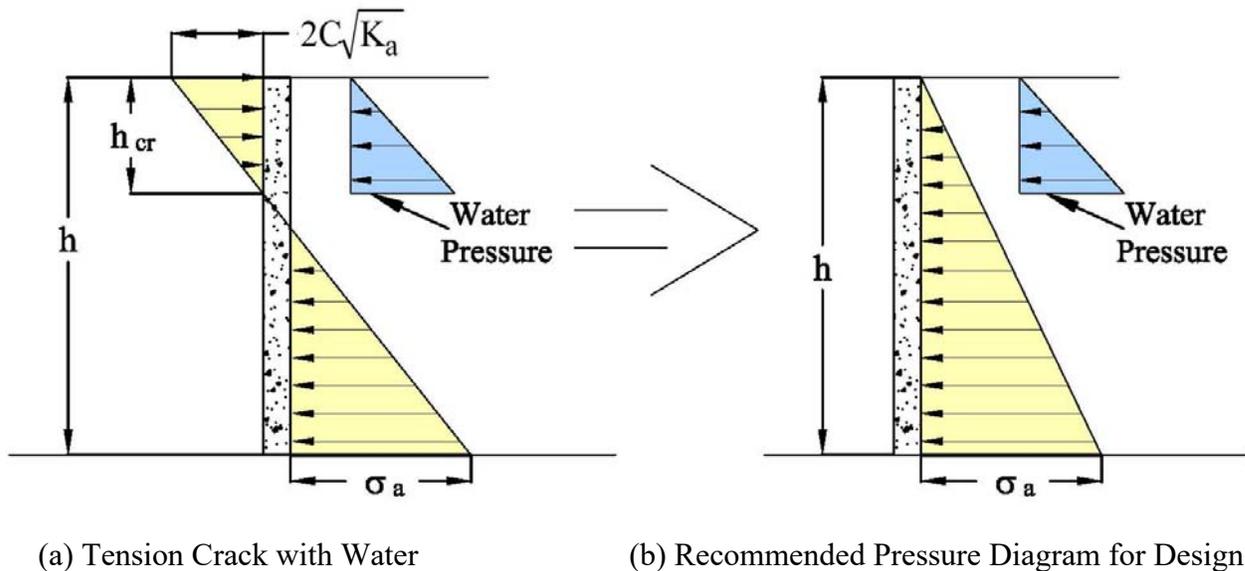


Figure 4-12: Load Distribution for Cohesive Backfill

The apparent active earth pressure coefficient,  $K_{apparent}$ , may be determined by:

$$K_{apparent} = \frac{\sigma_a}{\gamma h} \geq 0.25 \quad \text{Eq. 4-39}$$

Where: (for Eq. 4-24 through Eq. 4-39)

- $h$  = height of pressure surface at back of wall.
- $P_a$  = active lateral earth pressure resultant per unit width of wall.
- $P_p$  = passive lateral earth pressure resultant per unit width of wall.
- $\phi$  = effective friction angle of soil.
- $C$  = effective soil cohesion.
- $K_a$  = coefficient of active lateral earth pressure.
- $K_p$  = coefficient of passive lateral earth pressure.
- $\gamma$  = unit weight of soil.
- $h_{cr}$  = height of the tension crack.

## EARTH PRESSURE THEORY AND APPLICATION

The active lateral earth pressure ( $\sigma_a$ ) acting over the wall height,  $h$ , should not be less than 0.25 times the effective vertical stress ( $\sigma_v = \gamma h$ ) at any depth. Any design based on a lower value must have superior justification such as multiple laboratory tests verifying higher values for "C", as well as time frames and other conditions that would not affect the cohesive value while the shoring is in place.

### 4.5 SHORING SYSTEMS AND SLOPING GROUND

There are many stable slopes in nature even though the slope angle  $\beta$  is larger than the soil friction angle  $\phi$  due to presence of cohesion  $C$ . None of the earth pressure theories will work when the slope angle  $\beta$  is larger than friction angle  $\phi$  even if the shoring system is to be installed in cohesive soil. Mohr Circle representation of the  $C$ - $\phi$  soil backfill with slope angle  $\beta > \phi$  is shown in Figure 4-13.

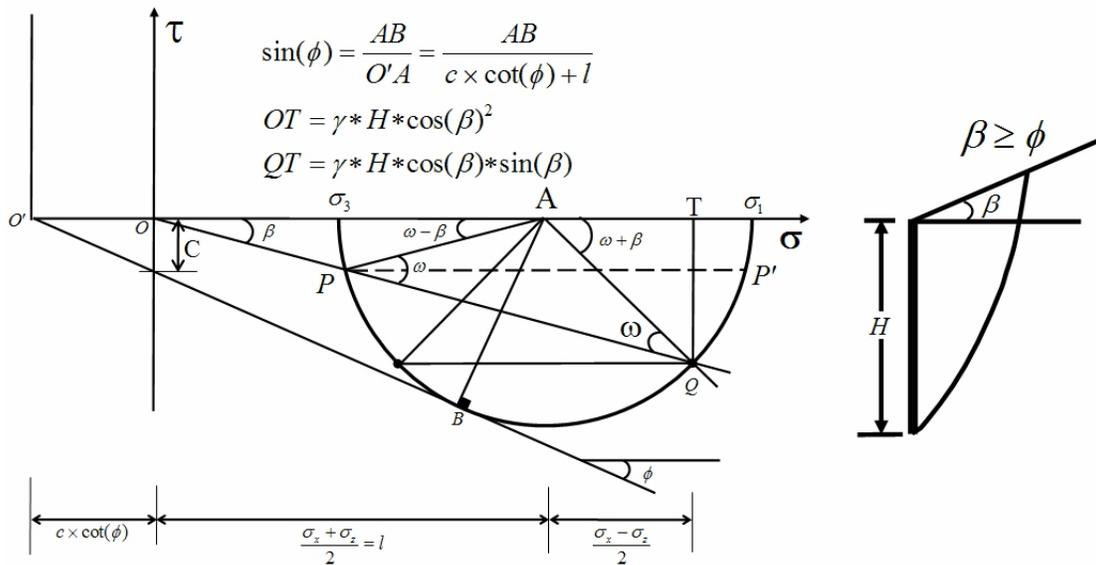


Figure 4-13. Sloping Ground

The following equations developed by the authors are based on ASCE Journal of Geotechnical and Geoenvironmental Engineering (February 1997) and are used to solve this problem.

$$\sin \beta \leq \sin \phi + \frac{c}{l} \cos \phi \tag{Eq. 4-40}$$

Where:

$$l = \frac{1}{\cos \phi^2} \left[ \sigma_x + \frac{1}{2} \sin(2\phi) - \sqrt{\sigma_v (\cos \beta^2 - \cos \phi^2) + \sigma_x [c \sin(2\phi)] + c^2 \cos \phi^2} \right] \tag{Eq. 4-41}$$

$$\sigma_v = \gamma (H \cos \beta)$$

$$\sigma_x = \gamma (H \cos^2 \beta)$$

The following outlines various methods for analyzing shoring systems that have sloping ground conditions.

### 4.5.1 Active Trial Wedge Method

Figure 4-14 shows the assumptions used to determine the resultant active pressure for sloping ground with an irregular backfill condition applying the wedge theory. This is an iterative process. The failure plane angle ( $\alpha_n$ ) for the wedge varies until the maximum value of the active earth pressure is computed using Eq. 4-42. The development of Eq. 4-42 is based on the limiting equilibrium for a general soil wedge. It is assumed that the soil wedge moves downward along the failure surface and along the wall surface to mobilize the active wedge. This wedge is held in equilibrium by the resultant force equal to the resultant active pressure ( $P_a$ ) acting on the face of the wall. Since the wedge moves downward along the face of the wall, this force acts with an assumed wall friction angle ( $\delta$ ) below the normal to the wall to oppose this movement.

For any assumed failure surface defined by angle  $\alpha_n$  from the horizontal and the length of the failure surface  $L_n$ , the magnitude of the wedge weight ( $W_n$ ) is the weight of the soil wedge plus the relevant surcharge load. For any failure wedge the maximum value of  $P_a$  can be determined using Eq. 4-42.

$$P_a = \frac{W[\tan(\alpha - \phi)] - C_o L_c [\sin \alpha \tan(\alpha - \phi) + \cos \alpha] - C_a L_a [\tan(\alpha - \phi) \cos(-\omega) + \sin \omega]}{[1 + \tan(\delta + \omega) \tan(\alpha - \phi)] \cos(\delta + \omega)}$$

Eq. 4-42

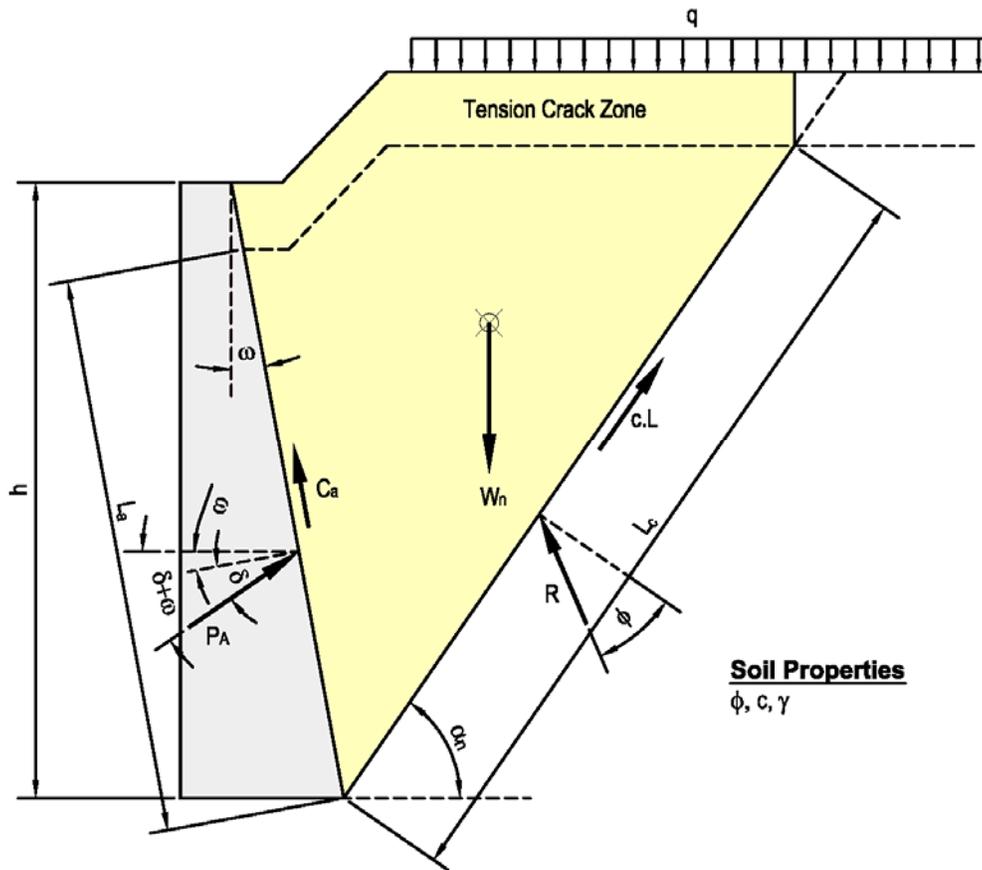


Figure 4-14. Active Trial Wedge

Where:

- $P_a$  = active lateral earth pressure resultant per unit width of wall.
- $W$  = weight of soil wedge plus the relevant surcharge loads.
- $\delta$  = friction angle between backfill material and back of wall.
- $\theta$  = effective friction angle of soil.
- $\alpha_n$  = failure plane angle with respect to horizontal.
- $C$  = soil cohesion.
- $L_n$  = length of the failure plane.



Where:

$P_p$  = passive lateral earth pressure resultant per unit width of wall.

$W_n$  = weight of soil wedge plus the relevant surcharge loads.

$\delta$  = friction angle between backfill material and back of wall.

$\phi$  = effective friction angle of soil.

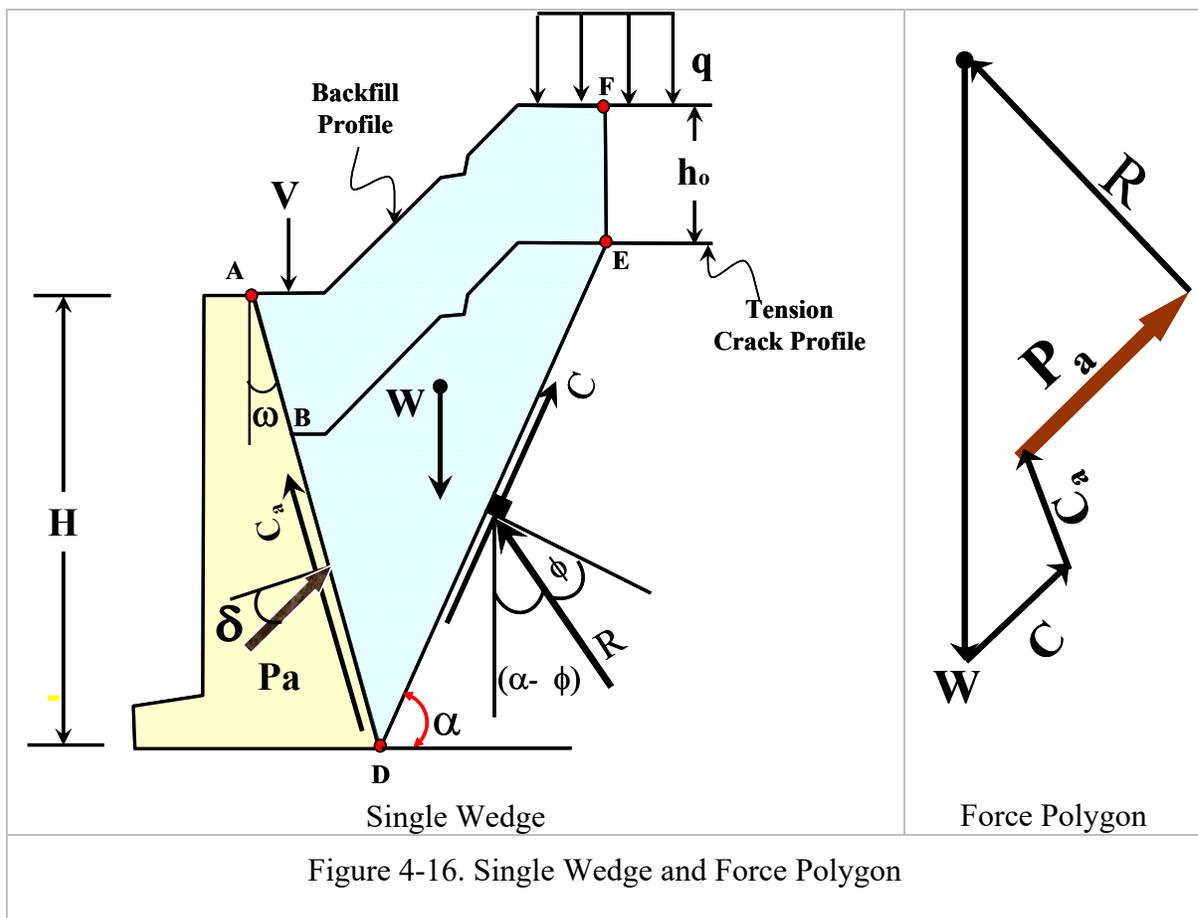
$\alpha_n$  = failure plane angle with respect to horizontal.

$C$  = soil cohesion.

$L_n$  = length of the failure plane.

### 4.5.3 Culmann's Graphical Solution for Active Earth Pressure

Culmann (1866) developed a convenient graphical solution procedure to calculate the active earth pressure for retaining walls for irregular backfill and surcharges. Figure 4-16 shows a failure wedge and a force polygon acting on the wedge. The forces per unit width of the wall to be considered for equilibrium of the wedge are as follows:



## EARTH PRESSURE THEORY AND APPLICATION

1.  $W$  = Weight of the wedge including weight of the tension crack zone and the surcharges with a known direction and magnitude

$$W = \text{ABDEFA}_{\text{area}} (\gamma) + q(lq) + V \quad \text{Eq. 4-44}$$

2.  $C_a$  = Adhesive force along the backfill of the wall with a known direction and magnitude

$$C_a = c (\text{BD}) \quad \text{Eq. 4-45}$$

3.  $C$  = Cohesive force along the failure surface with a known direction and magnitude

$$C = c (\text{DE}) \quad \text{Eq. 4-46}$$

4.  $h_o$  = Height of the tension crack

$$h_o = \frac{2C\sqrt{K_a}}{g(K_a)}; \text{ Where } K_a = \tan^2\left(45 - \frac{\phi}{2}\right) \quad \text{Eq. 4-47}$$

5.  $R$  = Resultant of the shear and normal forces acting on the failure surface DE with the direction known only

6.  $P_a$  = Active force of wedge with the direction known only

Where:

$c$  = Soil cohesion value.

$K_a$  = Rankine active earth pressure coefficient.

$\phi$  = Soil friction angle.

$\gamma$  = Unit weight of soil.

To determine the maximum active force against a retaining wall, several trial wedges must be considered and the force polygons for all the wedges must be drawn to scale as shown Figure 4-17.

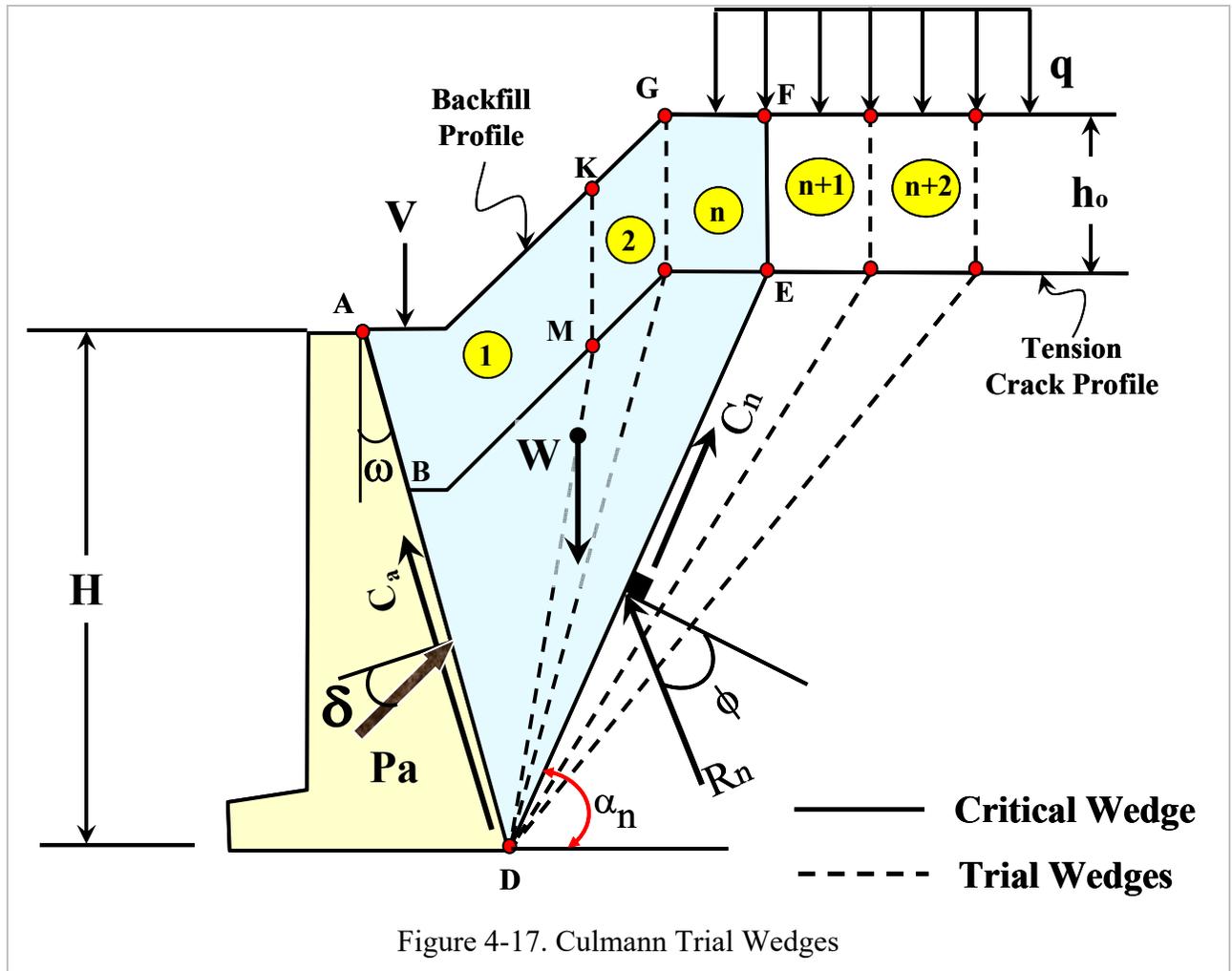


Figure 4-17. Culmann Trial Wedges

The procedure for estimating the maximum active force,  $P_a$ , as shown in Figure 4-17 and Figure 4-18, is described as follows:

1. Draw the lines for the tensile crack profile parallel to the backfill profile with height equal to  $h_0$ .
2. Draw several trial wedges to intersect the tension crack profile line.
3. Draw the vectors to represent the weight of wedges per unit width of the wall including the surcharges.
4. Draw adhesion force vector  $C_a$  acting along the face of the wall.
5. Draw cohesion force vector  $C_{oh}$  acting along the failure surfaces.
6. Draw the active force vector  $P_a$ .
7. Draw the resultant force vector  $R$  acting on the failure place
8. Repeat steps 2 through 7 until all trial wedges are complete

## EARTH PRESSURE THEORY AND APPLICATION

9. Draw a smooth curve through these points as shown in Figure 4-18. A cubic spline function is used in CT-Flex computer program to draw the smooth line between point  $P_1$  through point  $P_{n+2}$  as shown in Figure 4-18.
10. Draw dashed line  $TT'$  through the left end of force vectors  $P_a$  as shown in Figure 4-18.  
Draw a parallel line to line  $TT'$  that is tangent to the above curve to measure maximum active earth pressure length as shown in Figure 4-18.
12. Draw a line parallel to the force vectors  $P_a$  that begins at  $TT'$  and ends at the intersection point of the tangent line to the curved line above. This is the maximum active pressure force vector  $P_{a_{max}}$ .

The maximum active pressure shown in Figure 4-17 and Figure 4-18 is obtained as:

$$P_a = (\text{length of } L) \times (\text{load scale } \lambda)$$

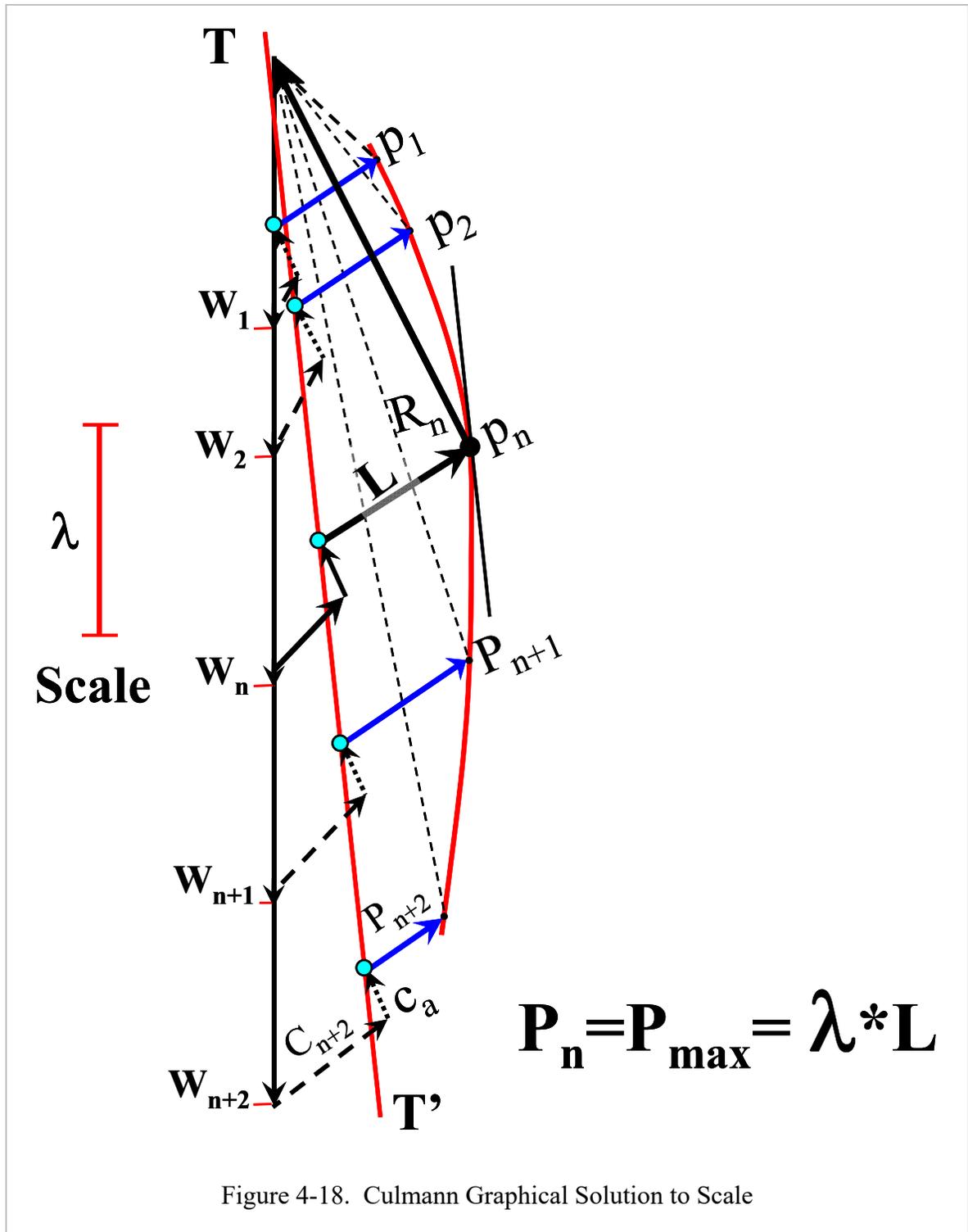


Figure 4-18. Culmann Graphical Solution to Scale

**4.5.3.1 Example 4-1 Culmann Graphical Method**

Calculate the maximum active earth pressure using the Culmann graphical method for a retaining wall given in Figure 4-19 using the following backfill properties.

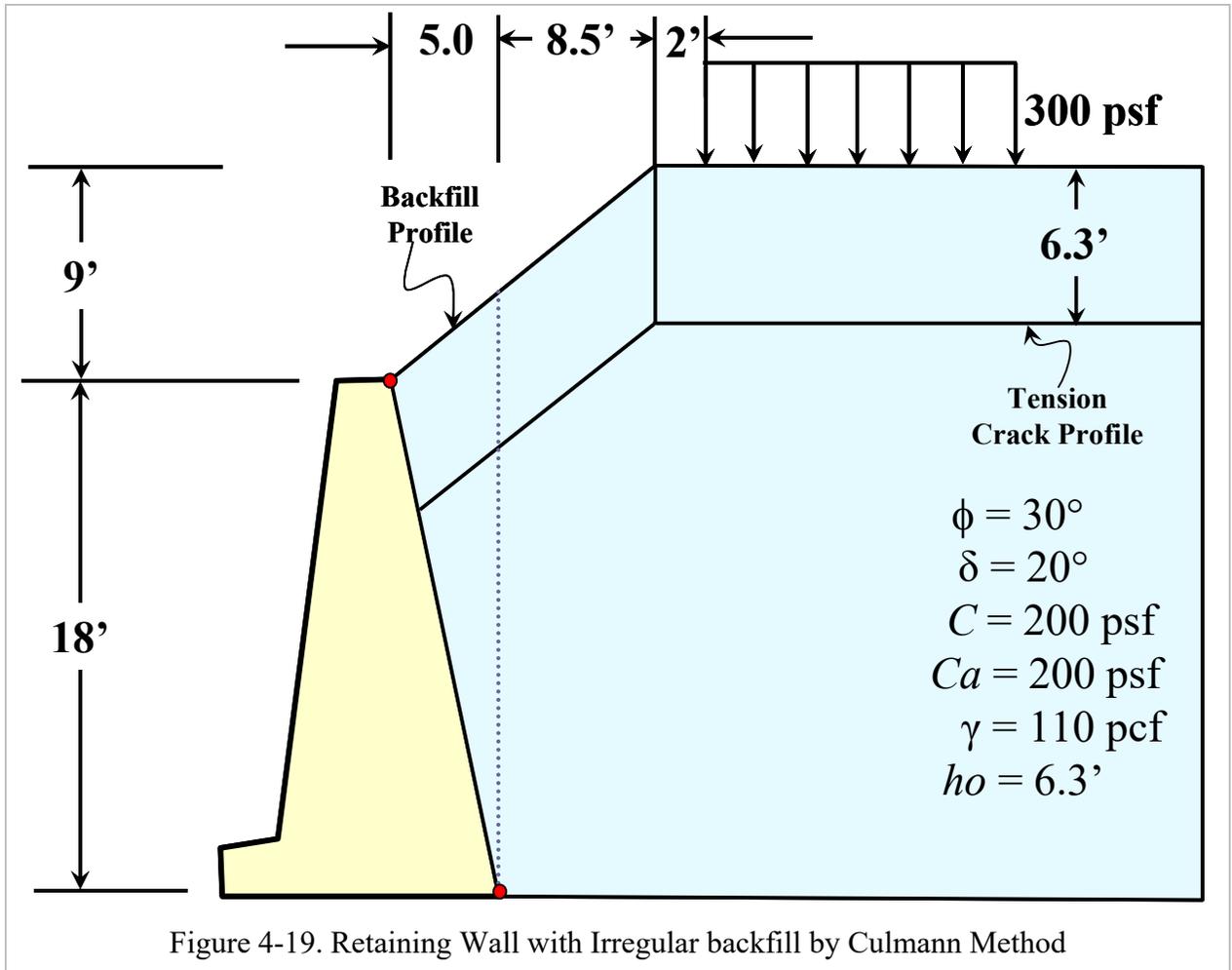


Figure 4-19. Retaining Wall with Irregular backfill by Culmann Method

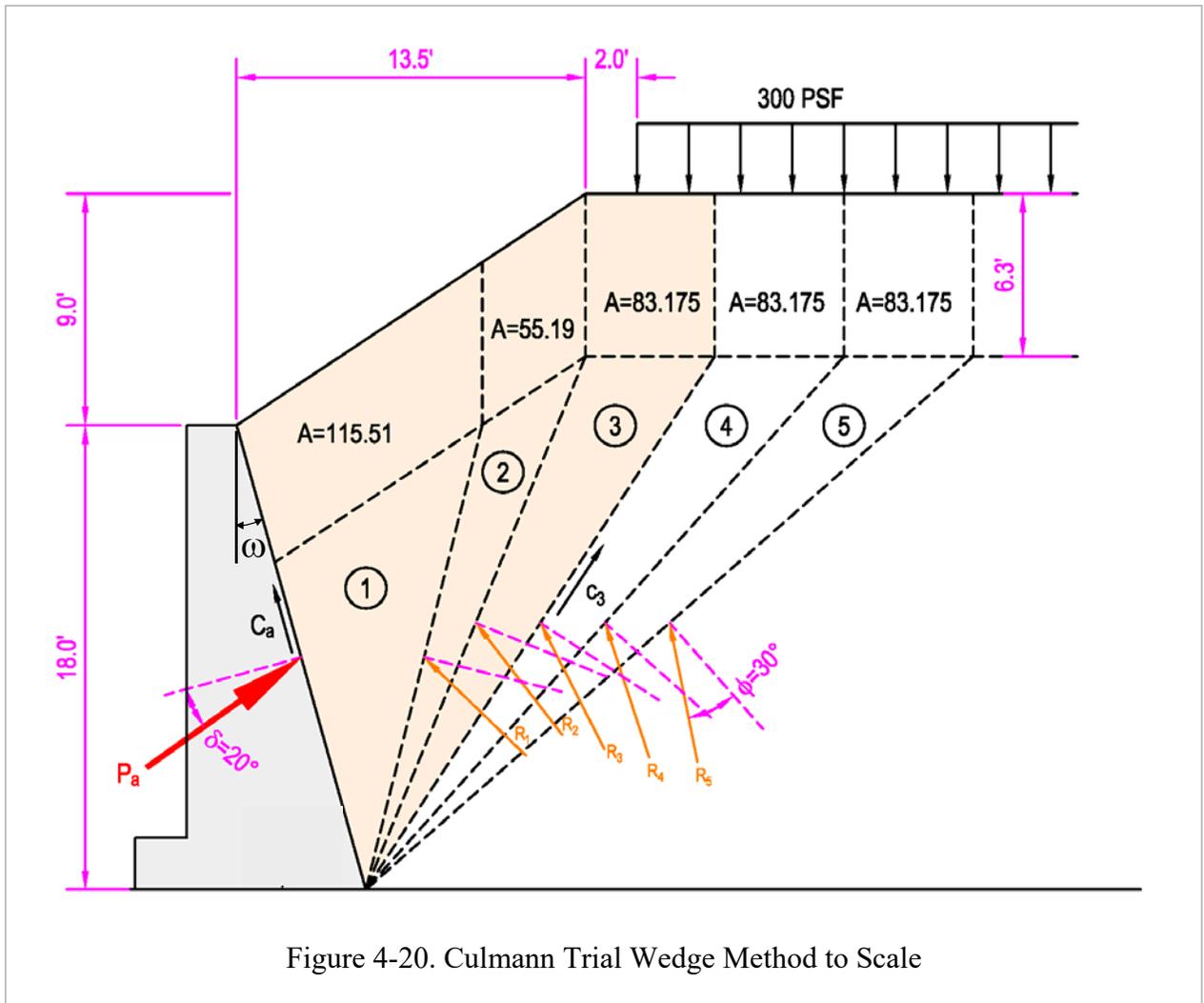


Figure 4-20. Culmann Trial Wedge Method to Scale

Solution:

As shown in Figure 4-20 several trial wedges are drawn. The weight of each of these wedges, the adhesive force at the wall interface and cohesive force along the failure surface are computed as is shown below.

The active earth pressure due to soil-wall interaction is constant for all wedges and is calculated as shown below.

$$\text{Determine wall angle: } \omega = \tan^{-1}\left(\frac{5'}{18'}\right) = 15.52^\circ$$

$$L_a = \frac{(18 - 6.3)}{\cos(15.52)} = 12.14 \text{ ft.}$$

$$C_a = c_a L_a = (2)(12.14) = 2.43 \text{ k/ft}$$

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Where  $L_a$  is the length of the active wedge along the backwall and  $C_a$  is the active earth pressure due to wall-backfill adhesion properties.

Table 4-3. Culmann Graphical Method Results

Geometry			Weight Components				Cohesion Comp.		
Wedge	Backfill Profile Coordinates		Wt (k)	$\Sigma$ Wt (k)	Sur. Wt (k)	$\Sigma$ Sur. (k)	Total Wt (k)	Length (ft)	Coh (k)
	X	Y							
1	4.51	18.04	12.72	12.72	0	0	12.72	18.6	3.72
2	8.51	20.70	6.09	18.81	0	0	18.81	22.38	4.48
3	13.52	20.70	9.17	27.98	0.91	0.91	28.89	24.72	4.94
4	18.53	20.70	9.17	37.15	1.50	2.41	39.56	27.78	5.56
5	23.54	20.70	9.18	46.33	1.50	3.91	50.24	31.34	6.27

The force polygon for all the wedges and maximum active force using scaling factors are shown in Figure 4-21. The maximum earth pressure is about 8.5 kips/ft.

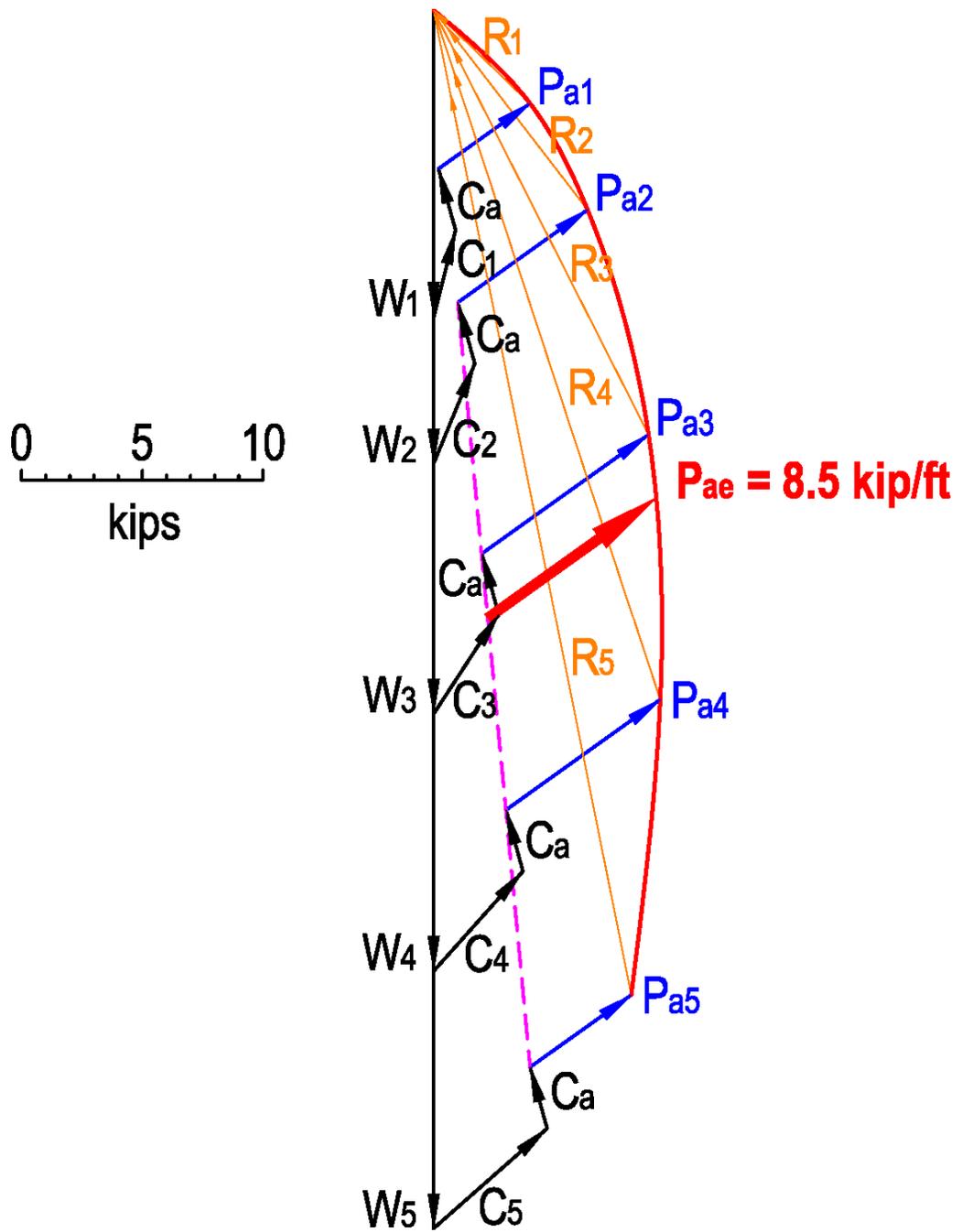
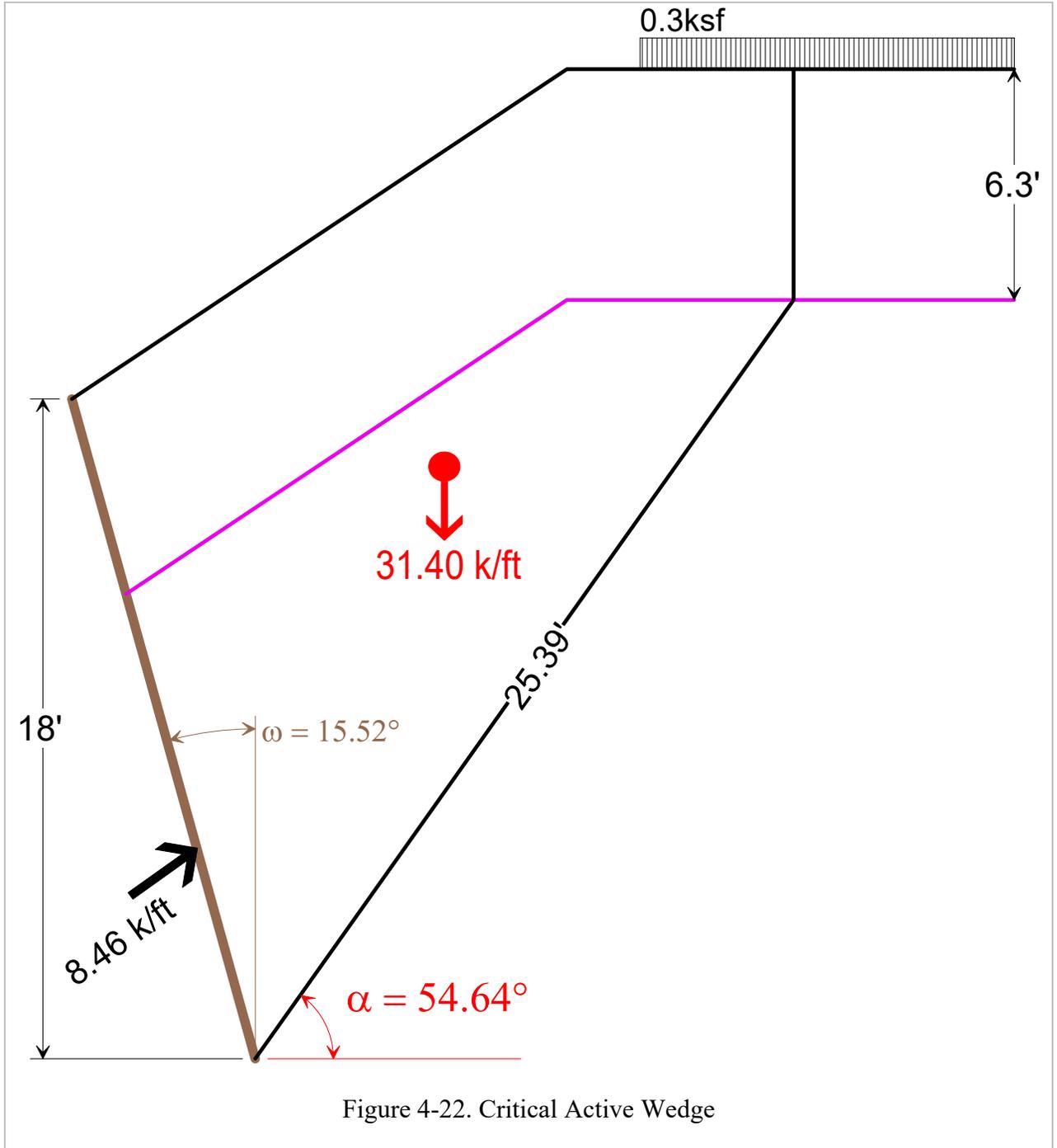


Figure 4-21. Culmann Graphical Solution Using Force Polygon

**4.5.3.2 Example 4-2 Trial Wedge Method**

Repeat Example 4-1 using trial wedge method. Figure 4-22 illustrates the most critical failure surface developed using the Caltrans Trenching and Shoring Check Program.



Eq. 4-42 is used to calculate the active earth pressure.

$$P_a = \frac{W[\tan(\alpha - \phi)] - C_o L_c [\sin \alpha \tan(\alpha - \phi) + \cos \alpha] - C_a L_a [\tan(\alpha - \phi) \cos(-\omega) + \sin \omega]}{[1 + \tan(\delta + \omega) \tan(\alpha - \phi)] \cos(\delta + \omega)}$$

CT-T&S Program is used to calculate the failure plane angle ( $\alpha$ ) and the length of the critical failure surface ( $L_c$ ).

$$P_a = \frac{WT - COH - ADH}{[1 + \tan(\delta + \omega) \tan(\alpha - \phi)] \cos(\delta + \omega)} \quad P1-1$$

Calculate the weight contribution from weight of the wedge and weight the surcharge (WT):

$$WT = W[\tan(\alpha - \phi)] = (31.40)[\tan(54.64 - 30)] = 14.40 \text{ k/ft}$$

Calculate adhesion (ADH) component.

$$L_a = \frac{(18 - 6.3)}{\cos(15.52)} = 12.14 \text{ ft.}$$

$$\begin{aligned} ADH &= C_a L_a [\tan(\alpha - \phi) \cos(-\omega) + \sin(-\omega)] \\ &= (0.2)(12.14)[\tan(54.64 - 30) \cos(-15.52) + \sin(-15.52)] = 0.422 \text{ k/ft} \end{aligned}$$

Calculate cohesion (COH) component.

$$\begin{aligned} COH &= C_o L_c [\sin \alpha \tan(\alpha - \phi) + \cos \alpha] \\ &= (0.2)(25.39)[\sin(54.64) \tan(54.64 - 30) + \cos(54.64)] = 4.837 \text{ k/ft} \end{aligned}$$

Substitute WT, COH and ADH in to P1-1.

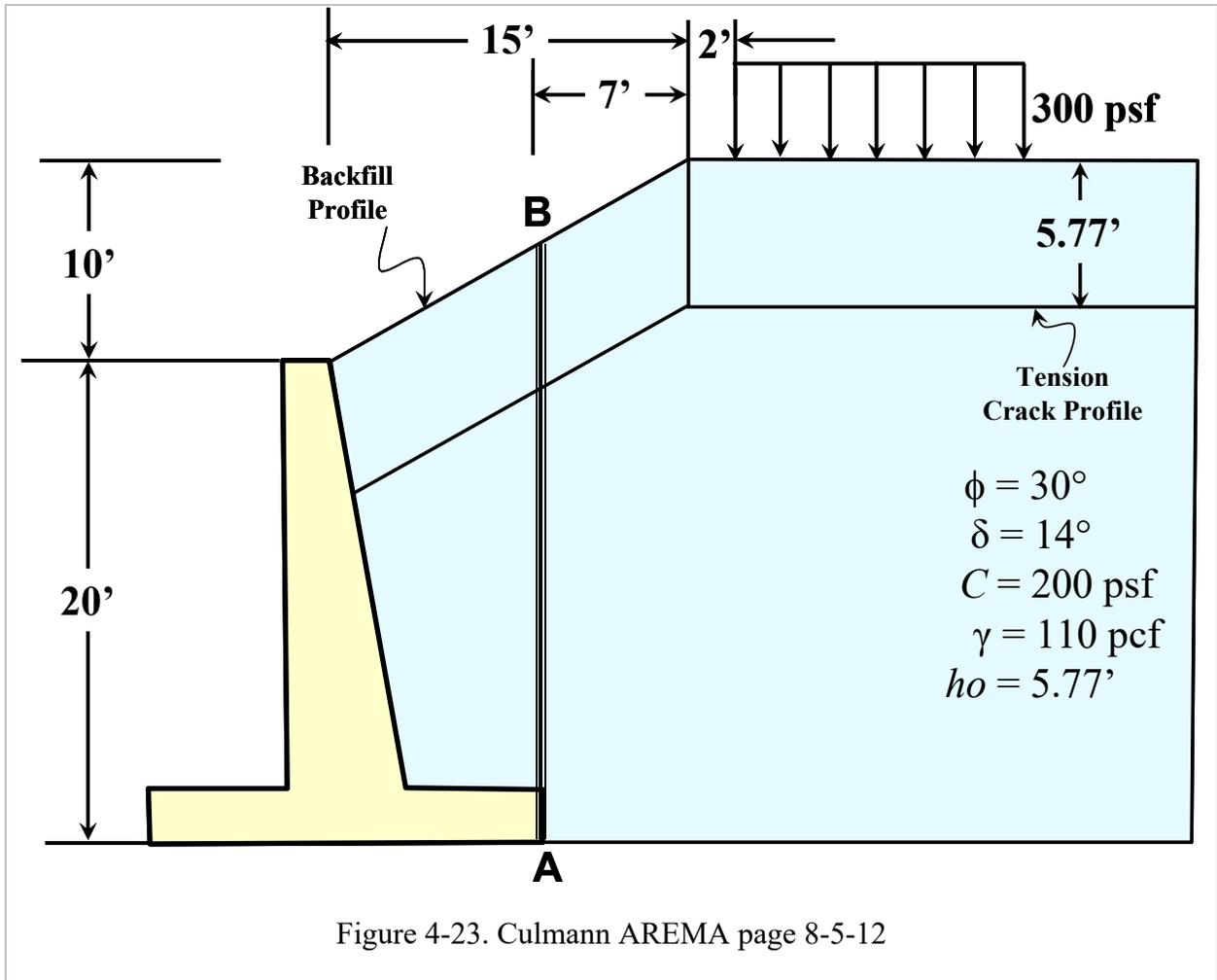
$$P_a = \frac{14.40 - 4.84 - .422}{[1 + \tan(20 + 15.52) \tan(54.64 - 30)] \cos(20 + 15.52)} = 8.46 \text{ k/ft}$$

## EARTH PRESSURE THEORY AND APPLICATION

The following examples are taken from AREMA (American Railway Engineering and Maintenance-of-Way Association) Manual for Railway Engineering.

### 4.5.3.3 Example 4-3 (AREMA Manual page 8-5-12)

Calculate the maximum active earth pressure using the Culmann graphical method for a retaining wall with a heel (earth pressure at line AB) given in Figure 4-23.



Solution:

As shown in Figure 4-24 several trial wedges are drawn. The weight of each of these wedges, the adhesive force at the wall interface and cohesive force along the failure surface are computed as is shown below.

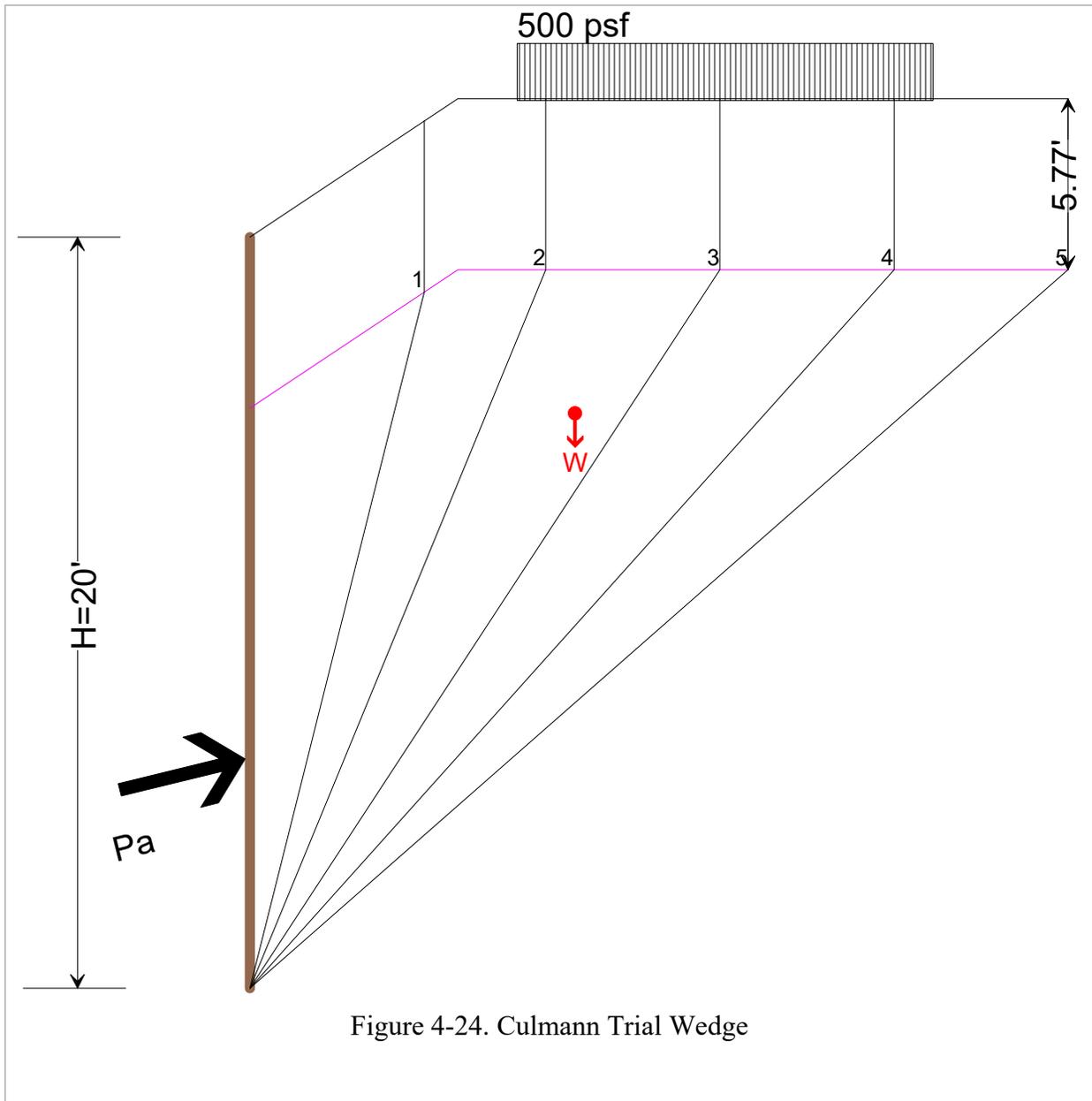


Figure 4-24. Culmann Trial Wedge

## EARTH PRESSURE THEORY AND APPLICATION

Table 4-4. Culmann Trial Wedge Method Results

Geometry			Weight Components				Cohesion Comp.		
Wedge	Backfill Profile Coordinates		Wt (k)	$\Sigma$ Wt (k)	Sur. Wt (k)	$\Sigma$ Sur. (k)	Total Wt (k)	Length (ft)	Coh (k)
	X	Y							
1	5.87	23.48	10.95	10.95	0	0	10.95	24.20	4.84
2	9.96	24.23	8.47	19.42	0.48	0.48	19.90	26.20	5.24
3	15.83	24.23	12.58	32.00	2.93	3.41	35.41	28.94	5.79
4	21.69	24.23	12.58	44.58	2.93	6.34	50.92	32.52	6.50
5	27.55	24.23	12.58	57.16	0.66	7.00	64.16	36.69	7.34

The force polygon for all the wedges and maximum active force using scaling factors are shown in Figure 4-25. The maximum earth pressure is about 11.35 kips/ft.

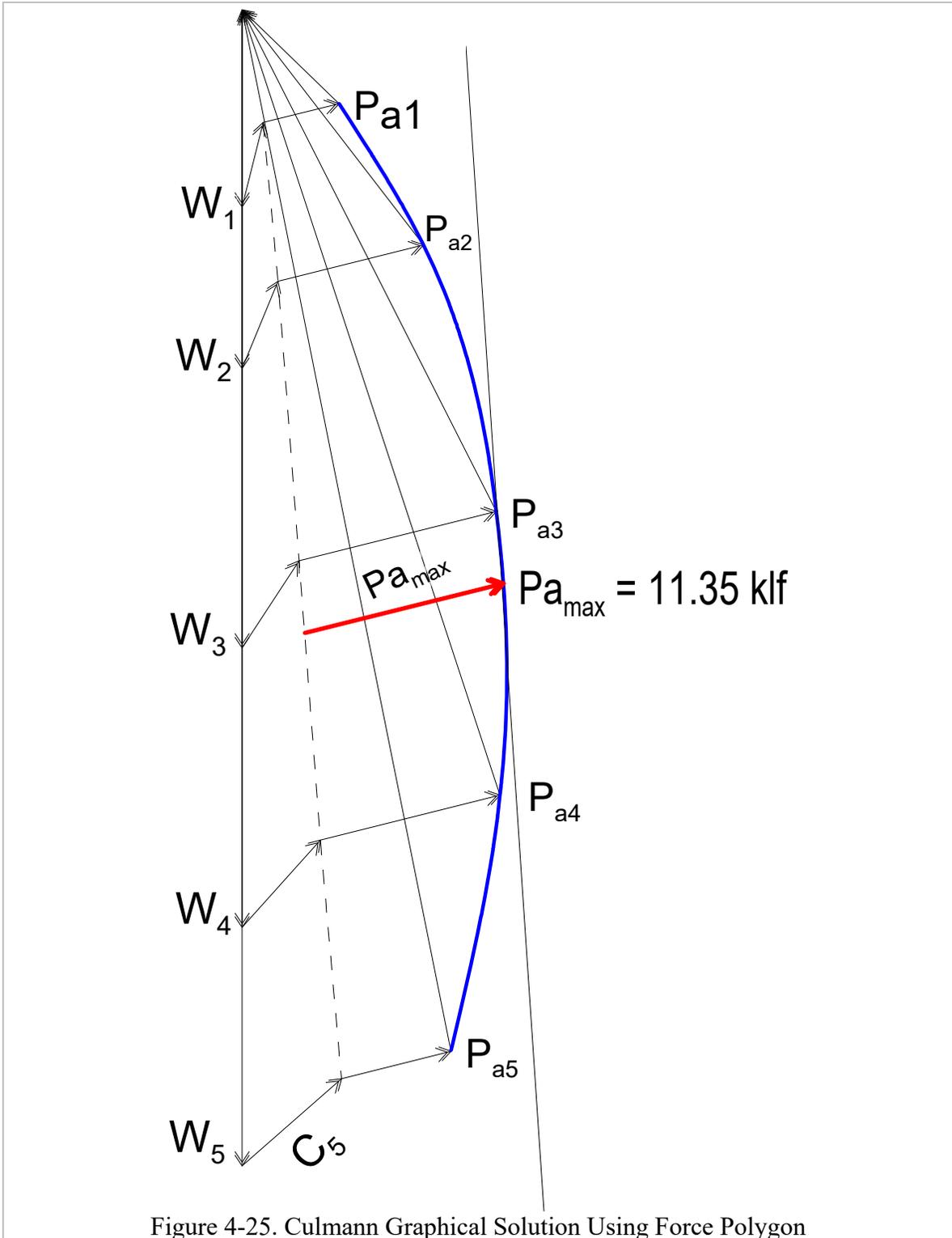


Figure 4-25. Culmann Graphical Solution Using Force Polygon

**4.5.3.4 Example 4-4 (AREMA Manual page 8-5-12)**

Repeat using the trial wedge method. Figure 4-26 illustrates the most critical failure surface developed using the Caltrans Trenching & Shoring Check Program.

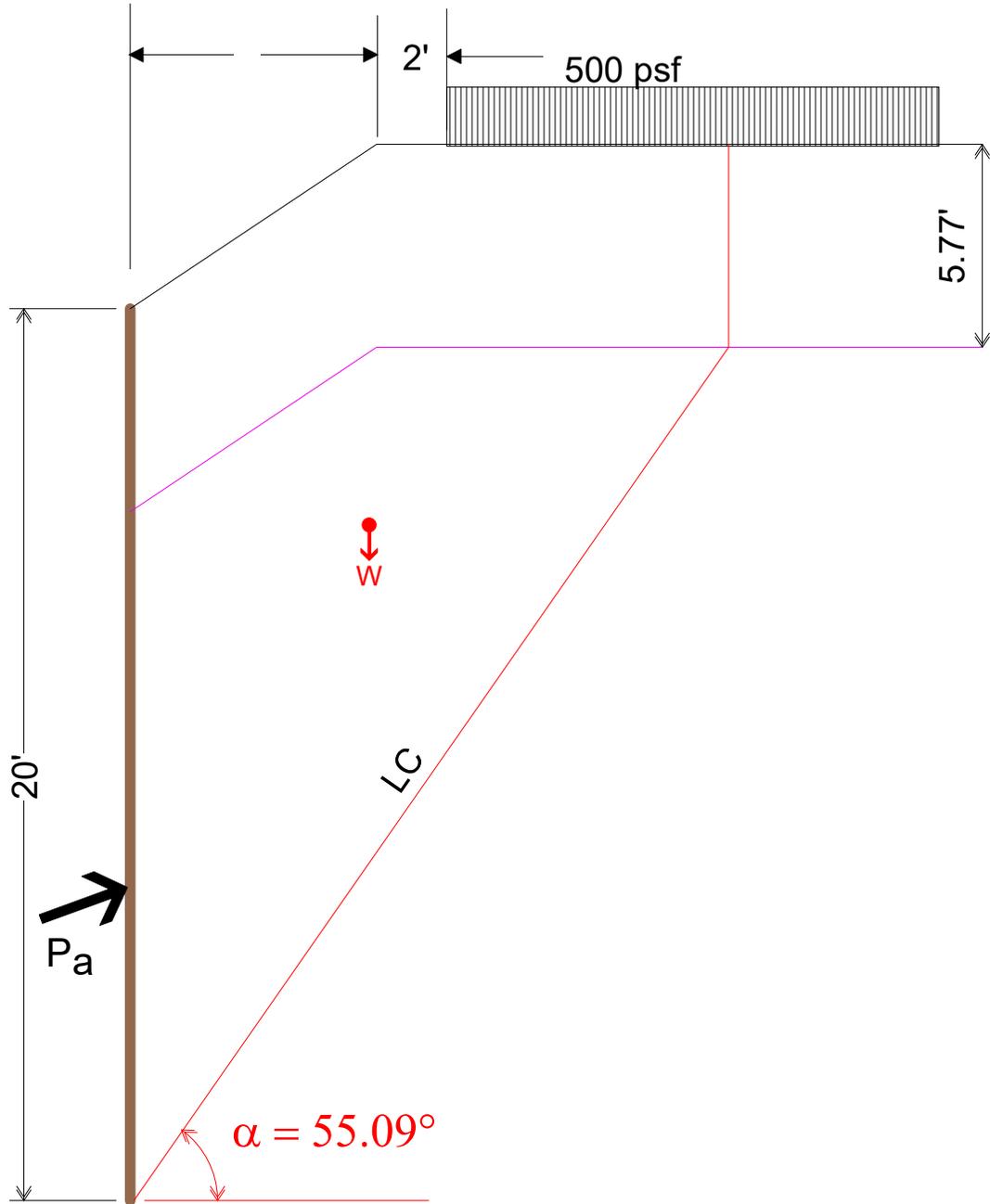


Figure 4-26. Critical Active Wedge Method

Eq. 4-42 without the adhesion component is used to calculate the active earth pressure ( $P_a$ ). Caltrans Trenching & Shoring Check Program is used to calculate the failure plane angle ( $\alpha$ ) and the length of the critical failure surface ( $L_c$ ).

$$P_a = \frac{W[\tan(\alpha - \phi)] - C_o L_a [\sin\alpha \tan(\alpha - \phi) + \cos\alpha]}{[1 + \tan\delta \tan(\alpha - \phi)] \cos\delta}$$

or

$$P_a = \frac{WT - COH}{[1 + \tan\delta \tan(\alpha - \phi)] \cos\delta} \quad P4.1$$

Calculate the weight contribution from weight of the wedge and weight of the surcharge (WT):

$$WT = W[\tan(\alpha - \phi)] = (38.31)[\tan(55.09 - 30)] = 17.92 \text{ k/ft}$$

Calculate cohesion (COH) component.

$$\begin{aligned} COH &= C_o L_a [\sin\alpha \tan(\alpha - \phi) + \cos\alpha] \\ &= (2)(29.55)[\tan(55.09 - 30)\sin(55.09) + \cos(55.09)] = 5.65 \text{ k/ft} \end{aligned}$$

Substitute WT and COH into equation P4.1.

$$P_a = \frac{(17.92 - 5.65)}{[1 + \tan(14)\tan(55.09 - 30)] \cos(14)} = 11.32 \text{ k/ft}$$

**4.5.3.5 Example 4-5 (AREMA Manual page 8-5-13)**

Calculate the maximum active earth pressure using the Culmann graphical method for a retaining wall with no heel given in Figure 4-26 using the following backfill properties.

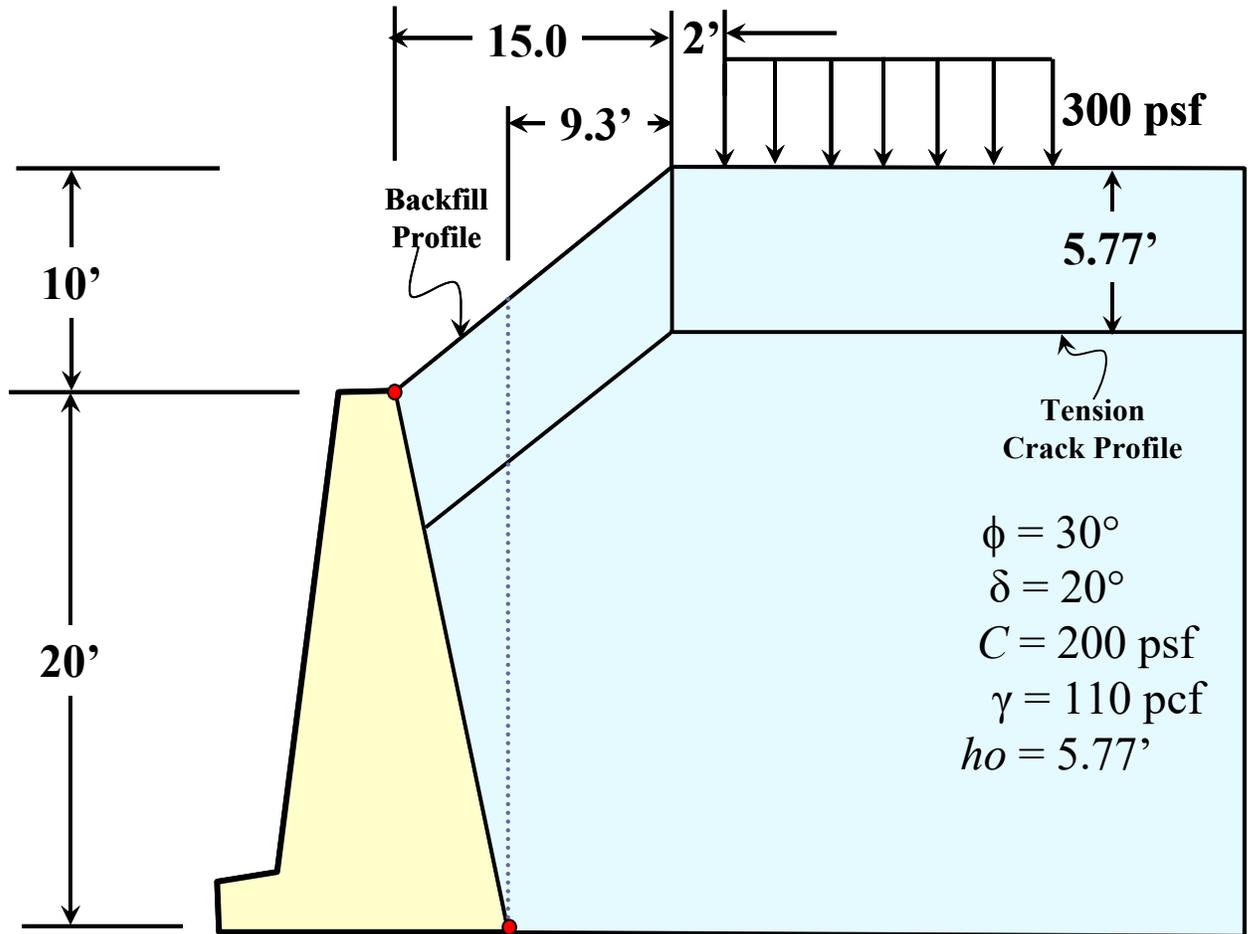


Figure 4-27. Retaining Wall with Irregular backfill

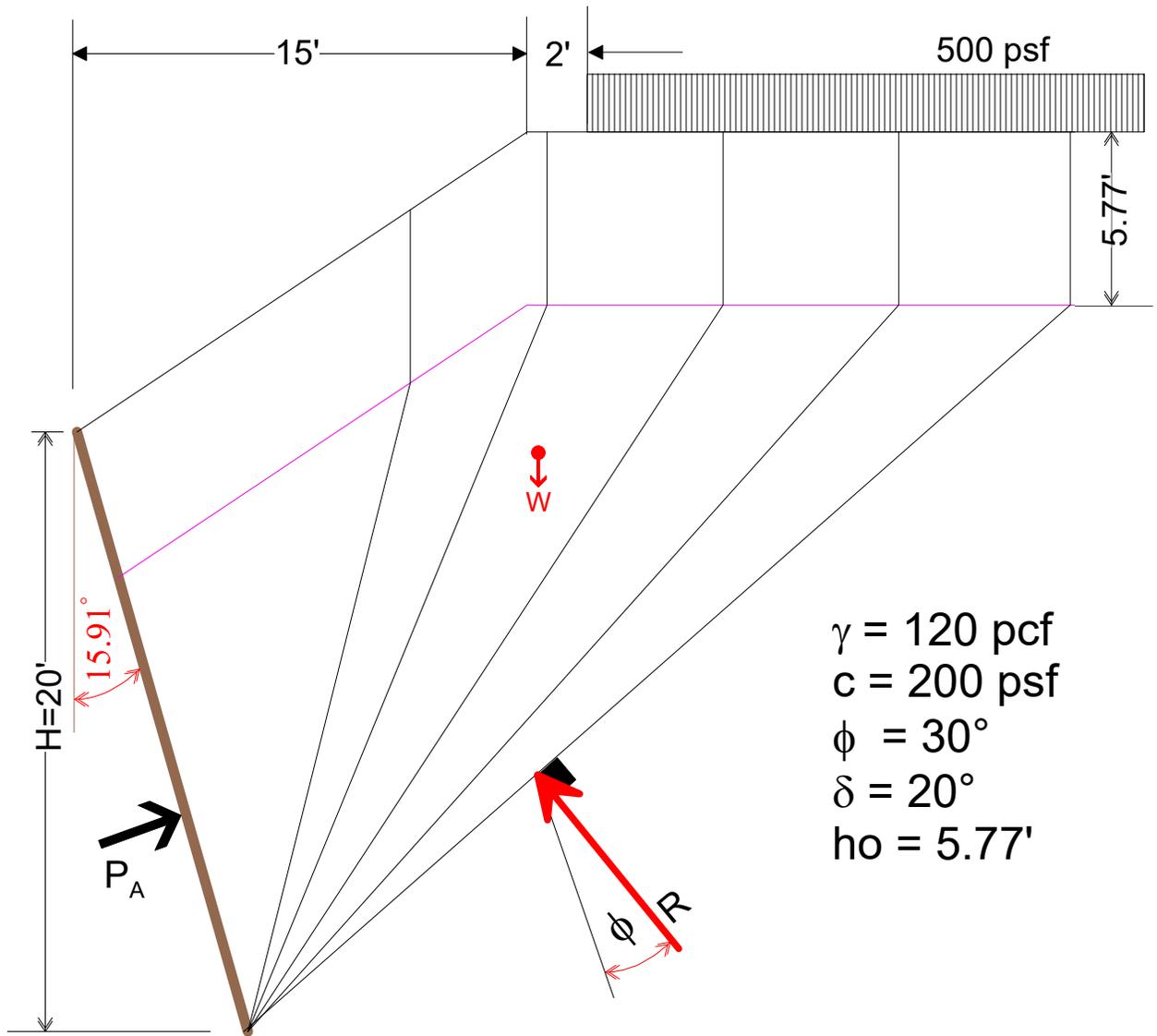
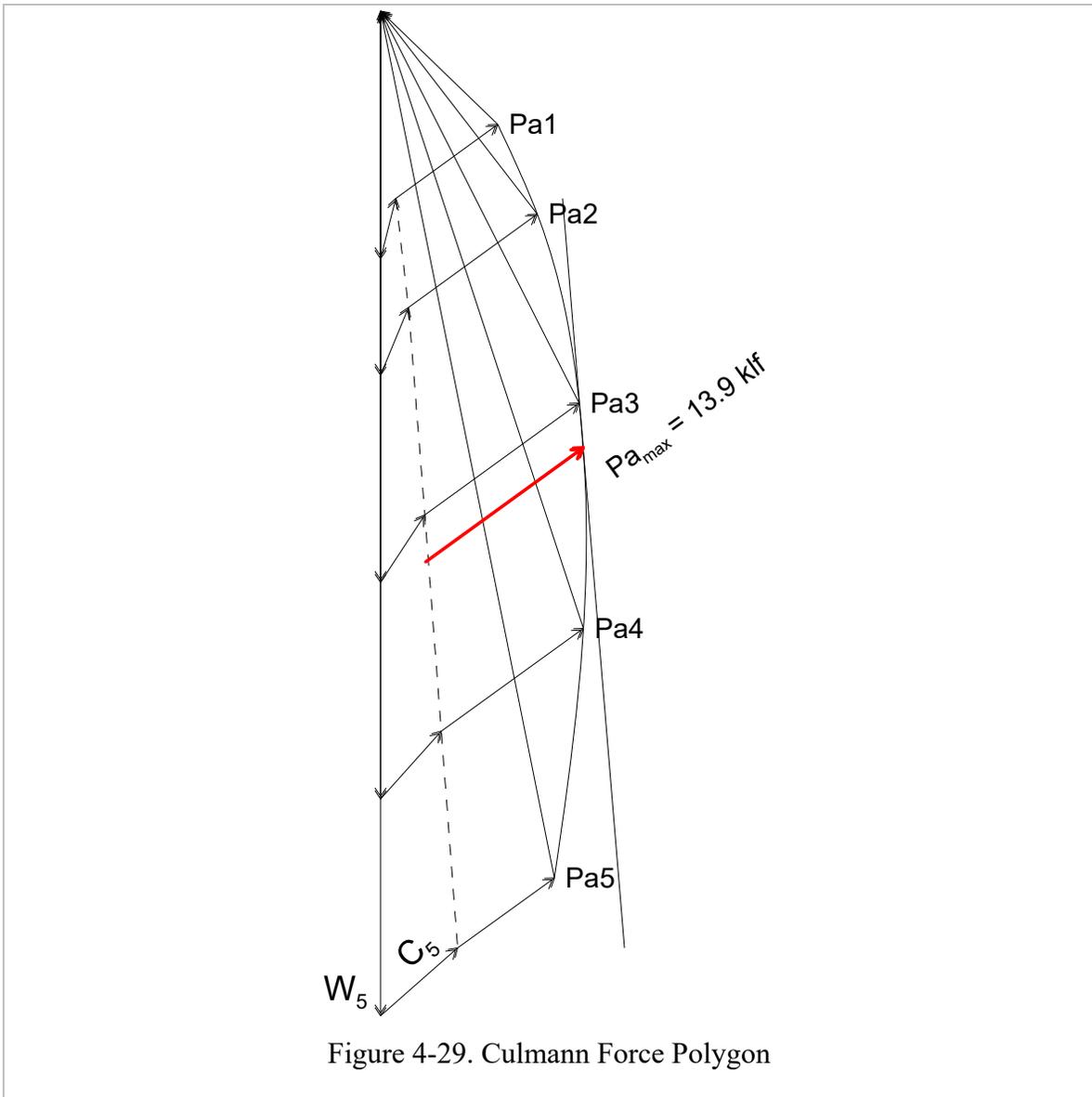


Figure 4-28. Culmann Trial Wedge

# EARTH PRESSURE THEORY AND APPLICATION

Table 4-5. Culmann Graphical Method Results

Geometry			Weight Components				Cohesion Comp.		
Wedge	Backfill Profile Coordinates		Wt (k)	$\Sigma$ Wt (k)	Sur. Wt (k)	$\Sigma$ Sur. (k)	Total Wt (k)	Length (ft)	Coh (k)
	X	Y							
1	5.41	21.64	17.74	17.74	0	0	17.74	22.30	4.46
2	9.96	24.23	8.33	26.07	0	0	26.07	26.20	5.24
3	15.83	24.23	12.58	38.65	2.26	2.26	40.91	28.94	5.79
4	21.69	24.23	12.58	51.23	2.93	5.19	56.42	32.52	6.50
5	27.55	24.23	12.58	63.81	1.81	7.00	64.16	36.69	7.34



**4.5.3.6 Example 4-6 (AREMA Manual page 8-5-13)**

Repeat using the trial wedge method. Figure 4-29 illustrates the most critical failure surface developed using the Caltrans Trenching & Shoring Check Program.

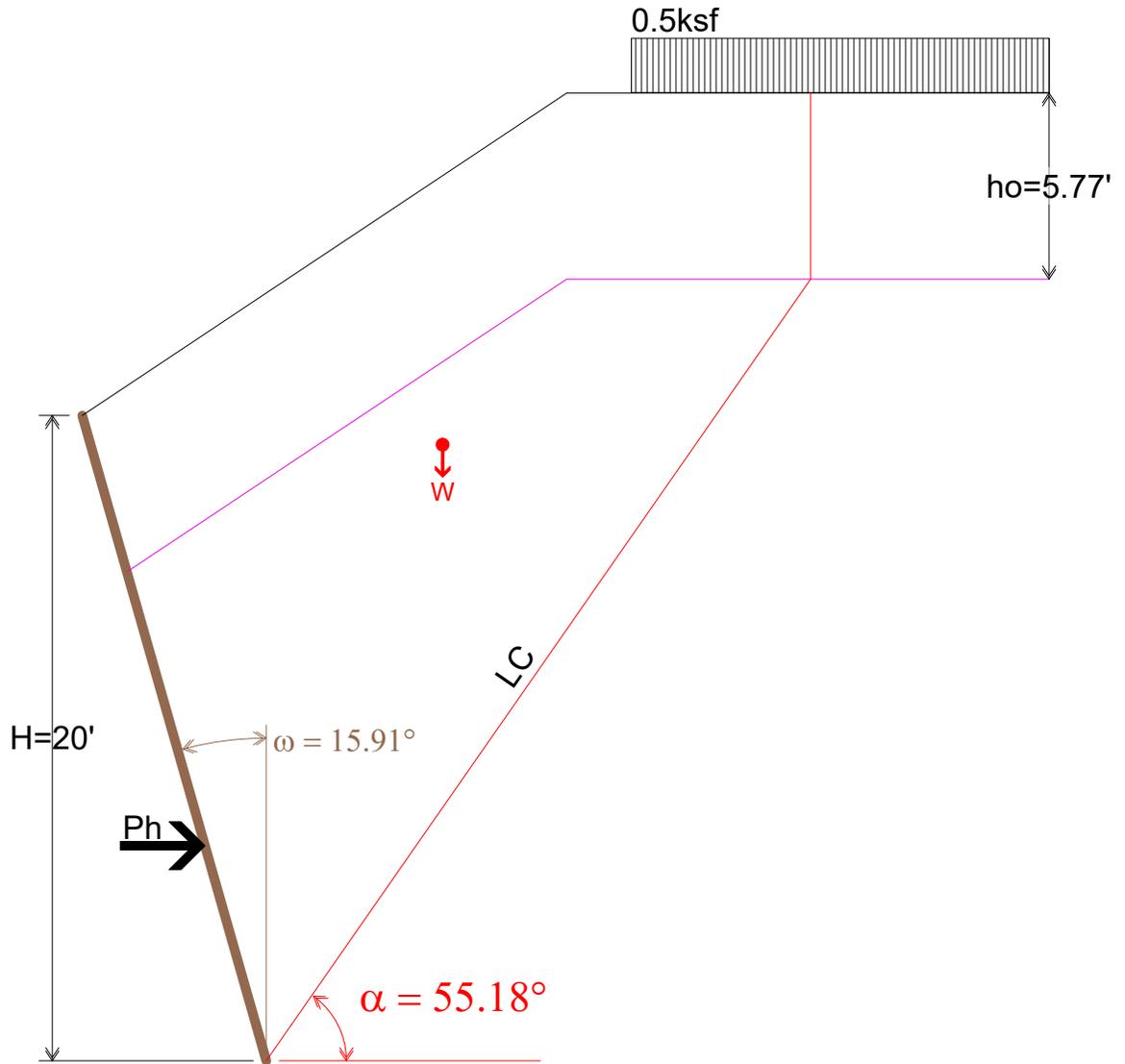


Figure 4-30. Critical Active Wedge

## EARTH PRESSURE THEORY AND APPLICATION

Eq. 4-42 without the adhesion component is used to calculate the active earth pressure ( $P_a$ ). Caltrans Trenching & Shoring Check Program is used to calculate the failure plane angle ( $\alpha$ ) and the length of the critical failure surface ( $L_c$ ).

$$P_a = \frac{W[\tan(\alpha - \phi)] - C_o L_c [\sin\alpha \tan(\alpha - \phi) + \cos\alpha]}{[1 + \tan(\delta + \omega)\tan(\alpha - \phi)]\cos(\delta + \omega)}$$

or

$$P_a = \frac{WT - COH}{[1 + \tan(\delta + \omega)\tan(\alpha - \phi)]\cos(\delta + \omega)} \quad \text{P5-1}$$

Calculate the weight contribution from weight of the wedge and weight of the surcharge (WT):

$$WT = W[\tan(\alpha - \phi)] = (43.65)[\tan(55.18 - 30)] = 20.51 \text{ k/ft}$$

Calculate cohesion (COH) component.

$$\begin{aligned} COH &= C_o L_c [\sin\alpha \tan(\alpha - \phi) + \cos\alpha] \\ &= 0.2(29.52)[\tan(55.18 - 30)\sin(55.18) + \cos(55.18)] = 5.65 \text{ k/ft} \end{aligned}$$

Substitute WT and COH into equation P5-1.

$$P_a = \frac{(20.51 - 5.65)}{[1 + \tan(20 + 12.91)\tan(55.09 - 30)]\cos(20 + 15.91)} = 13.70 \text{ k/ft}$$

#### 4.6 EFFECT OF WALL FRICTION

Figure 4-31 shows a shoring system with a wall-soil interface friction angle,  $\alpha$ , that has been sufficiently extended below the dredge line. The shoring system is stable when the active earth pressure developed on the high side of the wall is opposed by much higher passive earth pressure on the low side. It can be seen that the sliding surface, Figure 4-31, for active earth pressure is practically a straight line whereas a straight line cannot approximate the sliding surface for passive earth pressure. Computation of the passive earth pressure using the log spiral failure surface is presented in the following sections.

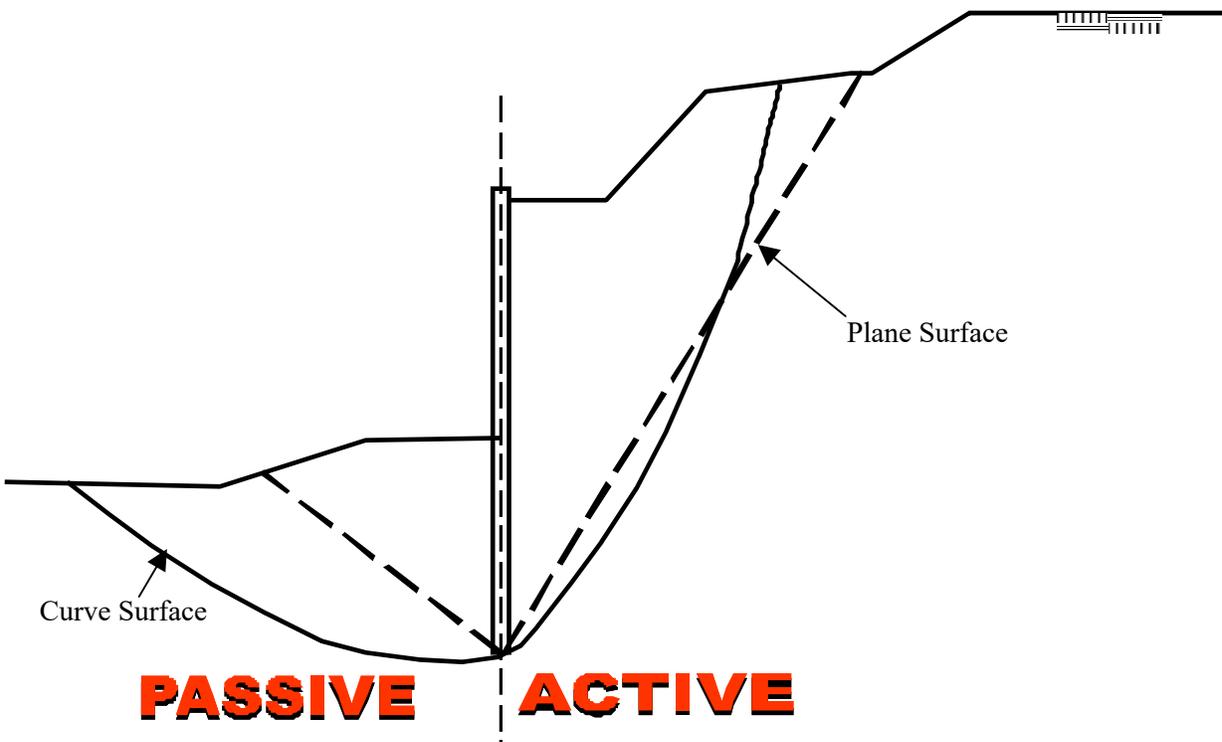


Figure 4-31. Passive Active failure surface; straight line versus spiral surface of sliding.

#### 4.7 LOG SPIRAL PASSIVE EARTH PRESSURE

As mentioned in previous sections, Rankine's theory should not be used to calculate the passive earth pressure forces for a shoring system because it does not account for wall friction. While Coulomb's theory to determine the passive earth pressure force accounts for the angle of wall friction ( $\delta$ ), the theory assumes a linear failure surface. The result is an error in Coulomb's calculated force due to the fact that the actual sliding surface is curved rather than planar.

Coulomb's theory gives increasingly erroneous values of passive earth pressure as the wall friction ( $\delta$ ) increases. Therefore, Coulomb's theory could lead to unsafe shoring system designs because the calculated value of passive earth pressure would become higher than the soil could generate. Terzaghi (1943) suggested that combining a logarithmic spiral and a straight line could represent the failure surface. Morison and Ebeling (1995) suggested a single arc of the logarithmic spiral could realistically represent the failure surface. Both methods, (Terzaghi 1943 composite failure surface and Morison and Ebeling 1995) are implemented in this Trenching and Shoring Manual.

The composite failure surface will be examined first. As seen in Eq. 4-48 and Figure 4-32 (Shamsabadi, *et al.*, 2005), the logarithmic spiral portion of the failure surface (BD) is governed by the height of the wall (AB), the location of the center of the logarithmic spiral arc (O), and the soil's internal friction angle ( $\phi$ ) in sand and c- $\phi$  soil. However, the curved failure surface will be circular ( $R = R_o$ ) in cohesive soil (for total stress analysis,  $\phi = 0$ ). The spiral surface is given as:

$$R = R_o e^{\theta \tan \phi} \tag{Eq. 4-48}$$

$R_o$  is obtained from triangle OAB. The upper portion of DE is a straight line, which is tangent to the curve BD at point D. DE makes an angle  $\alpha_1$  with the horizontal given in Eq. 4-19.

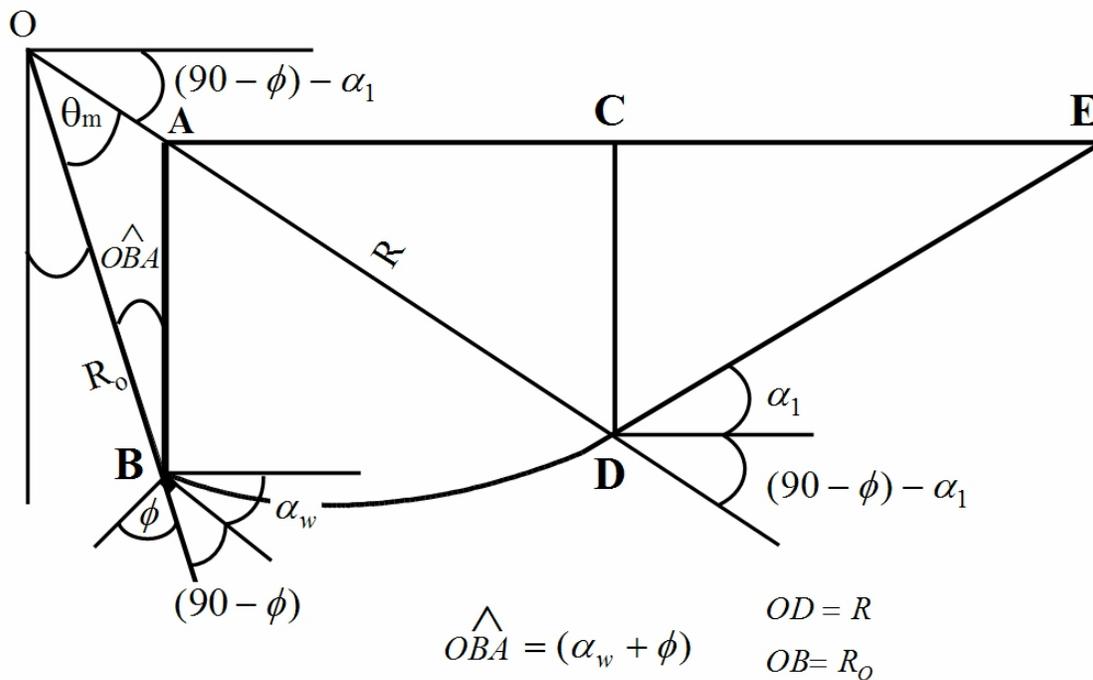


Figure 4-32. Geometry of the developing mobilized failure plane (Shamsabadi, *et al.*, 2005)

The logarithmic spiral leaves the wall at the takeoff angle ( $\alpha_w$ ) at radius OB, and intersects the conjugate failure surface wedge CDE. AD lies on a ray of the logarithmic spiral zone that must pass through the center of the logarithmic spiral arc. As a result, the location of the center of the Log-Spiral curve (O) can be accurately defined based on the subtended angle  $\theta_m$ . Either moment equilibrium or force equilibrium can be used to calculate the passive earth pressure force per unit length of the wall. Several authors have calculated passive earth pressure coefficients using log spiral failure surfaces (moment method and method of slices), circular failure surfaces and elliptical failure surfaces. The shoring engineer has the option to use any of these methods.

### 4.7.1 Composite Failure Surface

#### 4.7.1.1 Force Equilibrium Procedures

The log spiral surface at the bottom of the wall (Figure 4-32) starts with the takeoff angle ( $\alpha_w$ ), which is calculated as follows:

$$\alpha_w = \left(45 - \frac{\phi}{2}\right) - \alpha_p \quad \text{Eq. 4-49}$$

The angle  $\alpha_w$  has a positive value when it is above the horizontal and a negative when it is below the horizontal.

$$\alpha_p = \frac{1}{2} \tan^{-1} \left[ \frac{2K(\tan \delta)}{K-1} \right] \quad \text{Eq. 4-50}$$

Where  $\delta$  is the wall interface friction angle that varies from zero to its full value  $\delta$  (where  $\delta = \delta_{ult}$ ) as a function of  $\phi$ . The coefficient K is the horizontal to vertical stress ratio given in Eq. 4-51.

$$K = \frac{A1 + A2}{A3} \quad \text{Eq. 4-51}$$

Where:

$$A1 = 1 + \sin^2 \phi + \frac{C}{\sigma_z} \sin(2\phi) \quad \text{Eq. 4-52}$$

$$A_2 = 2 \cos \phi \left( \sqrt{\left( \tan \phi + \frac{C}{\sigma_z} \right)^2 + \tan^2 \delta \left[ 4 \left( \frac{C}{\sigma_z} \right)^2 + \frac{C}{\sigma_z} \tan \phi \right] - 1} \right) \quad \text{Eq. 4-53}$$

$$A_3 = \cos^2 \phi + 4 \tan^2 \delta \quad \text{Eq. 4-54}$$

$$\sigma_z = \gamma H$$

The value of  $\theta_m$  can be obtained from the following relationships:

$$\theta_m = \alpha_1 - \alpha_w \quad \text{Eq. 4-55}$$

Where  $\alpha_1$  is the failure angle of slice 1 (Figure 4-33).

Therefore, the value of  $\theta_m$ , can be obtained both from the geometry of the composite failure surface and/or from the state of the stresses of a soil element at the bottom of the wall. The geometry of the failure surface presented in Figure 4-32 can be established using Eq. 4-49, Eq. 4-50 and Eq. 4-55. It should be noted that the direction of the takeoff angle ( $\alpha_w$ ) is a function of the wall-soil interface friction angle ( $\delta$ ), the angle of internal friction ( $\phi$ ), the cohesion of the soil ( $C$ ), and the wall height ( $H$ ). Once the geometry of the failure plane is established then the failure mass can be divided into slices as shown in Figure 4-33. Earth pressure  $P_{ph}$  is then calculated by summation of forces in the vertical and horizontal direction for all slices using Eq. 4-56.

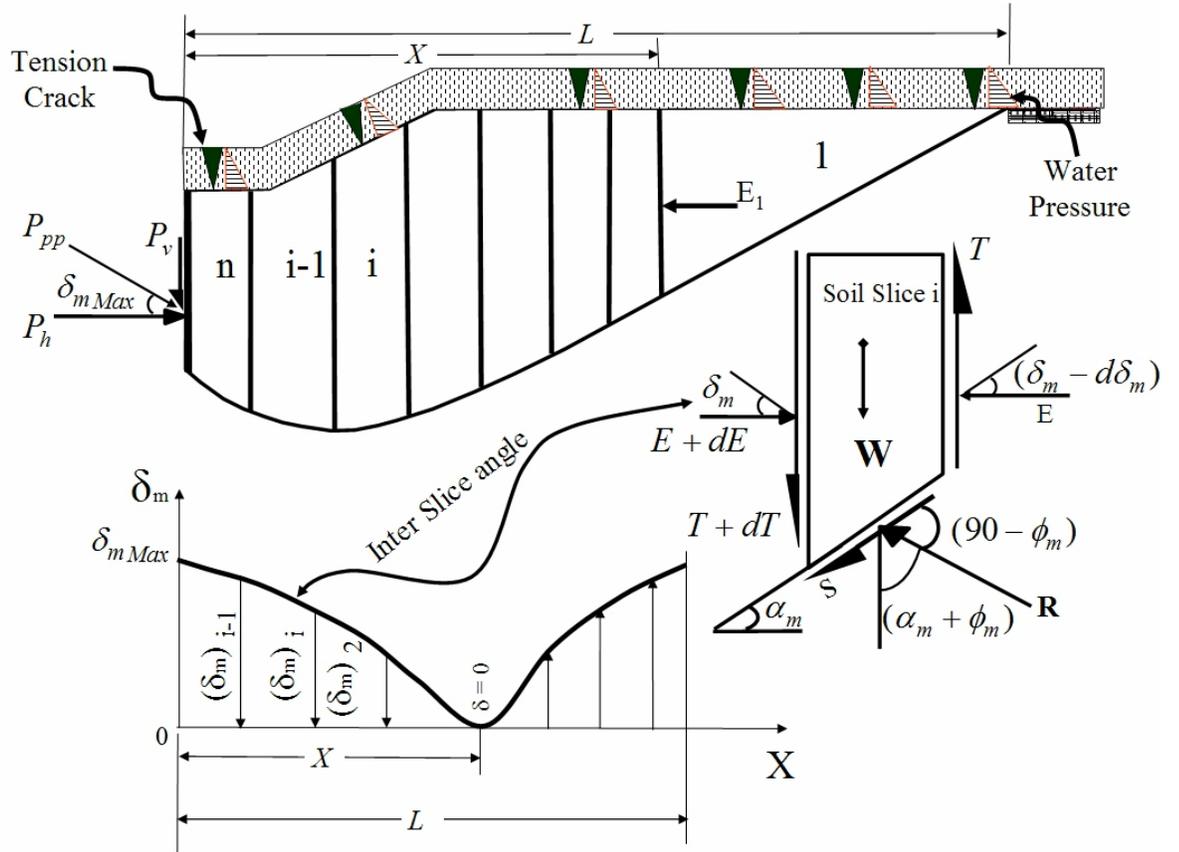


Figure 4-33. Geometry of the failure surface and associated interslice forces.

$$P_h = \frac{\sum_{i=1}^n dE}{1 - \tan \delta \tan(\alpha_w + \phi)}$$

Eq. 4-56

Where:

$$dE = \frac{W \tan(\alpha + \phi) + (C)(L) [\sin \alpha \tan(\alpha + \phi) + \cos \alpha]}{1 - \tan \delta \tan(\alpha + \phi)}$$

Eq. 4-57

By dividing the resisting wall force  $P_h$  by  $0.5\gamma H^2$ , one obtains the horizontal passive pressure coefficient ( $K_{ph}$ ) which is expressed as:

$$K_{ph} = \frac{2 P_{ph}}{\gamma H^2}$$

Eq. 4-58

**4.7.1.2 Moment Equilibrium Procedures**

The passive earth pressure  $P_p$  can be determined by summing moments (rather than forces as described above) about the center of the log spiral point  $O$  considering those forces acting on the free body associated with the weight and cohesion respectively. This is a two-step process, which is solved by method of superposition. Considering the weight of the free body diagram shown in Figure 4-34,  $P_p$  can be determined as follows:

$$E_w = \frac{(W_{ABDF})(L_2) + (P_R)(L_3)}{L_1}$$

Eq. 4-59

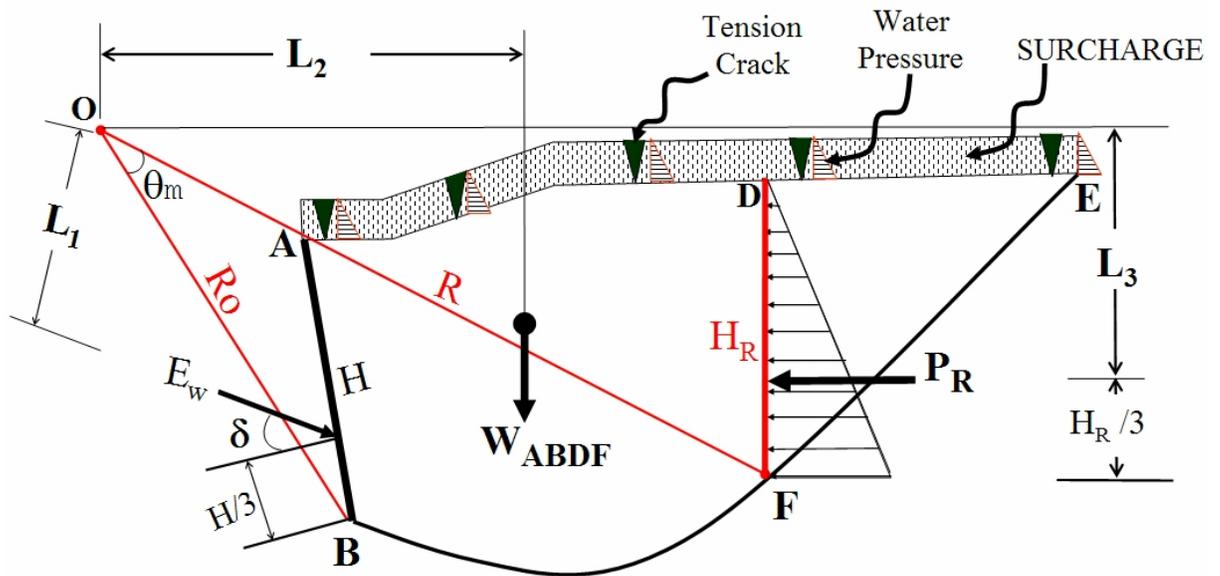


Figure 4-34. Geometry of the failure surface due to weight.

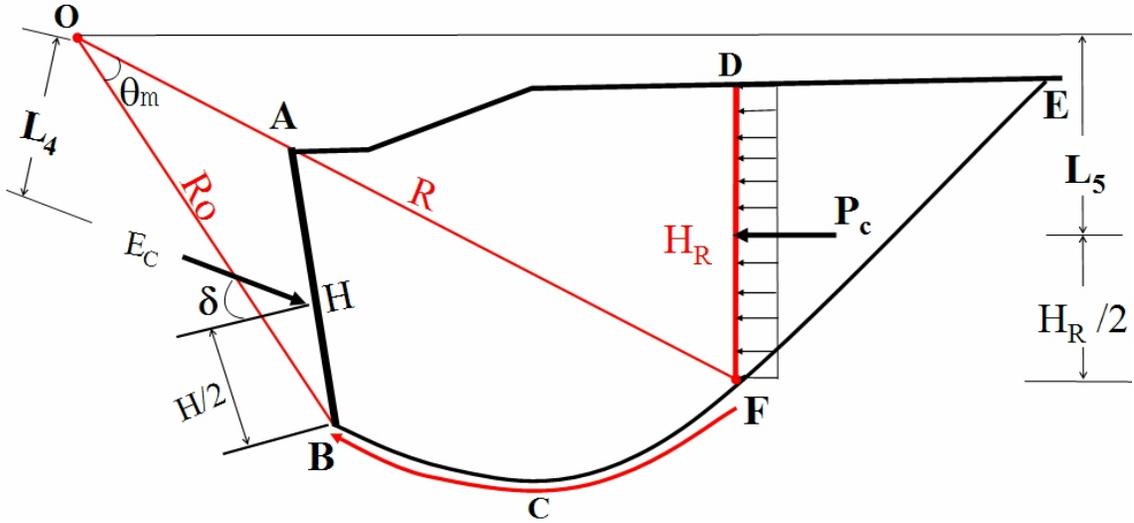


Figure 4-35. Geometry of the failure surface due to cohesion.

Considering the cohesion part of the backfill only as shown in Figure 4-34 the passive earth pressure due to cohesion ( $E_C$ ) can be determined by the summation of moments about the center of log spiral point O as follows:

$$E_C = \frac{M_C + (P_C)(L_5)}{L_4} \quad \text{Eq. 4-60}$$

Where:

$$M_C = \frac{C + P_C}{\tan \phi} (R^2 - R_0^2) \quad \text{Eq. 4-61}$$

For the cohesive soil where the soil friction is equal to zero:

$$M_C = (C)(\theta)(R^2) \quad \text{Eq. 4-62}$$

$W_{ABDF}$  = Weight of log spiral section and the surcharge weight.

$H_R$  = Height of left side of Rankine section.

$P_R$  = Horizontal force component of Rankine Section DFE.

$P_C$  = Horizontal force component of Rankine Section DFE due to Cohesion.

$E_w$  = Total lateral earth pressure due to weight.

$E_C$  = Total lateral earth pressure due to Cohesion.

$M_c$  = Moment due to Cohesion due to log spiral section.

Eq. 4-60 and Eq. 4-61 are obtained using the following procedures:

1. Calculate earth pressure on vertical face of DEF using Rankine's equation.

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2. Calculate weight of the zone ABDF including the surcharge.
3. Take the moment about point O.

The total lateral earth pressure due to weight and cohesion is the summation of Eq. 4-59 and Eq. 4-60.

$$P_p = E_w + E_c \quad \text{Eq. 4-63}$$

The passive earth pressure force  $P_p$  is obtained by summing  $E_w$  and  $E_c$ . However, this may not be the unique solution to the problem as only one trial surface is examined. The value of passive pressure  $P_p$  must be determined for several trial surfaces as shown in Figure 4-36 until the minimum value of  $P_p$  is attained. Note that failure surface 2 is the critical failure surface.

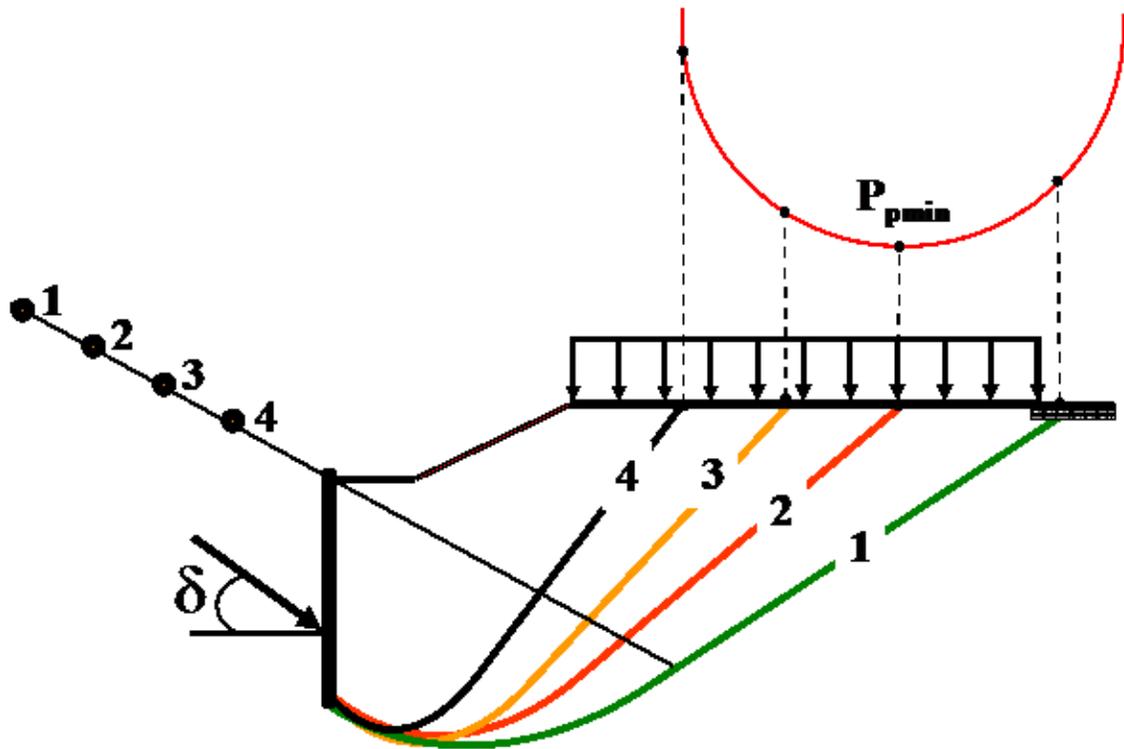


Figure 4-36. Moment Method

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For non-cohesive soils, values for the passive lateral earth pressure may be taken from Figure 4-37 using the following procedure:

- Given  $\delta$ ,  $\beta$ , and  $\phi$ .
- Calculate ratios  $\delta/\phi$  and  $\beta/\phi$ .
- Determine initial  $K_p$  for  $\beta/\phi$  from Figure 4-37.
- Determine reduction factor  $R$  using the ratio of  $\delta/\phi$ .
- Calculate final  $K_p = R * K_p$ .

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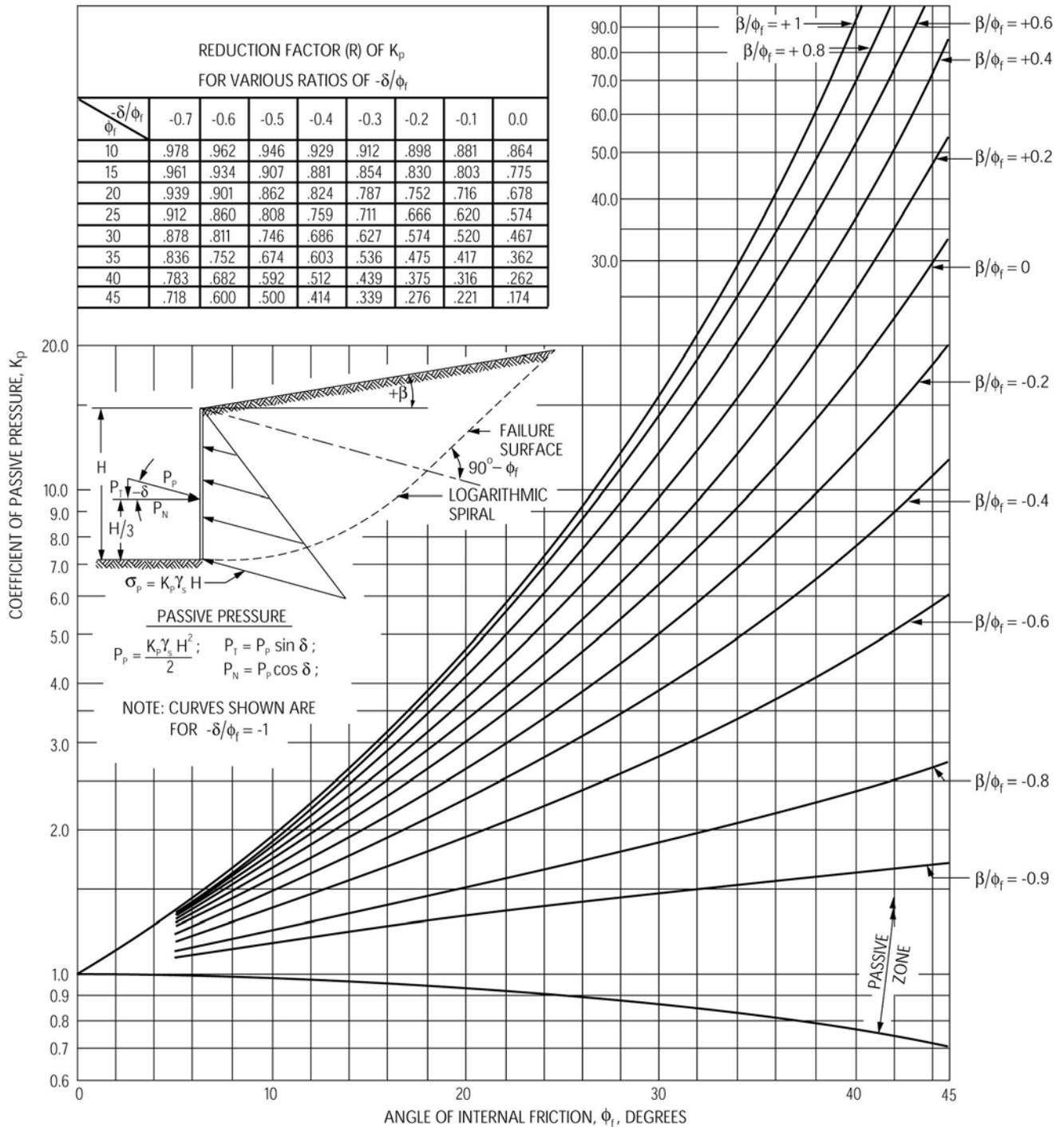


Figure 4-37. Passive earth pressure coefficient (Caquot and Kerisel, 1948)

For conditions that deviate from those described in Figure 4-37, the passive pressure may be calculated by using a trial procedure based on the trial wedge theory or a logarithmic spiral method.

### 4.7.2 Noncomposite Log Spiral Failure Surface

As shown in Figure 4-38, it is assumed that a single arc of the log spiral curve can represent the entire failure surface. Note, do not confuse this discussion with a "Global Stability Check" which is covered in CHAPTER 9 Section 9.4 SLOPE STABILITY. As described previously, the equations of the force equilibrium and or moment equilibrium methods are applied to calculate the passive or active force directly without breaking the failure surface into an arc of logarithmic spiral zone and a Rankine zone.

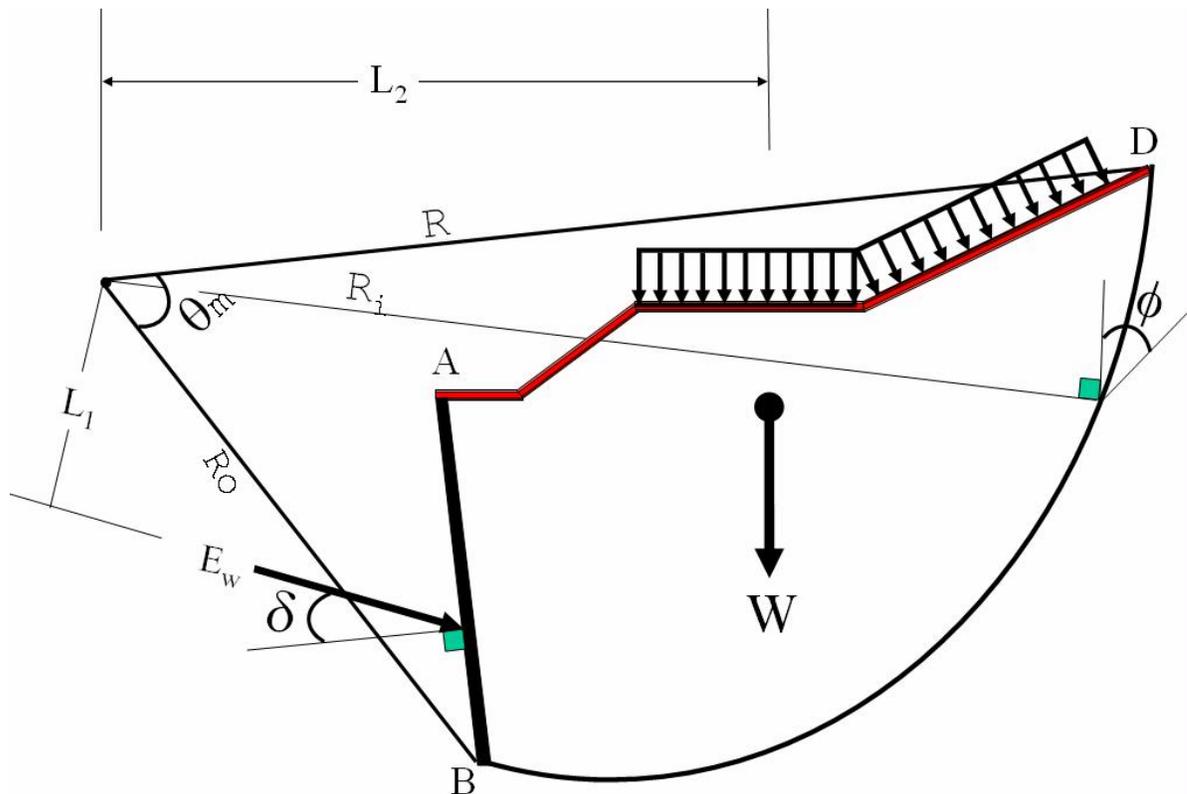


Figure 4-38. Mobilized full log spiral failure surface

### 4.7.2.3 Force Equilibrium Method

The formulation regarding the limit equilibrium method of slices (Shamsabadi, *et al.*, 2007, 2005) does not change since Rankine's zone was treated as a single slice. The entire mass above the failure surface BD is divided into vertical slices and Eq. 4-56 through Eq. 4-58 are used to calculate the magnitude of the earth pressure.

**4.7.2.4 Moment Equilibrium Method**

Only slight modifications are done to the moment limit equilibrium equations by removing the Rankine components of Eq. 4-59 and Eq. 4-60. From Figure 4-38:

$$E_w = \frac{(W_{ABD})(L_2)}{L_1} \tag{Eq. 4-64}$$

And for the cohesion component from Figure 4-38:

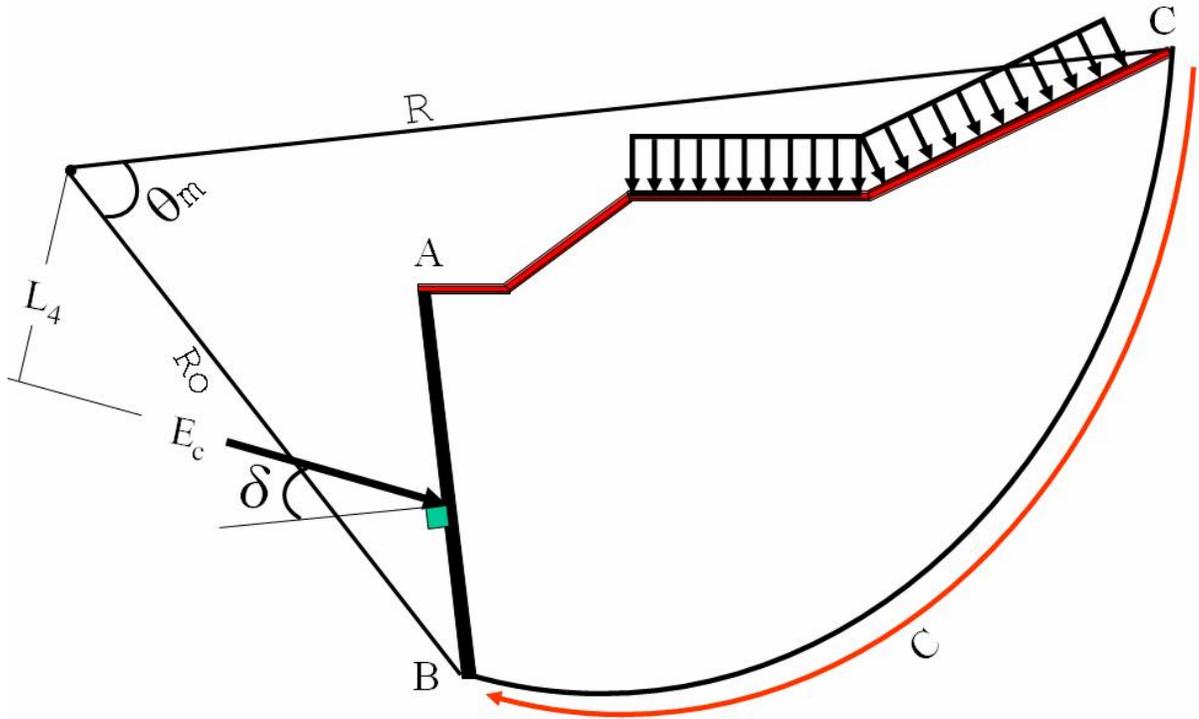


Figure 4-39. Mobilized full log spiral failure surface for cohesion component

$$E_c = \frac{M_c}{L_4} \tag{Eq. 4-65}$$

The passive earth pressure coefficients for various methods are listed in the following tables for zero slope backfill ( $\beta = 0^\circ$ ). For sloping backfill, the value of  $K_{ph}$  should be determined by the using Figure 4-37.

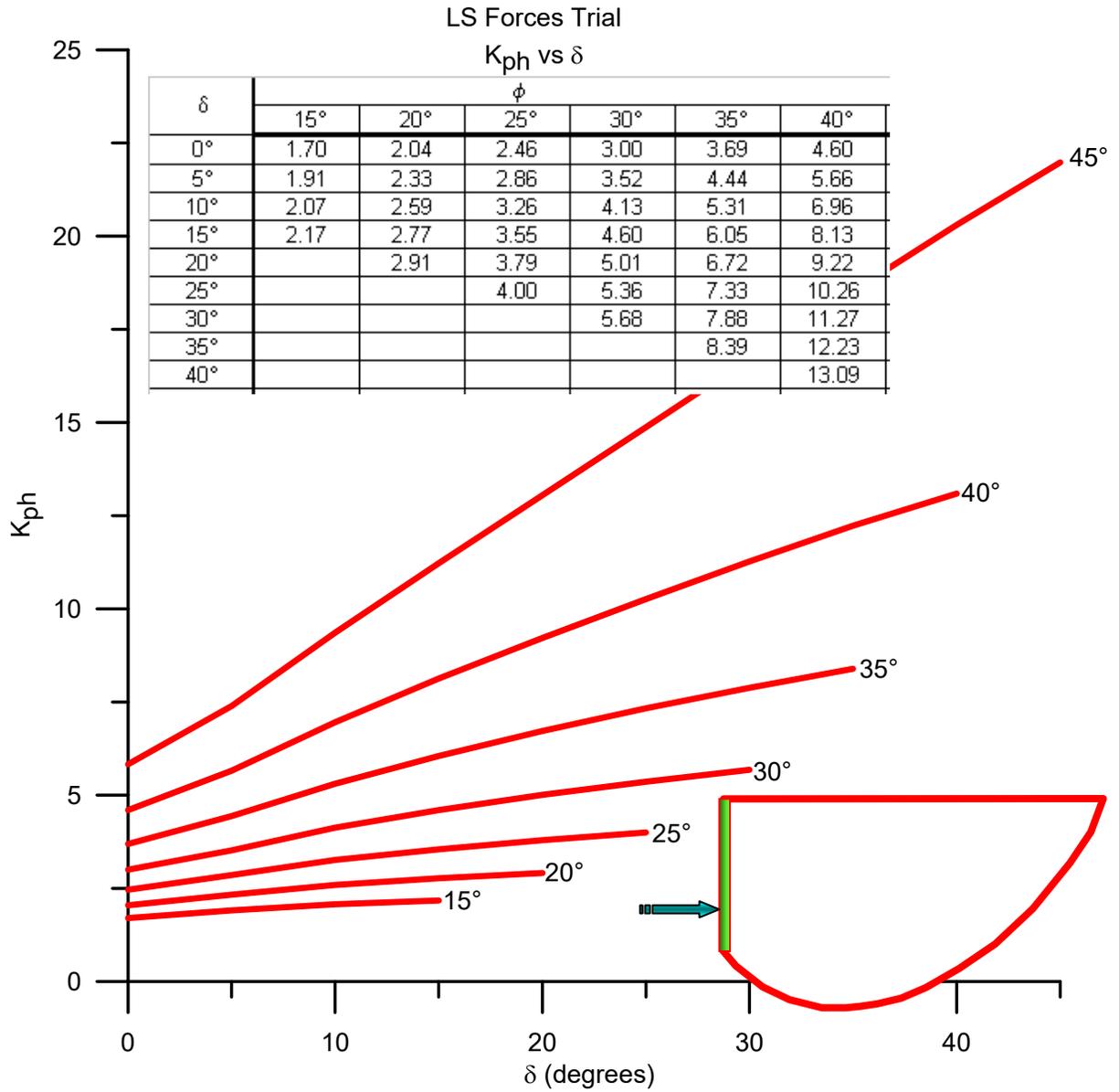


Figure 4-40. Log Spiral – Forces Method – Full Log Spiral – Trial

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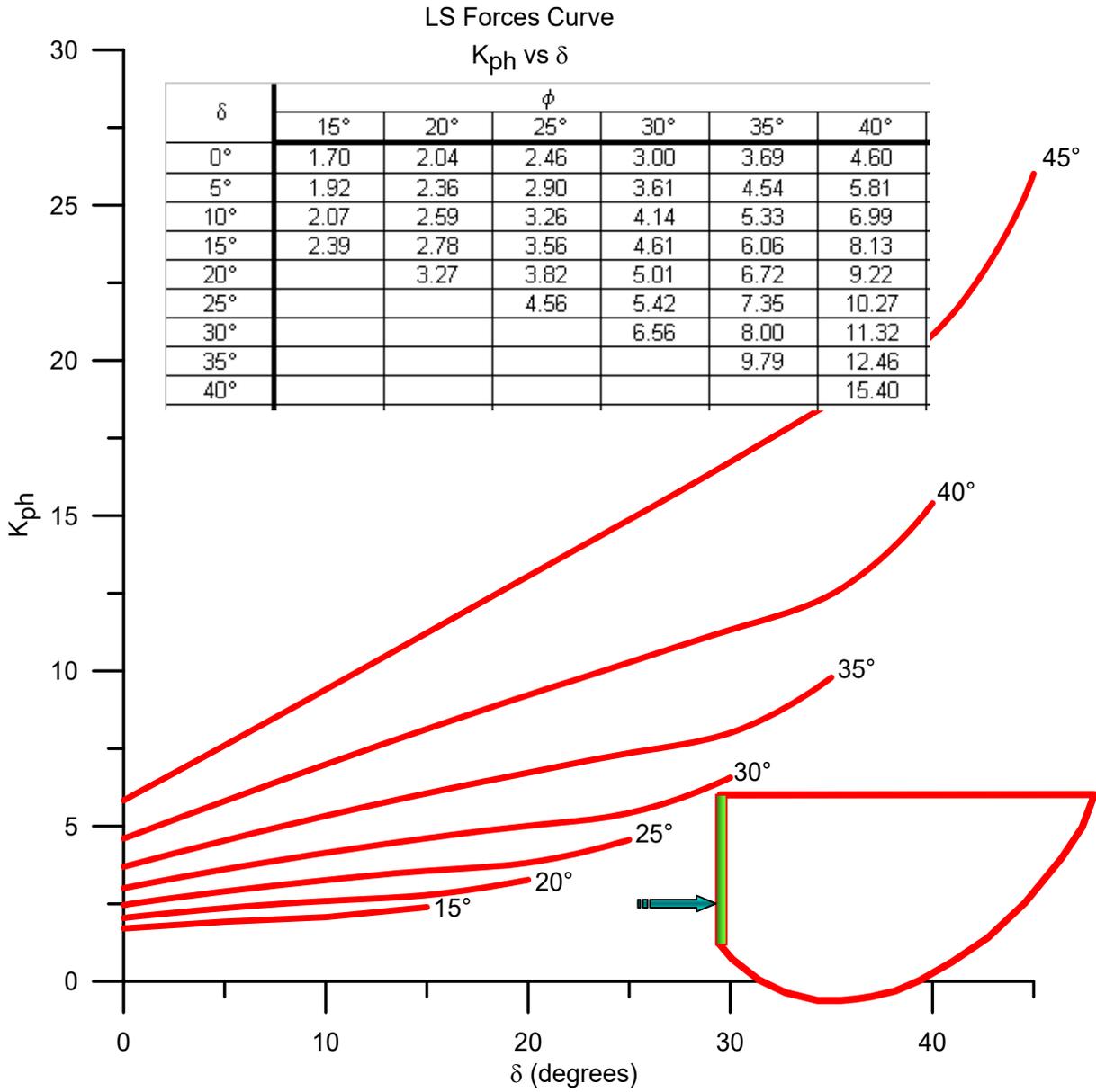


Figure 4-41. Log Spiral – Forces Method – Full Log Spiral – No Trial

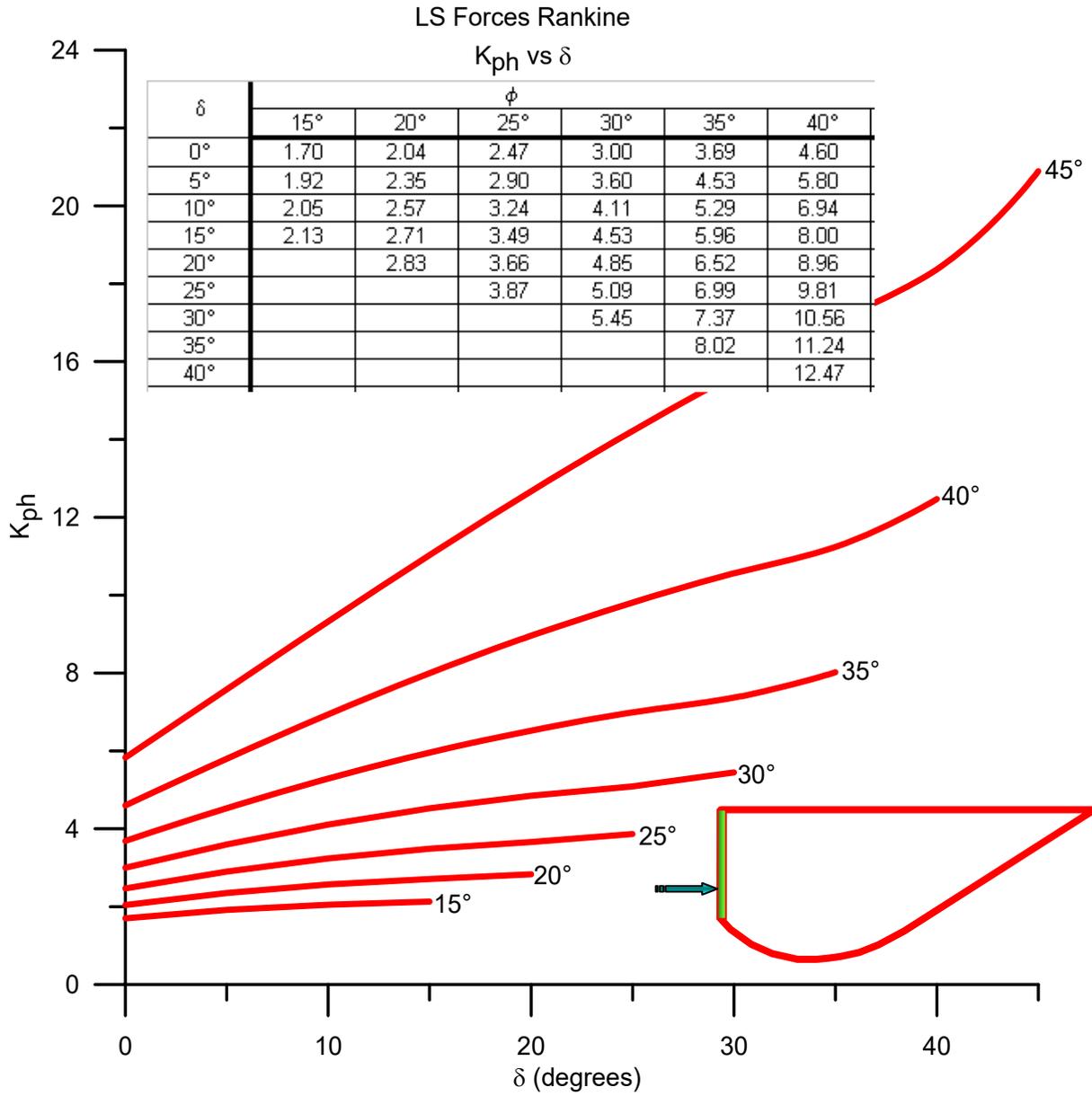


Figure 4-42. Log Spiral – Forces Method – Composite Failure Surface

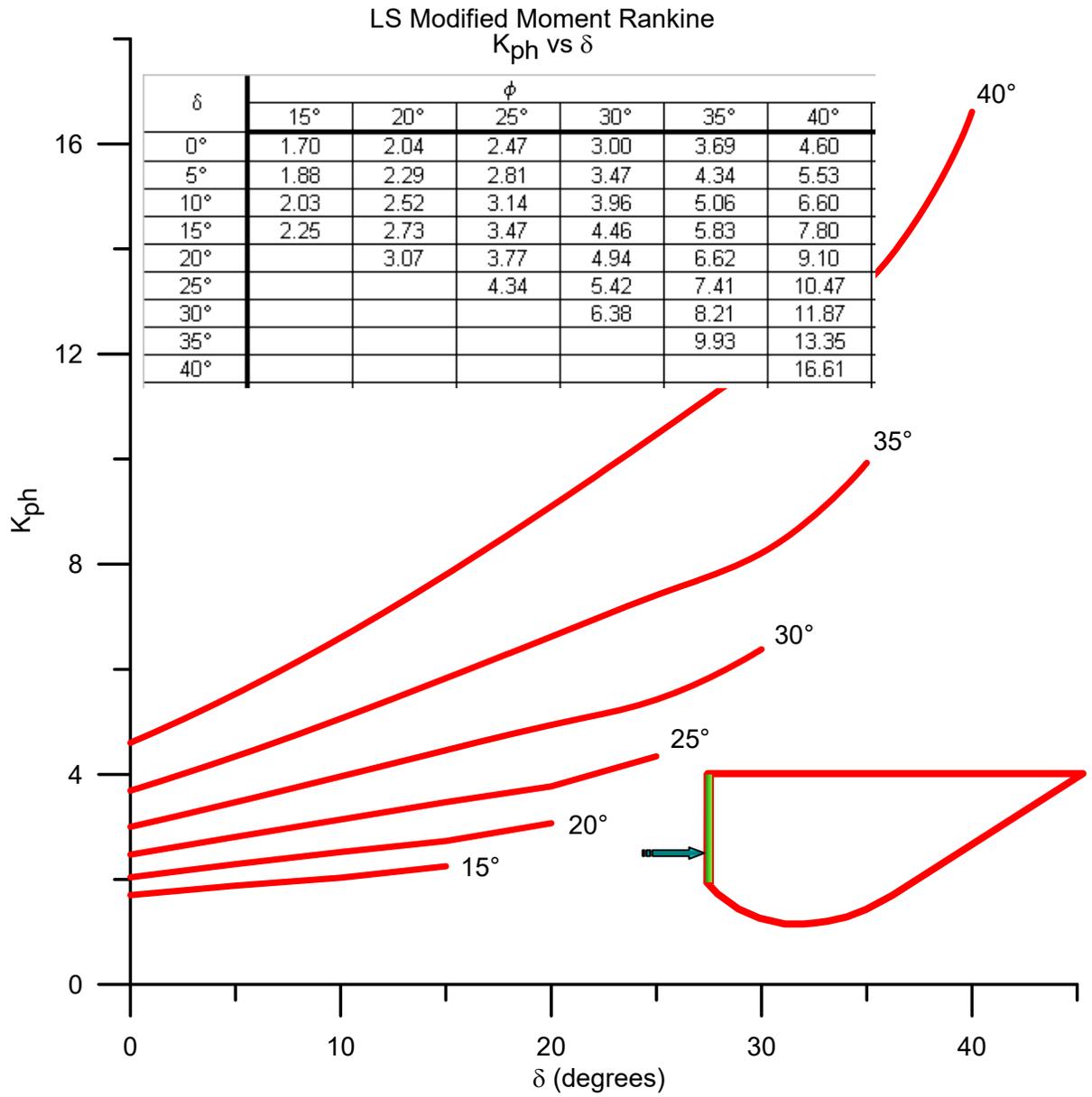


Figure 4-43. Log Spiral – Modified Moment Method – Composite Failure Surface

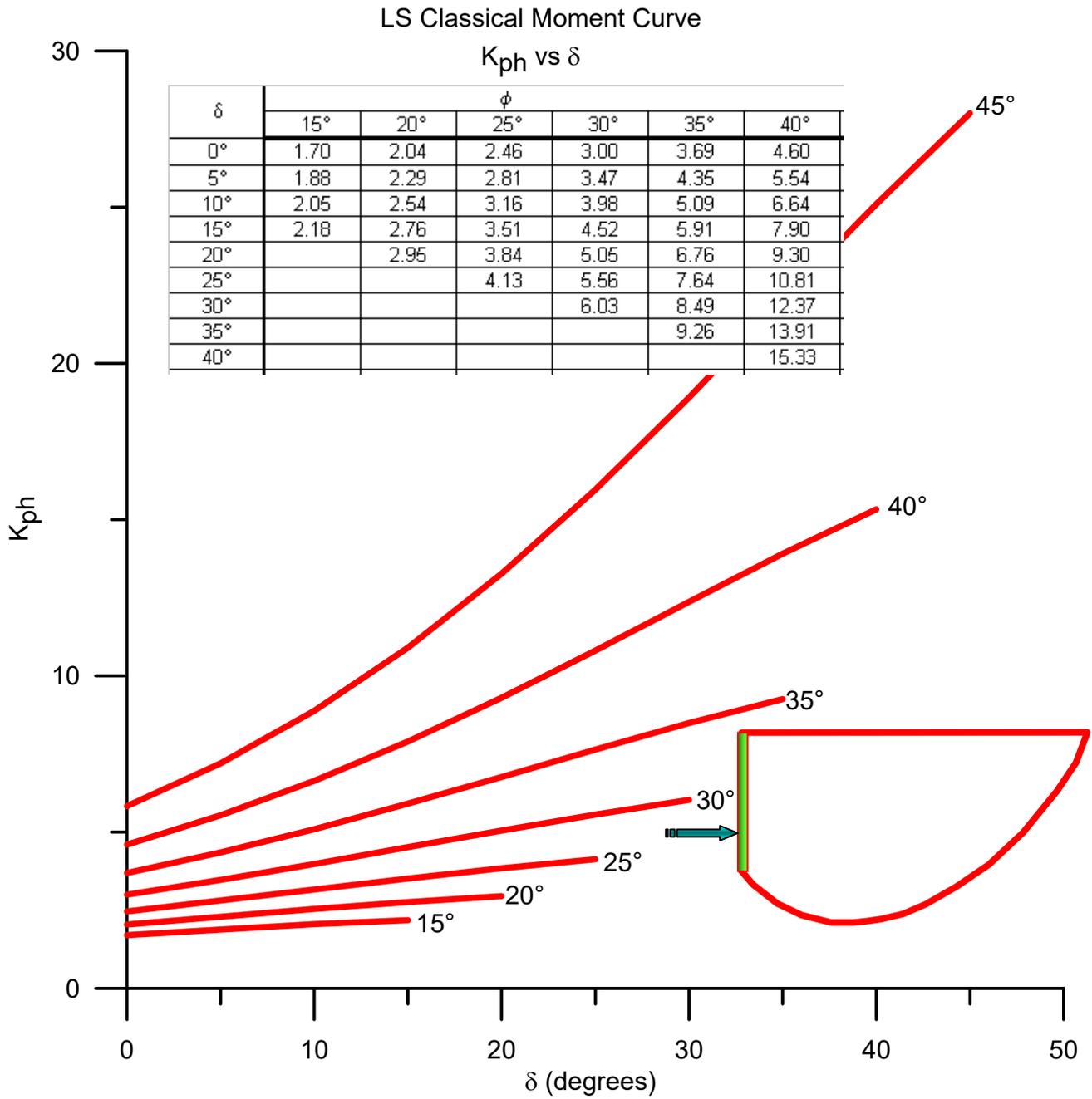


Figure 4-44. Log Spiral – Moment Method – Full Log Spiral Failure Surface

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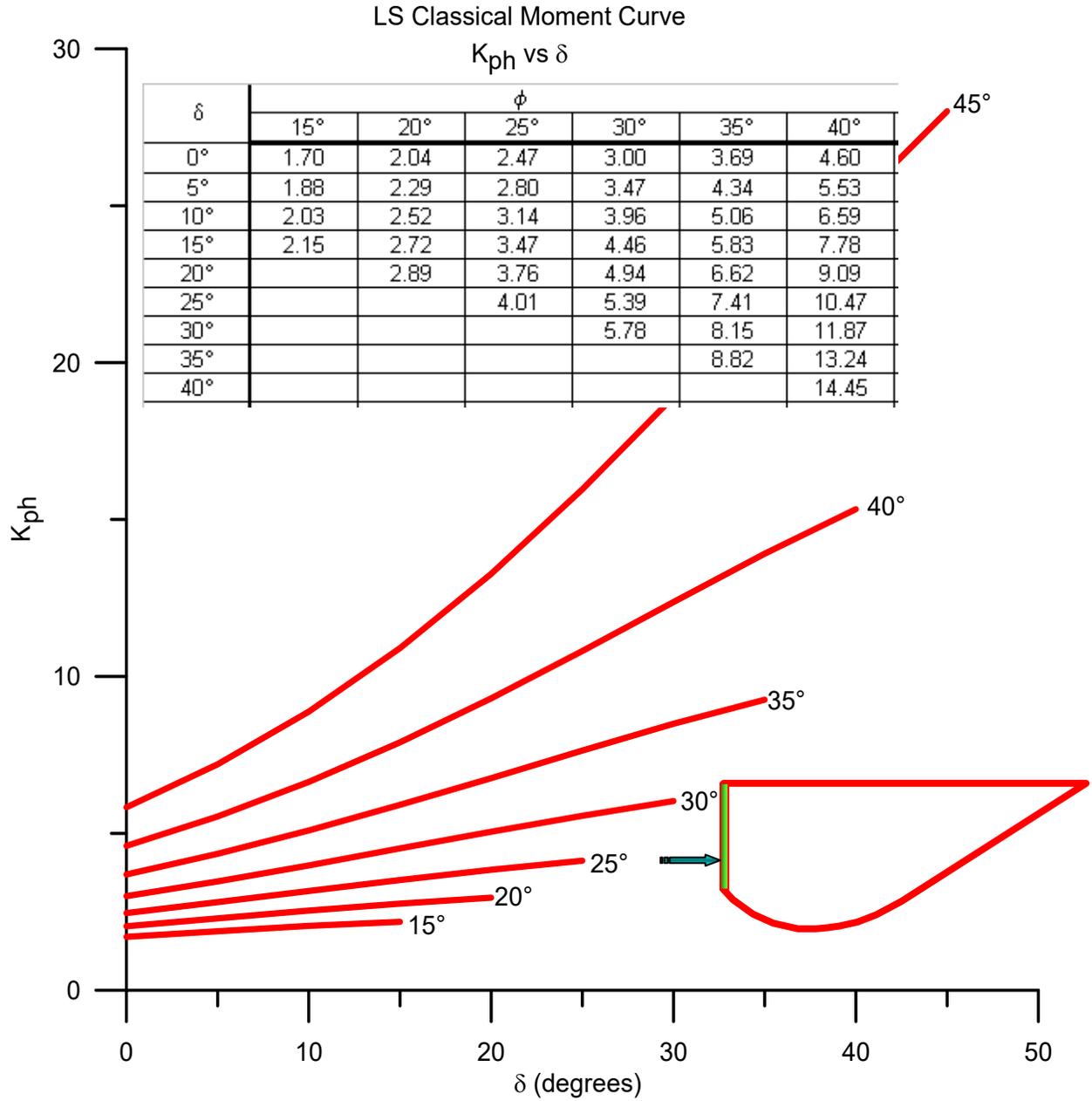


Figure 4-45. Log Spiral – Moment Method – Composite Failure Surface

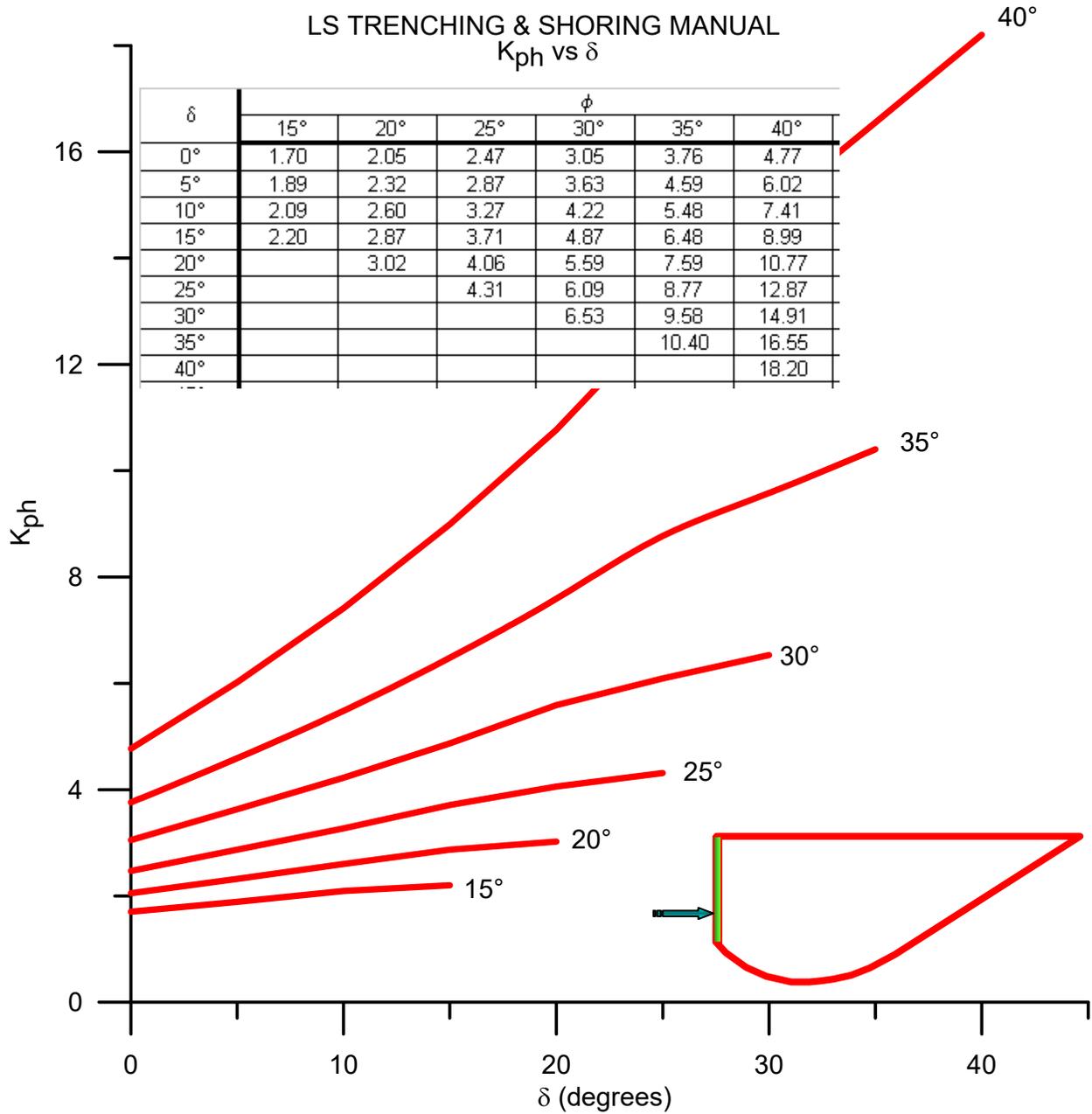


Figure 4-46. Log Spiral – see Figure 4-37

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Table 4-6 shows the values of  $K_{ph}$  computed by the various log spiral and straight line methods described above and shown in Figure 4-31 on page 4-50. When the wall interface friction ( $\delta$ ) less than about 1/3 of the backfill soil friction angle ( $\phi$ ) the value of  $K_{ph}$  does not differ significantly. However, for large values of wall interface friction angle ( $\delta$ ), the values of  $K_{ph}$  should be determined by the using the log spiral methods. Note that the values listed in the following table are for the purposes of comparison of the various methods with zero slope backfill ( $\beta = 0^\circ$ ).

Table 4-6.  $K_{ph}$  based on straight or curved rupture lines with zero slope backfill ( $\beta = 0^\circ$ )

$\phi$	$\delta/\phi$	$K_{ph}$						
		Straight Line		Log Spiral				
		Rankine	Coulomb/ Trial Wedge	Forces			Moment	
				Trial	Curve	Composite	Curve	Composite
30°	0	3.00	3.00	3.00	3.00	3.00	3.00	3.00
	1/3	3.00	4.14	4.13	4.14	4.11	3.98	3.96
	2/3	3.00	6.11	5.01	5.01	4.85	5.05	4.94
	1	3.00	10.10	5.68	6.56	5.45	6.03	5.78
35°	0	3.69	3.69	3.69	3.69	3.69	3.69	3.69
	1/3	3.69	5.68	5.57	5.58	5.53	5.36	5.31
	2/3	3.69	9.96	7.13	7.14	6.84	7.35	7.15
	1	3.69	22.97	8.39	9.79	8.02	9.26	8.82
40°	0	4.60	4.60	4.60	4.60	4.60	4.60	4.60
	1/3	4.60	8.15	7.75	7.76	7.66	7.46	7.37
	2/3	4.60	18.72	10.60	10.62	10.07	11.33	10.94
	1	4.60	92.59	13.09	15.40	12.47	15.33	14.45

## 4.8 SURCHARGE LOADS

A surcharge load is any load which is imposed upon the surface of the soil close enough to the excavation to cause a lateral pressure to act on the system in addition to the basic earth pressure. Groundwater will also cause an additional pressure, but it is not a surcharge load.

Examples of surcharge loads are spoil embankments adjacent to the trench, streets or highways, construction machinery or material stockpiles, adjacent buildings or structures, and railroads.

### 4.8.1 Minimum Construction Surcharge Load

The minimum lateral construction surcharge of 72 psf ( $\sigma_h$ ) shall be applied to the shoring system to a depth of 10 feet ( $H_s$ ) below the shoring system or to the excavation line whichever is less. See Figure 4-47. This is the minimum surcharge loading that shall be applied to any shoring system regardless of whether or not the system is actually subjected to a surcharge loading. Surcharge loads which produce lateral pressures greater than 72 psf would be used in lieu of this prescribed minimum.

This surcharge is intended to provide for the normal construction loads imposed by small vehicles, equipment, or materials, and workmen on the area adjacent to the trench or excavation. It should be added to all basic earth pressure diagrams. This minimum surcharge can be compared to a soil having parameters of  $\gamma = 109$  pcf and  $K_a = 0.33$  for a depth of 2 feet [ $(0.33)(109)(2) = 72$  psf]

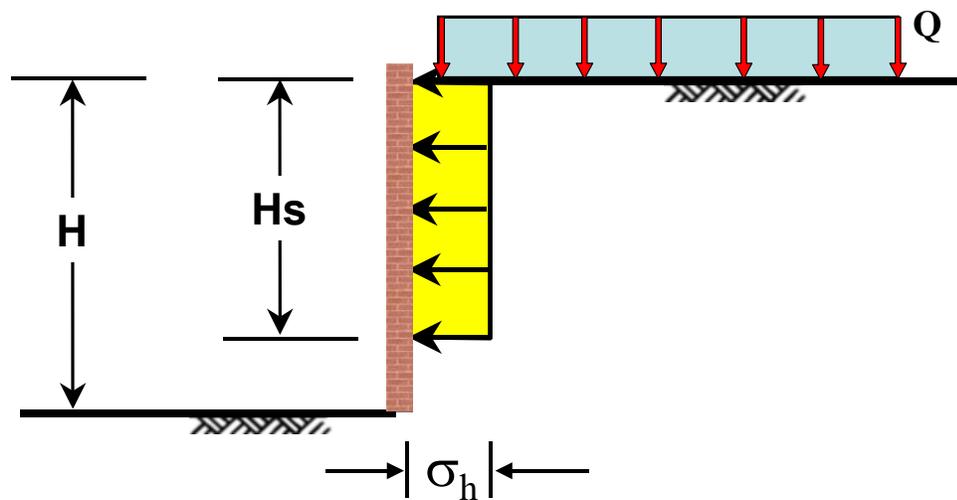


Figure 4-47. Minimum Lateral Surcharge Load

### 4.8.2 Uniform Surcharge Loads

Where a uniform surcharge is present, a constant horizontal earth pressure must be added to the basic lateral earth pressure. This constant earth pressure may be taken as:

$$\sigma_h = (K)(Q) \quad \text{Eq. 4-66}$$

Where:

$\sigma_h$  = constant horizontal earth pressure due to uniform surcharge

K = coefficient of lateral earth pressure due to surcharge for the following conditions:

- Use  $K_a$  for active earth pressure.
- Use  $K_o$  for at-rest earth pressure.

Q = uniform surcharge applied to the wall backfill surface within the limits of the active failure wedge.

### 4.8.3 Boussinesq Loads

Typically, there are three (3) types of Boussinesq Loads. They are as follows:

#### 4.8.3.1 Strip Load

Strip loads are loads such as highways and railroads that are generally parallel to the wall.

The general equation for determining the pressure at distance  $h$  below the ground line is (See Figure 4-48):

$$\sigma_h = \frac{2Q}{\pi} [\beta_R - \sin \beta \cos(2\alpha)] \quad \text{Eq. 4-67}$$

Where  $\beta_R$  is in radians.

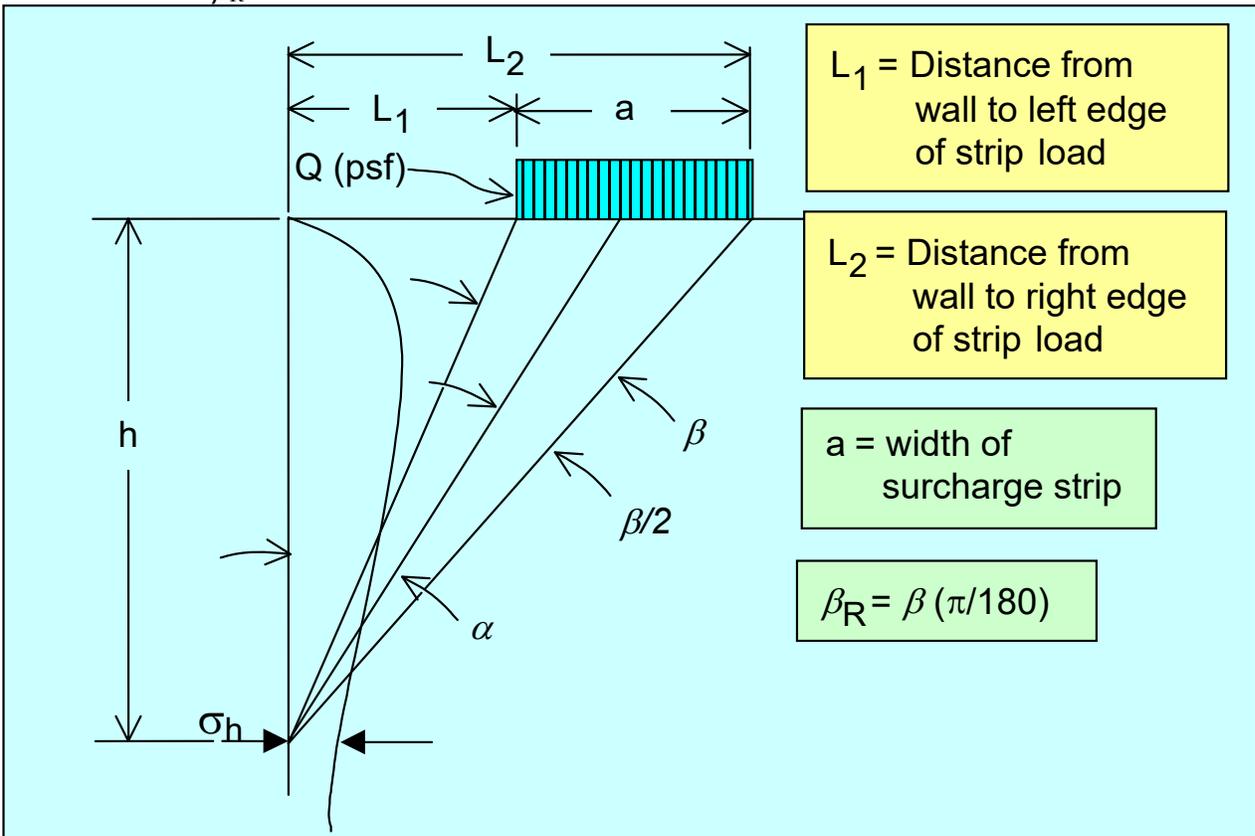


Figure 4-48. Boussinesq Type Strip Load

**4.8.3.2 Line Load**

A line load is a load such as a continuous wall footing of narrow width or similar load generally parallel to the wall. K-Railing could be considered to be a line load.

The general equation for determining the pressure at distance h below the ground line is:  
(See Figure 4-49)

For  $m \leq 0.4$ :

$$\sigma_h = \frac{Q_l}{H} \frac{0.2n}{(0.16 + n^2)^2} \quad \text{Eq. 4-68}$$

For  $m > 0.4$

$$\sigma_h = 1.28 \frac{Q_l}{H} \frac{m^2 n}{(m^2 + n^2)^2} \quad \text{Eq. 4-69}$$

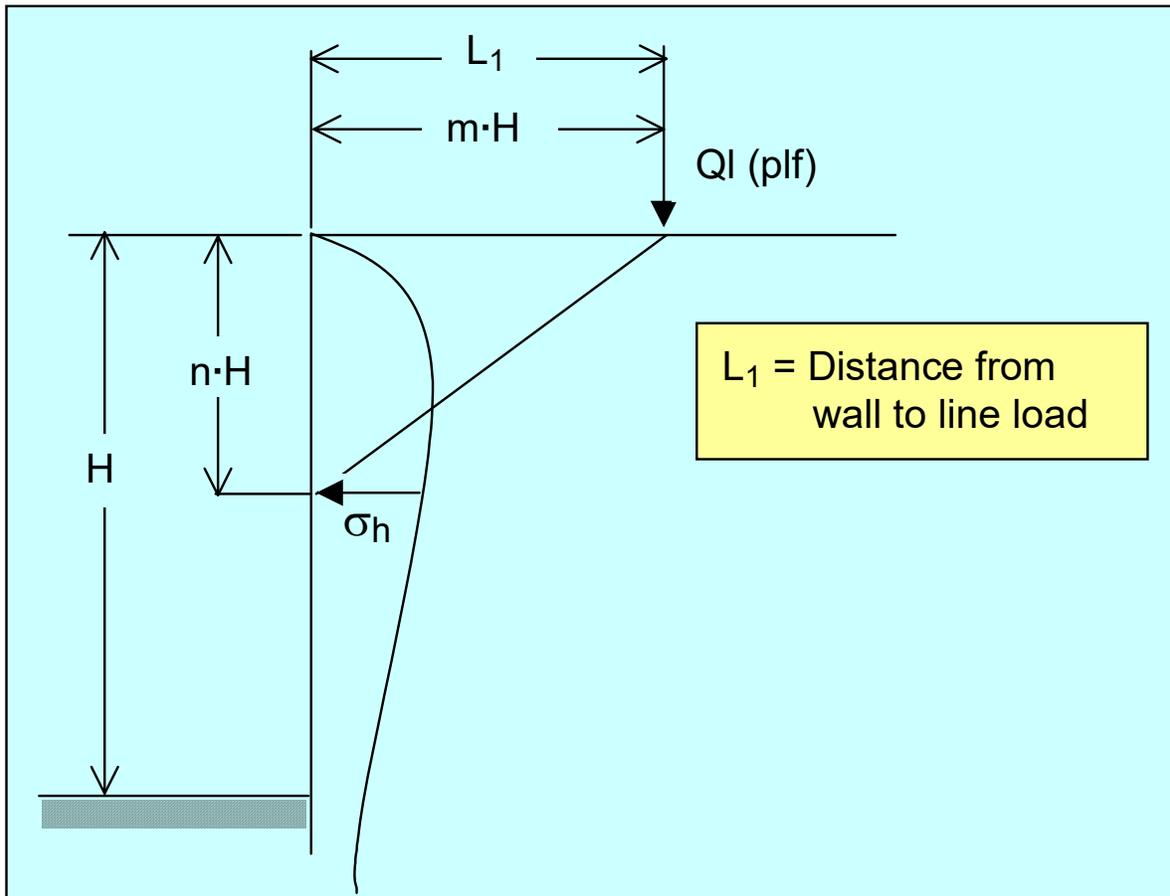


Figure 4-49. Boussinesq Type Line Load

### 4.8.3.3 Point Load

Point loads are loads such as outrigger loads from a concrete pump or crane. A wheel load from a concrete truck may also be considered a point load when the concrete truck is adjacent an excavation and in the process of the unloading. The truck could be positioned either parallel or perpendicular to the excavation.

The general equation for determining the pressure at distance h below the ground line is:  
(See Figure 4-50)

For  $m \leq 0.4$ :

$$\sigma_h = 0.28 \frac{Q_p}{H^2} \frac{n^2}{(0.16 + n^2)^3} \quad \text{Eq. 4-70}$$

For  $m > 0.4$

$$\sigma_h = 1.77 \frac{Q_p}{H^2} \frac{m^2 n^2}{(m^2 + n^2)^3} \quad \text{Eq. 4-71}$$

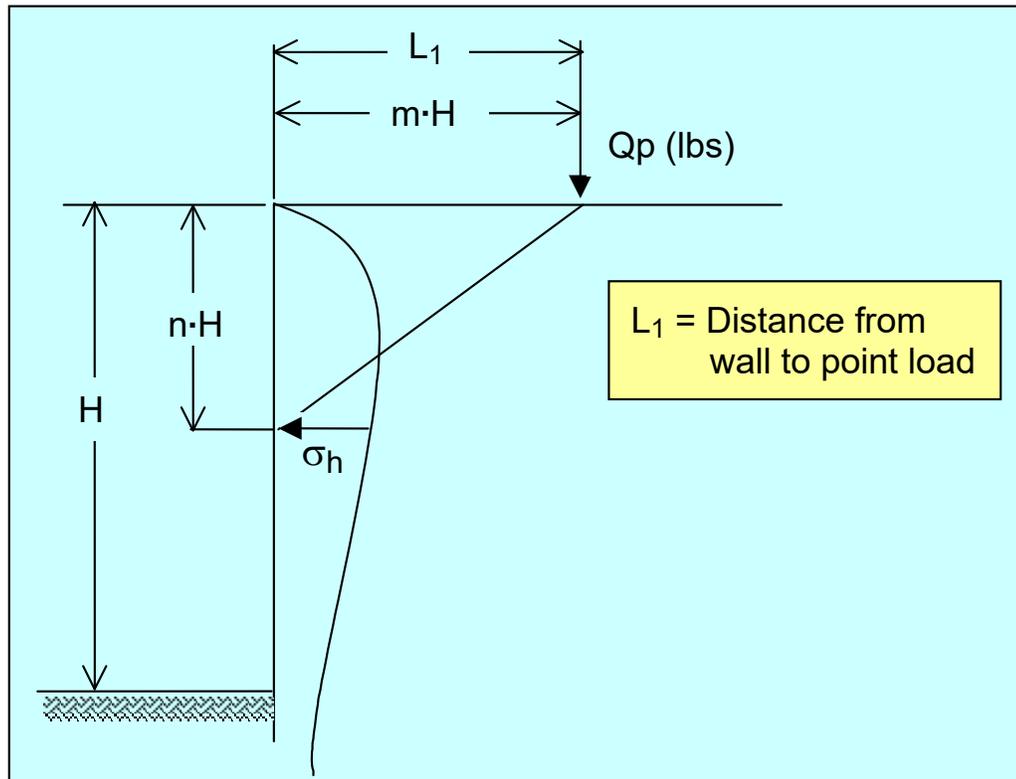


Figure 4-50. Boussinesq Type Point Load

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In addition,  $\sigma_h$  is further adjusted by the following when the point is further away from the line closest to the point load: (see Figure 4-51)

$$\sigma'_h = \sigma_h \cos^2[(1.1)\theta] \quad \text{Eq. 4-72}$$

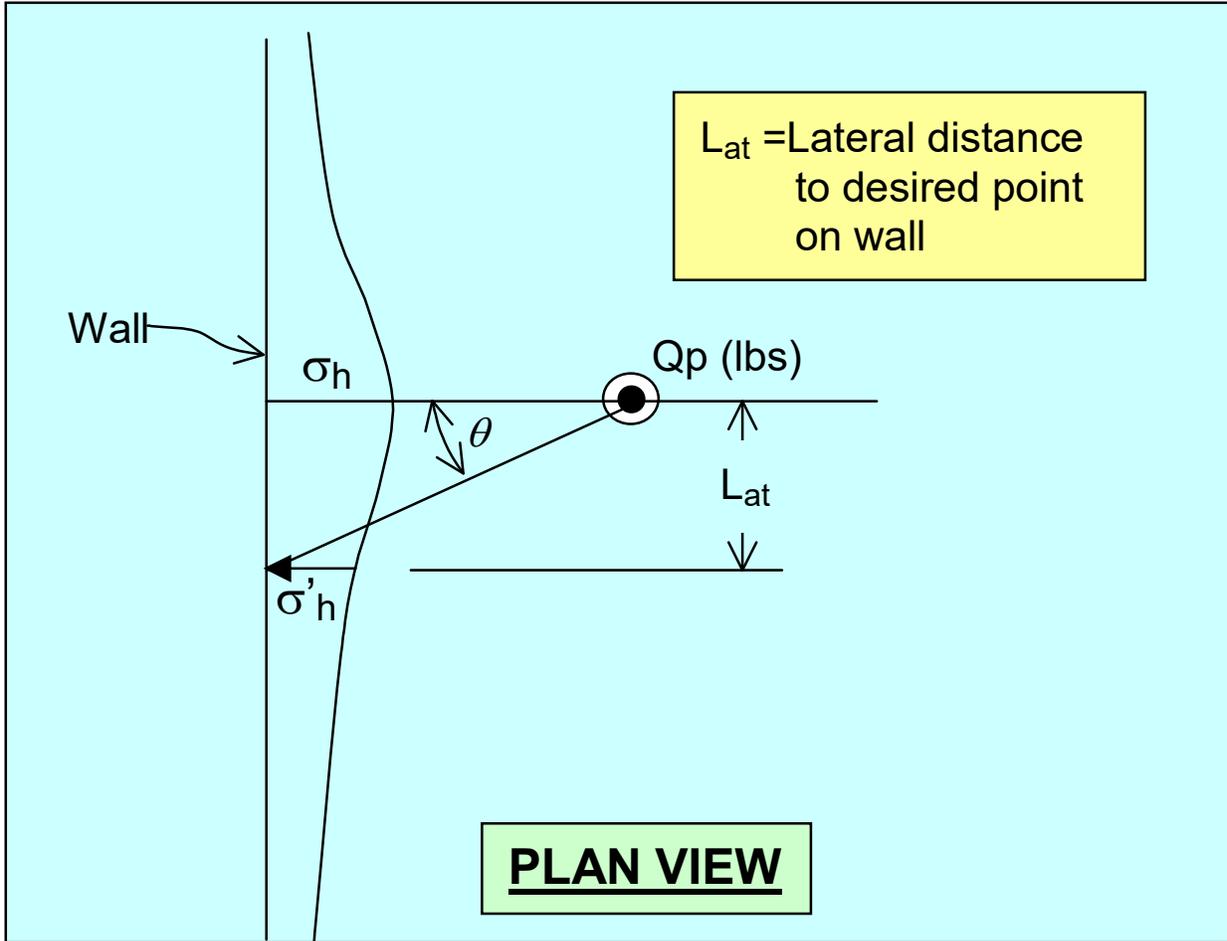


Figure 4-51. Boussinesq Type Point Load with Lateral Offset

### 4.8.4 Traffic Loads

Traffic near an excavation is one of the more commonly occurring surcharge loads. Trying to analyze every possible scenario would be time consuming and not very practical. For normal situations, a surcharge load of 300 psf spread over the width of the traveled way should be sufficient.

The following example compares the pressure diagrams for a HS20 truck. (using point loads) centered in a 12' lane to a load of  $Q = 300$  psf (using the Boussinesq Strip method). The depth of excavation is 10'.

$$x_1 = m_1 H$$

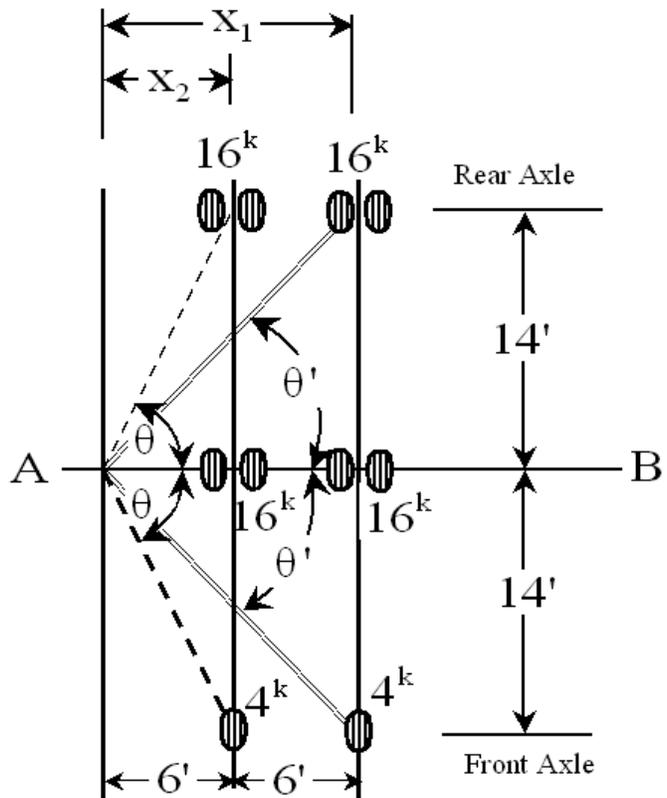
$$\therefore m_1 = \frac{12}{10} = 1.2$$

$$x_2 = m_2 H$$

$$\therefore m_2 = \frac{6}{10} = 0.6$$

$$n = \frac{\text{depth}}{H}$$

Depth	n
2'	0.2
4'	0.4
6'	0.6
8'	0.8
10'	1.0



For line AB, see Eq. 4-71. For loads at an angle to AB, see Eq. 4-72.

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Front and rear right wheels:  $\theta = 66.8^\circ, \therefore \cos^2[(1.1)(66.8^\circ)] = 0.08$

Front and rear left wheels:  $\theta = 49.4^\circ, \therefore \cos^2[(1.1)(49.4^\circ)] = 0.34$

- 1.) Right rear wheels: 
$$\sigma_h = \frac{(0.08)(1.77)(16,000)(0.6^2)(n^2)}{10^2(0.6^2 + n^2)^3}$$
- 2.) Left rear wheels: 
$$\sigma_h = \frac{(0.34)(1.77)(16,000)(1.2^2)(n^2)}{10^2(1.2^2 + n^2)^3}$$
- 3.) Right center wheels: 
$$\sigma_h = \frac{(1.77)(16,000)(0.6^2)(n^2)}{10^2(0.6^2 + n^2)^3}$$
- 4.) Left center wheels: 
$$\sigma_h = \frac{(1.77)(16,000)(1.2^2)(n^2)}{10^2(1.2^2 + n^2)^3}$$
- 5.) Right front wheels: 
$$\sigma_h = \frac{(0.08)(1.77)(4,000)(0.6^2)(n^2)}{10^2(0.6^2 + n^2)^3}$$
- 6.) Left front wheels: 
$$\sigma_h = \frac{(0.34)(1.77)(4,000)(1.2^2)(n^2)}{10^2(1.2^2 + n^2)^3}$$

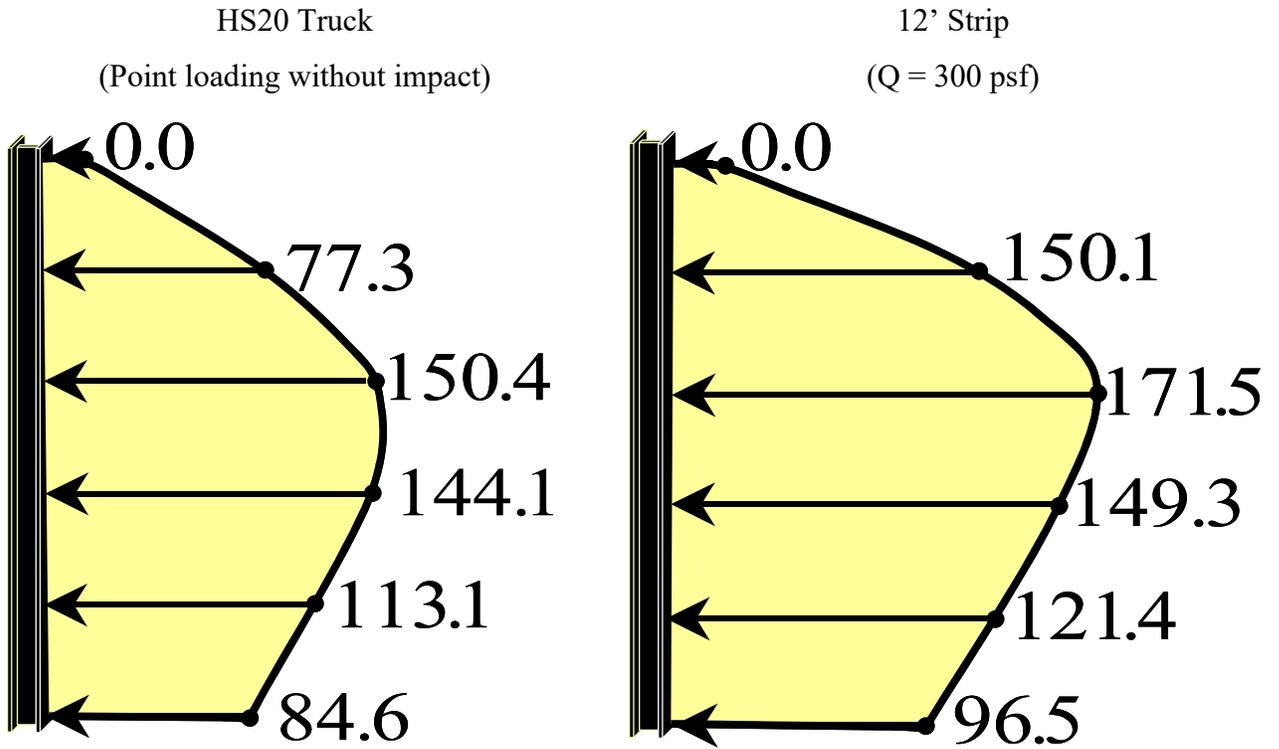
Combine and simplify similar equations:

$$\text{a.) } \sigma_H = \frac{(112.2)(n^2)}{(0.36 + n^2)^3}$$

$$\text{b.) } \sigma_H = \frac{(581.1)(n^2)}{(1.44 + n^2)^3}$$

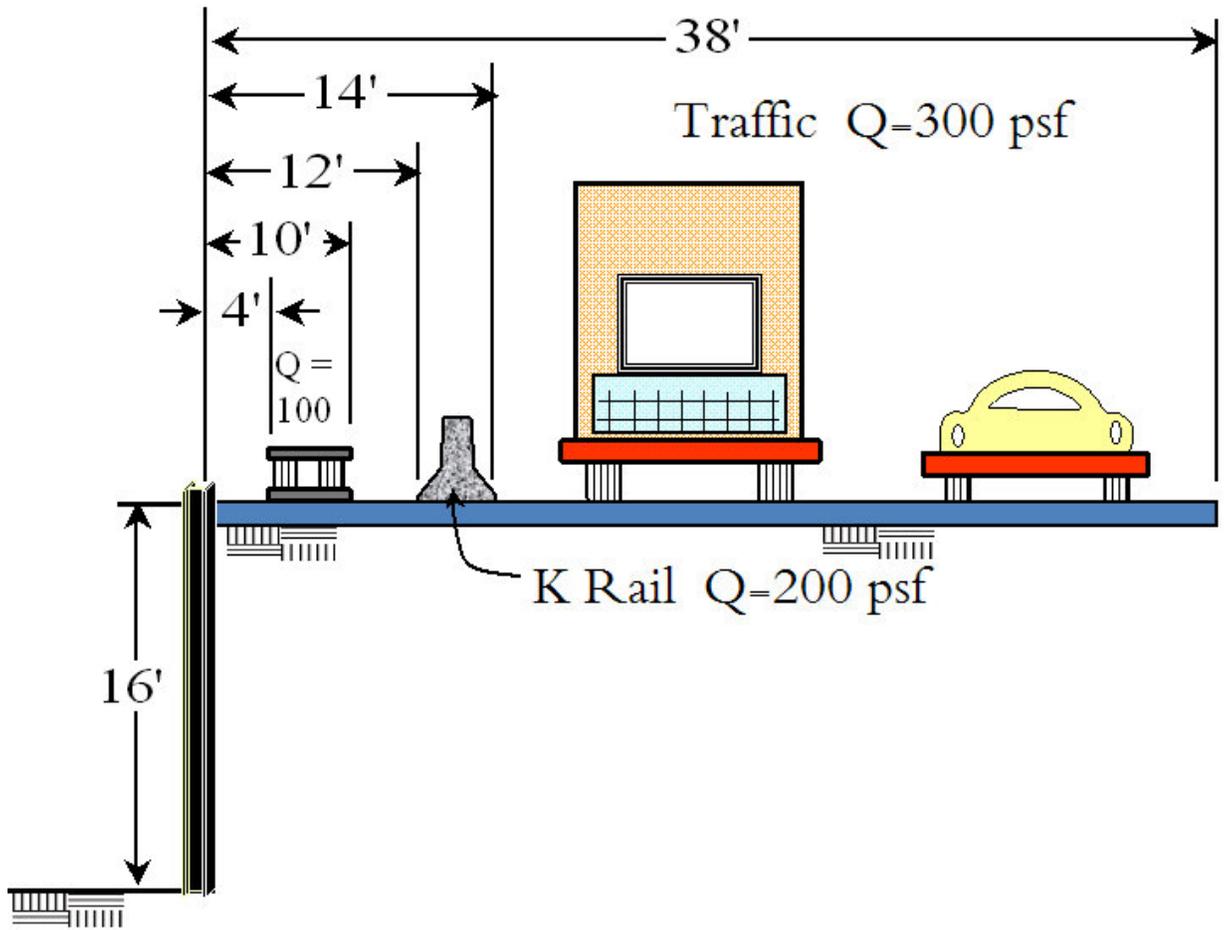
Depth	n	a.) $\sigma_H$	b.) $\sigma_H$	$\sum \sigma_H$
0'	0.0	0.0	0.0	0.0
2'	0.2	70.1	7.2	77.3
4'	0.4	127.7	22.7	150.4
6'	0.6	108.2	35.9	144.1
8'	0.8	71.8	41.3	113.1
10'	1.0	44.6	40.0	84.6

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CONCLUSION: Strip load of  $Q = 300$  psf compares favorably to a point load evaluation for HS20 truck loadings.

4.8.4.1 Example 4-7 Sample Problem – Surcharge Loads



Example 4-7. Surcharge Loads

Surcharge Lateral Pressures (psf)

Depth	Q = 100	Q = 200	Q = 300	Sum
0.1	1.9	0.3	1.7	72*
1	17.9	3.0	17.1	72*
2	30.2	5.8	33.8	72*
4	35.7	10.1	63.7	109.5
6	29.5	12.3	87.1	128.9
8	21.9	12.7	103.3	137.9
10	15.9	11.9	112.6	140.4
12	11.5	10.5	116.4	138.4
14	8.5	9.0	116.1	133.6
16	6.3	7.6	112.9	126.8

\* Minimum construction surcharge load.

Add soil pressures of surcharge loads to derive combined pressure diagram.

#### 4.8.5 Alternate Surcharge Loading (Traffic)

An acceptable alternative to the Boussinesq analysis described below consists of imposing imaginary surcharges behind the shoring system such that the resulting pressure diagram is a rectangle extending to the computed depth of the shoring system and of a uniform width of 100 psf. Generally, alternative surcharge loadings are limited to traffic and light equipment surcharge loads. Other loadings due to structures, or stockpiles of soil, materials or heavy equipment will need to be considered separately.

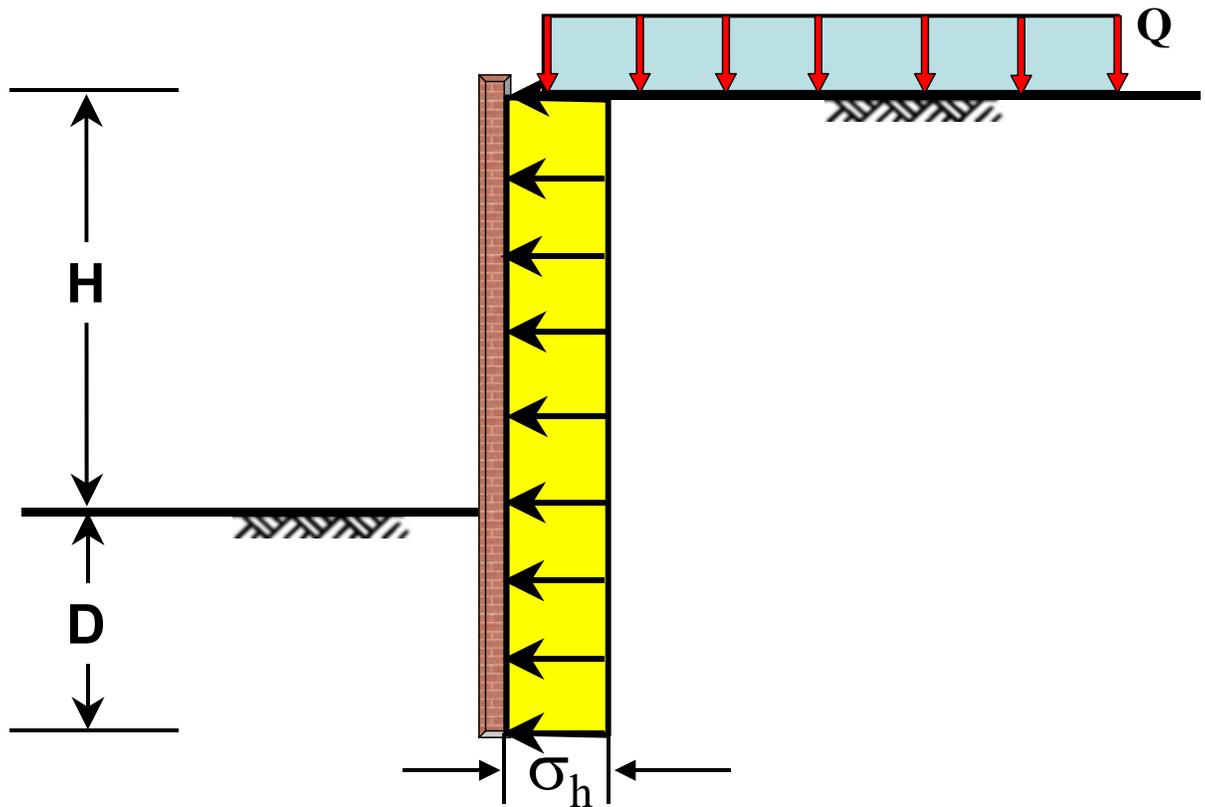


Figure 4-52. Alternate Traffic Surcharge Loading